

DOCUMENT RESUME

ED 428 937

SE 061 994

AUTHOR Bradsby, Larry; Bradsby, Shirley
 TITLE Resources for Computation. Book 1, Grades K-6. The Math Rescue Series.
 ISBN ISBN-1-57035-191-0
 PUB DATE 1998-00-00
 NOTE 585p.
 AVAILABLE FROM Sopris West, 4093 Specialty Place, Longmont, CO 80504; Tel: 800-547-6747 (Toll-Free); Web site: <http://www.sopriswest.com> (\$75).
 PUB TYPE Guides - Classroom - Teacher (052)
 EDRS PRICE EDRS Price MF03 Plus Postage. PC Not Available from EDRS.
 DESCRIPTORS Addition; *Arithmetic; *Computation; Division; Educational Assessment; Elementary Education; *Mathematics Activities; Mathematics Instruction; *Multiple Intelligences; Multiplication; Subtraction; Teaching Methods

ABSTRACT

This book provides regular classroom and special education teachers with alternative, individualized methods of teaching math computation skills. Learning activities are designed to appeal to auditory, visual, and kinesthetic learning modalities. Six mathematics topics addressed include modeling addition and subtraction, basic facts for addition and subtraction, addition and subtraction with regrouping, modeling of multiplication and division, basic facts of multiplication and division, and algorithms for multiplication and division. The activities in each topic are identified by appropriate grade level and include assessments. (ASK)

 * Reproductions supplied by EDRS are the best that can be made *
 * from the original document. *

The Math Rescue Series

Resources

FOR

Computation

by
Larry Bradsby
Shirley Bradsby

PERMISSION TO REPRODUCE AND
 DISSEMINATE THIS MATERIAL IN MICROFORM
 AND IN ELECTRONIC MEDIA FOR ERIC
 COLLECTION SUBSCRIBERS ONLY
 HAS BEEN GRANTED BY

J. Laubauer

2A TO THE EDUCATIONAL RESOURCES
 INFORMATION CENTER (ERIC)

U.S. DEPARTMENT OF EDUCATION
 Office of Educational Research and Improvement
 EDUCATIONAL RESOURCES INFORMATION
 CENTER (ERIC)

☒ This document has been reproduced as
 received from the person or organization
 originating it.

☐ Minor changes have been made to
 improve reproduction quality.

• Points of view or opinions stated in this
 document do not necessarily represent
 official OERI position or policy.

BEST COPY AVAILABLE

BOOK 1  **GRADES K-6**

Copyright © 1998 by Larry Bradsby and Shirley Bradsby
All rights reserved.

Permission is granted to the purchasing teacher to reproduce the forms and worksheets for use in his or her classroom only. No other part of this work may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopying or recording, or by any information storage and retrieval system, without the express written permission of the publisher.

ISBN #1-57035-191-0

Edited by Beverly Rokes
Text layout and design by Kimberly Harris
Cover design by Sherri Rowe
Production assistance by Jennifer Williams

Cuisenaire ® is a registered trademark of Cuisenaire Company of America, Inc.
Hot Wheels ® is a registered trademark of Mattel, Inc.
Invicta ® is a registered trademark of Invicta Plastics Limited
Learning Links ® is a registered trademark of Learning Links, Inc.
Post-it ® is a registered trademark of Minnesota Mining and Manufacturing Company
Popsicle ® is a registered trademark of Popsicle Industries, Inc.
Pepperidge Farm ® and Goldfish ® are registered trademarks of Pepperidge Farm Incorporated
Styrofoam ® is a registered trademark of The Dow Chemical Company
Teddy Grahams ® is a registered trademark of Nabisco, Inc.
UNIFIX ® is a registered trademark of Philograph Publications Limited
Vis-A-Vis ® is a registered trademark of Sanford Corporation
VELCRO ® is a registered trademark of Velcro Industries B.V.
Ziploc ® is a registered trademark of DowBrands Inc.

Printed in the United States of America
Published and Distributed by:



Sopris West

4093 Specialty Place • Longmont, CO 80504 • (303) 651-2829
www.sopriswest.com



About the Authors

Larry Bradsby, M.S.

Larry is currently a Mathematics Consultant and director of the Project Standards Teaching and Manipulative Mathematics. He was a teacher and a math supervisor for the Jefferson County Public Schools in Colorado. Larry has achieved national recognition in the field of math education and is a past president of the national Council of Supervisors of Mathematics. He has provided more than 500 workshops around the US and in Europe. He has also served as a National Assessment consultant and is the author of many math curriculum materials. Most of all, he loves to teach!

Shirley Bradsby, M.A.

Shirley has taught special needs children for more than 20 years from kindergarten through high school. She has been honored as the National Teacher of the Year in Learning Disabilities. She is past president of the Colorado Council for Learning Disabilities. She is currently a University Supervisor for Special Education teachers and a consultant for special needs education for several school districts, and holds workshops around special education topics both in the US and Europe.



Dedication

To Carrie, Darilyn, and Tim—always an inspiration and source of pride.



Introduction

Overview

The Math Rescue series was written to answer the question, "How do I meet the needs of ALL my students and make them successful?" The lessons are written for teachers who have a variety of student needs and abilities in their classroom. The activities are presented with K-5 grade level designations which can be used by teachers in elementary and middle school. Math Rescue can be easily used with any math program as it provides the flexibility of choosing lower level activities that parallel what is being taught. All skills are broken down to step-by-step activities to help students be successful regardless of the setting (inclusion classroom, pull out, regular classroom).

Organization

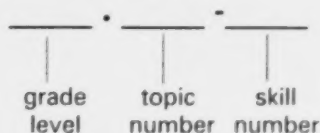
The Math Rescue series is written around the National Mathematics Standards. *Resources for Computation*, the first book in the series, deals with *computation* of whole numbers. The teacher can select activities from K-5 computation depending on the needs of each child. If there is a need to diagnose a student to determine their level of work, an assessment section is provided. Therefore, a teacher may work on computation and differentiate the activities to meet individual students' needs. The activities are designed in such a way that the student can be successful regardless of his modality strengths: auditory, visual, or kinesthetic.

This book is divided into six **Math Topics** as listed in the Table of Contents. Each **Math Topic** includes appropriate grade level activities. For example, the topic *Modeling Addition and Subtraction* has activities for students at the kindergarten, first, and second grade levels. Students at other grade levels, however, may need these activities. The grade level designations are used so teachers can transfer that information to IEPs or other documents needed to show a child's growth in mathematics. Each topic is broken down into grade level skills to better assess and monitor student growth.

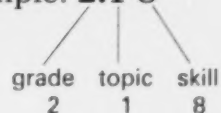


How to use this manual

Each section is coded by



Example: **2.1-8**



Section 2.1-8 is grade 2, topic 1, skill number 8. This coding is at the top of each page to be used as a quick reference. If a teacher wants to use a third grade activity, he/she should look in a section with a code starting with a 3.

Activities

There are three different kinds of activities that form a learning hierarchy:



Concept Manipulative Activities: Develop the math concept using a manipulative material.



Let's Practice: Ways to practice the skills other than through worksheets.



Problem Solving: Requires the students to use thinking skills to apply the skill to new situations.

Pick and Choose: There are several activities of each kind for every skill. There are too many for any student to complete them all. Teachers must pick and choose those they like best. This book is like a cook book; many activities are presented, teachers pick and choose which ones to use.

 The light bulb following the title of any activity means that the activity has some problem solving built into it. Make the student figure it out! Don't tell them how it is done.

Follow these steps to use *Resources for Computation* with your students:

1. Identify the math topic you wish to teach from the Table of Contents.
2. Diagnose students using the assessment section to determine level of performance, if needed.
3. Select the appropriate grade level activities from the collection presented.
 - A. "Concept Manipulative Activities" should be presented first.
 - B. "Let's Practice" activities should be used after the student understands the model.
 - C. "Problem Solving" should be included for every skill whether you use an activity with a  or a Problem Solving category activity.
4. Evaluate the student's progress using the assessment on each skill taught.
5. Record data on appropriate forms to show growth. A record card for this program is provided in the Appendix.

Table of Contents

Topic 1:	Modeling Addition and Subtraction	1
	Assessment of Topic 1 Skills	79
Topic 2:	Basic Facts for Addition and Subtraction	85
	Assessment of Topic 2 Skills	159
Topic 3:	Addition and Subtraction With Regrouping	163
	Assessment of Topic 3 Skills	249
Topic 4:	Modeling of Multiplication and Division	253
	Assessment of Topic 4 Skills	319
Topic 5:	Basic Facts of Multiplication and Division	323
	Assessment of Topic 5 Skills	395
Topic 6:	Algorithms for Multiplication and Division	397
	Assessment of Topic 6 Skills	571
Appendix		575

SKILL Joining sets to model addition***Skill Analysis***

- A. Can construct sets
- B. Can count members of sets with 0–10 objects
- C. Can conserve number
- D. Understands the concepts of put together, join, and combine
- E. Can write numerals for 0–10 objects

**Concept Manipulative Activities*****Joining Sets***

Materials: Counters (beans, UNIFIX cubes, etc.)
Math mat (one per student; any smooth place mat will do)

- Directions:**
1. Give each student 5–10 objects.
 2. Say, "Using the objects I've given you, make two sets on your math mat."
 3. Say, "Tell me about your sets." (Accept any descriptions. Examples: "These are blue and these are red." "These are big and these are small." "They are all beans." "There are three on this side and four on this side.") Have each student tell how many objects are in each of his/her sets.
 4. Ask the students, "Can you combine your sets?"
 5. Say, "Describe your new set." (Accept any description that is reasonable.)
 6. Ask, "How many objects do you have altogether?"
 7. Write the number sentence.

Yours Plus Mine Makes Ours

Materials: Counters (UNIFIX cubes, Pattern Blocks, etc.)
Math mat (one per student; any smooth place mat will do)

- Directions:**
1. Give each student a set of 0–5 objects.
 2. In turn, have each child describe his/her set. (Examples: “They are all blue.” “They are all big.” “There are three objects.”)
 3. Next, have pairs of students work together. Ask, “Can your sets be joined together? What happens when you join your sets? What can you tell me about your new set?”
 4. Have the students join their sets and describe how many objects they have altogether.
 5. Have different pairs of students combine their sets.

Cube Count

Materials: UNIFIX cubes (all of one color)
Math mat (one per student; any smooth place mat will do)

- Directions:**
1. Give each student a set of 1–5 cubes.
 2. Ask, “How many members are in your set?”
 3. Give each student another set of 1–5 cubes. Ask, “How many cubes are in this set?”
 4. Ask the students to combine their two sets.
 5. Ask, “How many objects are in your new set?”
 6. Repeat the activity many times using different colored cubes each time.
 7. Write the number sentence.

Stringing Beads

Materials: Beads of different colors
Wire or string for stringing beads

- Directions:**
1. Have the children work various problems using the beads and the wire or string. For example, say, "String three red beads. Now string two yellow beads. How many beads did you string altogether?"
 2. Repeat the activity having students make strings with different numbers of different colored beads.

Stories

Materials: Objects for addition (Popsicle sticks, tongue depressors, buttons, rods, etc.)
Math mat (one per student; any smooth place mat will do)

- Directions:**
1. Give each student 10 objects.
 2. Tell a variety of story problems that involve addition up to 10 and allow the students to work out the answers using the objects.
For example, say, "Pretend that you had three popsicles, and when you went to the store you bought three more. How many do you have now?"
 3. See if some of the students can make up and share their own story problems.



Let's Practice

Number Machine

Materials: Square box
Counters (buttons, beans, etc.)

- Directions:**
1. Remove the top of the box. Turn the box over so the open end of the box is on the bottom.
 2. Cut a hole in the box, like the coin slot in a soda machine.

3. Give each child 0–10 counters.
4. Call on a volunteer to drop one set of objects in the slot, say how many were in the set, and then drop in another set of objects, again saying the number.
5. Ask the child how many objects he/she dropped in altogether.
6. To check the answer, have the child pick up the box and count the total number of objects.
7. Repeat the activity with other volunteers.

Pattern Blocks

Materials: Bags containing two different colors of up to 10 Pattern Blocks (one bag per student)

- Directions:**
1. Give each student a bag of Pattern Blocks.
 2. Have the students separate the blocks into a set of each color.
 3. Ask the students to describe the sets (e.g., "There are two green blocks and four orange blocks. There are six altogether.")
 4. You may want to have the students make a design with their blocks.
 5. Have the students exchange bags and repeat the activity.

Hang Them Up to Dry

Materials: Cutouts of shirts, pants, socks, etc., such as those for paper dolls (several for each student)
Pinch-type clothespins
String

- Directions:**
1. Give each student several clothing cutouts, several clothespins, and a string.
 2. Have the children hang the clothes on the line to dry and then describe their clothesline. (Examples: "My clothesline has two shirts and four pairs of jeans, which makes six things on the line altogether." "My clothesline has three blouses and nothing else, which is three altogether.")

Swimming Ducks

Materials: Blue and green construction paper
Counters (beans, paper clips, etc.)

- Directions:**
1. Cut out a "pond" shape from blue construction paper. Glue it onto a piece of green construction paper to represent a pond and grass. Prepare enough of these for each child in a small working group. Explain that the paper represents a pond and grass and the counters represent ducks.
 2. Have the children toss 1–10 counters onto the paper and then tell you the number of "ducks" that landed in the pond, the number that landed in the grass, and the total. (Example: "Two ducks landed in the pond and three landed in the grass, which makes five ducks altogether.")
 3. Repeat the activity using a different number of counters.
 4. Have the student write the number sentence.

***TEACHER NOTE:** Students will get zero for one of their addends if none of their "ducks" land on the water or on the grass.

Animal Count

Materials: Workspace that resembles a corral
Play animal Counters (bears, Teddy Grahams®, etc; 10 per student)

- Directions:**
1. Put four animals in one set and three in another. Ask, "How many animals are there altogether?" Have the students use counters to help them solve the problem.
 2. Continue using different numbers for the sets.
 3. Variation: Have each child build his/her own story problems and share them with the group.



Problem Solving

Company's Coming

Materials: Chairs
Rectangular table

- Directions:
1. Give the children a problem like the following: Five people are having dinner together. How can you place five chairs at the table so that the chairs are on opposite sides?
 2. Ask children to demonstrate (one at a time) different ways to solve the problem.
 3. You might want to begin by having the children give predictions. Ask, "How might you do it?" (Possible response: "Three chairs on the right, two chairs on the left.")
 4. Continue the process with different numbers of chairs.

Candles on the Cake

Materials: Styrofoam cups inverted to represent cupcakes (one per student)
Birthday candles or objects to represent them (e.g., pegs, cocktail toothpicks, etc.) in two colors

- Directions:
1. Say, "We are going to put candles on a birthday cake for a boy who is five years old. The candles come in two colors. Choose five of the candles I have given you to decorate the cake."
 2. Ask several children what combination they used. (Possible response: "I used one red candle and four green candles. I used five altogether.")
 3. Change the number of candles and repeat the process.
 4. Include practice with zero as an addend. (e.g., "I used six red candles and no other color. I used six altogether.")

SKILL Removing a set to model subtraction**Skill Analysis**

- A. Can construct sets
- B. Can count members of sets with 0–10 objects
- C. Can conserve number
- D. Understands the concepts of take away and remove

**Concept Manipulative Activities****Removing Sets**

Materials: Objects such as beans, buttons, or paper pieces

- Directions:**
1. Give each child a certain number of similar objects to form a set. Different children may have different amounts, up to a total of 10.
 2. Ask the children to take away a certain number of objects from their set.
 3. Repeat the exercise, having the children remove a different number of objects each time.
 4. Write the number sentence.

How Many Are Left?

Materials: Flannel board or magnetic board
Flannel or magnetic objects

- Directions:**
1. Put a set of 1–10 objects on the flannel or magnetic board.
 2. Ask child to remove some of the objects from the set.
 3. Ask the class how many objects are left.

Take Down

Materials: Flannel board or magnetic board
Flannel or magnetic objects

- Directions:**
1. Ask a child to make a set of 1–10 objects on the flannel or magnetic board and name the number of objects in the set.
 2. Have another child remove a certain number of the objects (specify the number) and tell the number that remain.
 3. Replace the objects that were removed from the set and repeat the activity with different sets.

Story Problems

Materials: Objects such as beans, buttons, paper pieces, or rods

- Directions:**
1. Tell the children story problems like the following. Have the children use objects to find the answers.

Examples:

- a. There were four kittens in a basket. One kitten got tired of sitting and went to look for some fun. How many kittens are in the basket now?
- b. There were six fish swimming in a pond. One fish got hooked. How many fish are left in the pond?
- c. I had four cookies. Then the “Cookie Monster” arrived at my house and ate three of them. How many do I have left?
- d. Mom went bowling. She rolled her ball and knocked over 8 of the 10 pins. How many pins were left standing?
- e. There were five birds in a nest and two birds flew away. How many are left in the nest?
- f. There were seven apples in a tree. Three apples fell off. How many are left?

***TEACHER NOTE:** Incorporating children’s names from your classroom into the story problems may arouse greater interest.

2. Ask the children to make up their own story problems for the rest of the class to solve.



Let's Practice

Animal Watch

Materials: Workspace that resembles a forest
Play animals (bears, birds, horses, etc.)

- Directions:**
1. Put seven animals in the forest.
 2. Say, "Two of the animals left. How many animals are now in the forest?"
 3. Continue using different numbers.
 4. Variation: Have each child invent his/her own story problem and share it with the group.

Falling Off

Materials: Cutouts of shirts, pants, socks, etc., such as those for paper dolls (several for each student)
Pinch-type clothespins
String

- Directions:**
1. Give each student several clothing cutouts, several clothespins, and a string.
 2. Have the children hang the clothes on the line to dry.
 3. Tell them that two pieces of clothing came unhooked and ask them to model and describe the problem for their clothesline. (Possible response: "I had five pairs of jeans on the line. Two pairs came unhooked. There are three pairs left.")
 4. Repeat the exercise using different numbers.

Swimming Fish

Materials: Blue and green construction paper
Pepperidge Farm Goldfish crackers

- Directions:**
1. Cut out a "pond" shape from blue construction paper. Glue it onto a piece of green construction paper to represent a pond and grass. Prepare enough of these for each child in a small working group.
 2. Give each child 1–10 Goldfish crackers to place in the pond.
 3. Have the children count the number of "fish" that are swimming in the pond.
 4. Tell the children that they are going to go fishing. Say, "You caught ____ fish." Ask, "How many fish are left in the pond?" The students may eat the fish they caught.
 5. Repeat the activity using a different number.



Problem Solving

Missing Bees

Materials: Counters (5–10 per student)
Workspace (9" x 12" construction paper, place mat, carpet square, etc.)

- Directions:**
1. Have the students work in pairs.
 2. Give each pair of students the same number of counters. Say that the counters will represent bees.
 3. Ask the students to count the number of "bees" they have been given.
 4. Say that one student in each pair will be the "beekeeper." He/she will remove some bees while the other student closes his/her eyes.
 5. When the beekeeper is done, the other student will open his/her eyes and try to identify how many bees the beekeeper removed.

6. The beekeeper will then show the bees that were removed and return them to the "hive."
7. After completing the activity, the students should exchange roles.
8. The activity should be repeated several times before the beginning number of bees is changed.

OK Corral

Materials: Workspace that resembles a corral
 Play animals from a farm set
 Counters (5–10 per student)

- Directions:**
1. Put a set of animals in the corral (e.g., three horses).
 2. Say, "Two of the animals ran away yesterday. How many were in the corral to begin with?" Allow the students to use objects to help them solve the problem.
 3. Repeat the activity using different numbers.

SKILL Joining and separating sets

***TEACHER NOTE:** This section combines the two previous objectives of joining and separating sets. The exercises may provide the first experience some of the children have had where they have to discriminate between operations and choose the correct procedure. Many opportunities will be provided for the children to develop language skills that will help them decide which operation to use.

Skill Analysis

- A. Can construct sets
- B. Can count members of sets with 0–10 objects
- C. Can conserve number
- D. Understands the concept of addition
- E. Understands the concept of subtraction
- F. Recognizes the difference between the concepts of addition and subtraction



Concept Manipulative Activities

Bead Sets

Materials: Ten beads on a string or wire (one set per child)
One large floor abacus

- Directions:**
1. Call out a number from 1 to 10.
 2. Have the children slide that number of beads to one side on their string or wire.
 3. Say, "Change this set to a set of six. How did you do that?"
 4. Say, "Change this set to a set of two. How did you do that?"
 5. Continue with other numbers.

Changing Sets

Materials: Flannel board or magnetic board
Flannel or magnetic objects and numerals

- Directions:**
1. Put a set of 1–10 objects on the flannel or magnetic board.
 2. Ask a child to put the corresponding numeral under the objects.
 3. Ask a second child to replace that numeral with another one.
 4. Have a third child add or remove objects to make the number of objects the same as that indicated by the new numeral. Ask the student what he/she did to form the new set.
 5. Repeat the activity several times.



Let's Practice

Making Sets

Materials: Objects for addition or subtraction (tongue depressors, buttons, beans, etc.)
Two sets of cards numbered 0–10

- Directions:**
1. Give each student 10 objects.
 2. Have one student draw a numeral card and announce the number to the rest of the class.
 3. Each child should make a set with that many members (e.g., 7).
 4. Have a second student draw a card and announce the number (e.g., 5).
 5. Each child should make the new set from the previous set.
 6. Ask the students how the new set was formed. (Example: 7 to 5 by removing 2.)
 7. Variation: Divide the class into groups of two or three students. Have the students conduct the activity in their groups with one student serving as the leader.

21

Story Problems

Materials: Material objects or cutouts to match items in the story

Directions: 1. Tell addition and subtraction story problems and have the children model the problems with relevant objects. The objects could be physical materials or cutouts.

Examples:

- a. In a large pasture, there were nine cows. (Have students model this.) The next day there were six cows. How can you change this set (point to the nine cows) to show six cows? (Ask a student to demonstrate.)
- b. I looked in a tree and saw two birds. (Have a student model the set of birds.) Later, I looked in the tree and saw six birds. How can you change the set of two birds to show six birds?
- c. There were five cookies in the cookie jar before Mary came home. There is now one cookie in the cookie jar. Show what happened.
- d. Peter was playing in the sandbox. He had 8 cars in the sand. Jimmy came to play. Peter counted the cars and found there were 10. Show what happened.

**Problem Solving****Pupil Matching**

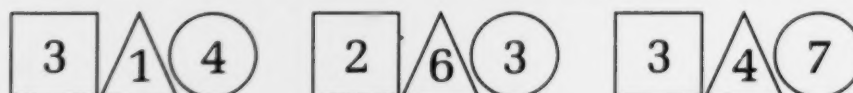
Materials: Dot cards (one per student; blackline masters may be found in the Appendix)

- Directions:**
1. Give each pupil a dot card that has 0–5 dots.
 2. Call out a number between 5 and 10.
 3. Have each pupil match his/her card with someone else's so that the total number of dots matches the numeral called. It is possible that not all pupils will be able to match with someone else. Also, more than two cards could be used to form a match.
 4. Call out a different number and repeat the activity.

Shape Count

Materials: Attribute Blocks

- Directions:
1. Give each child a set of Attribute Blocks.
 2. Have the children make separate sets of the squares, triangles, and circles (of any size).
 3. Then write a pattern like the example below on the board.



Have the children construct sets based on the pattern sentence. For the example above, the first set would contain three squares, one triangle, and four circles. Each time the pattern repeats, the students need to change the number of objects in their set to match the number in the pattern. For the example above, the second set would contain two squares, six triangles, and three circles.

4. Have the children continue to work with their sets until all the patterns have been completed. Then have the children make their own final pattern.
5. Repeat the activity with different pattern sentences and shapes.

SKILL Constructing and joining disjoint sets to find the sum

Skill Analysis

- Can construct a set of up to 20 members when given the number of members
- Understands the meanings of "set" and "joining sets"
- Can write the numbers 0-20
- Understands the "+" sign and "=" sign



Concept Manipulative Activities

Number Sentences

Materials: Large plus sign and large equals sign drawn on separate pieces of tagboard
1" ceramic tiles or cards labeled 0-9, +, and = (1 set per child)

- Directions:**
- Call two groups of students to the front of the class. (Say, e.g., "I want all students wearing red tops to come forward. I also want all students with curly hair to come forward.") Be sure to call groups that have nothing in common.
 - Ask for two volunteers to come forward. Give one the "+" sign and the other the "=" sign.
 - Tell the "sign" people to stand in their correct positions.
Example:



- Discuss what belongs on the other side of the equals sign.

5. Have students use their tiles or cards to make the number sentence.

Example:

$$\boxed{3} \boxed{+} \boxed{2} \boxed{=} \boxed{5}$$

Dominoes

Materials: Dominoes or domino cutouts (blackline master "Dominoes" may be found in the Appendix)

Domino record sheets (one horizontal and one vertical sheet per student; blackline masters follow this activity)

- Directions:**
1. Have each child take a domino or a domino cutout and draw the dots on one of the record sheets.
 2. Then have the children write the addition sentence that corresponds to the sets of dots. The record sheets provide both horizontal and vertical practice.



Domino Record (Horizontal)

Example:

•	•	•
•	•	•
•	•	•

--	--	--

--	--	--

--	--	--

--	--	--

--	--	--

--	--	--

$$\underline{6} + \underline{3} = \underline{9}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

Use with "Dominoes" Activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Domino Record (Vertical)

Example:

$$\begin{array}{|c|} \hline \bullet \bullet \bullet \\ \hline \bullet \bullet \bullet \\ \hline \bullet \bullet \bullet \\ \hline \end{array} + \begin{array}{|c|} \hline 6 \\ \hline 3 \\ \hline 9 \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} + \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} + \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} + \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} + \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array}$$

$$\begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array} + \begin{array}{|c|} \hline \\ \hline \\ \hline \\ \hline \end{array}$$

Use with "Dominoes" Activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Stringed Pasta or Beads

Materials: Colored pasta (recipes may be found in the Appendix)
String
Box

- Directions:
1. Make strings of 0–9 pieces of colored pasta (one color per string). Use a different amount of pasta for each string.
 2. Tie each string together and put the pasta strings in a box.
 3. Have each child pull out two strings.
 4. The children should count each set and determine how many pieces of pasta there are altogether.
 5. Have the children write the number sentence (e.g., $5 + 6 = 11$).

Joining Cubes

Materials: UNIFIX cubes or Connecting links

- Directions:
1. Give each child 10 UNIFIX cubes of one color and 10 more UNIFIX cubes of another color.
 2. Tell the students to make a set of a certain number of one color and another set of a certain number of the other color. (Say, for example, "Make a set of four yellow cubes. Make a set of three green cubes.")
 3. Say, "Join the two sets together. How many cubes do you have altogether?" (For the example above, the response would be, "Four yellow and three green gives you seven cubes altogether.")
 4. Variation: Have the students use 1" ceramic tiles or cards labeled 0–20, +, and = to make the number sentence (e.g., $\boxed{4} + \boxed{3} = \boxed{7}$).

Adding With the Empty Set

Materials: Attribute Blocks
Three loops of string or yarn for each pair of students

- Directions:**
1. Have each child pick a partner. Give each pair a set of Attribute Blocks and three loops.
 2. Ask each pair of children to form a set of six blocks enclosed in a loop.
 3. Now have the pairs enclose an empty set in a loop.
 4. Ask, "Who can join these two sets to form a new set? How many members are there when you join the empty set and the other set?"
 5. Repeat the activity several times, each time using one number and an empty set.
 6. Let the children take turns giving the directions.

*TEACHER NOTE: You may want to demonstrate this activity first.

Treasure Hunt

Materials: Objects in the classroom

- Directions:**
1. Tell the students that they are going to go on a treasure hunt.
 2. Name an object that can be found in the room and select a group of students to find it. (Example: "Find the volumes of the encyclopedia.") Choose an object that has 20 or fewer parts.
 3. Select another group to find an object that is not in the room (e.g., lions, watermelons, or tigers).
 4. Ask each group for the number that names the members of their set.
 5. Have a student write the number sentence that would result if the sets were joined.
 6. Repeat the procedure with other groups of students and other objects.



Let's Practice

Colored Beans

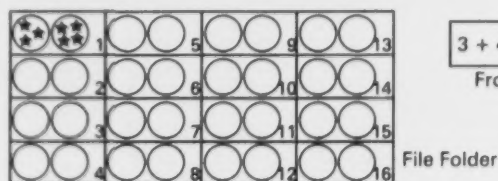
Materials: Plastic cups (one per child)
Lima beans painted on one side or two different kinds of beans

- Directions:**
1. Fill each cup with 20 lima beans or with 10 each of the two different kinds of beans.
 2. Give each child a cup of beans.
 3. Have the children take some beans from their cups and sort them into two groups (painted and not painted or different types of beans).
 4. Ask the children to discuss their findings. (Example: "I have a set of 8 beans that I am joining to a set of 5 beans. I have 13 altogether.")
 5. Have the children write the number sentence.

Self-Checking Cards

Materials: Colored file folder (divided into 16 sections)
Tagboard cards (cut to the size of the sections in the folder) with number sentences written on them and then laminated
Stickers (dots, stars, etc.) or stamps

- Directions:**
1. Draw two circles in each file folder section.
 2. Use dots (or stars, or stamps) to represent sets in each circle. The sets should match the numbers in the sentences on the tagboard cards. Example:



After filling in all the circles, laminate the file folder.

3. Have the children match the number sentence on each card to the relevant folder section. They can check their answers by comparing the folder section number with the number on the back of the tagboard card.

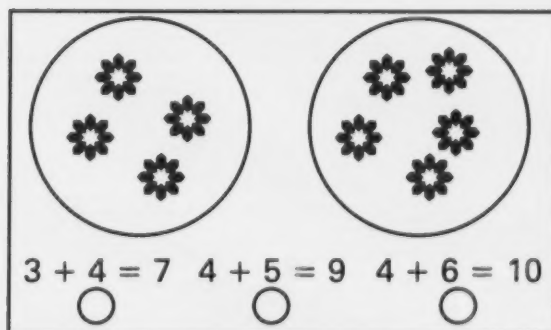
***TEACHER NOTE:** Here is another fun way to have the cards be self-checking. (1) Divide a sheet of tagboard into sections that are the same size as the file folder sections. (2) On the back of the sheet of tagboard, glue an attractive picture from a magazine or poster. (3) Write the number sentences in the sections on the front side of the tagboard. (4) Cut the tagboard into the pre-lined cards and laminate them. (5) Have the children match the cards to the sets. (6) When all of the cards have been matched, the students can turn them over. If all of the matches are correct, the students will see an intact picture.

Punch Through

Materials: 5" x 7" index cards
 Pictures or stickers
 Pipe cleaners

- Directions:** 1. Make 20 task cards. Each card should have two disjoint sets created with pictures or stickers and three number sentences, one of which matches the disjoint sets. A hole should be punched out underneath each number sentence. On the back of the card, put an "X" above the hole for the correct number sentence. Laminate the cards.

Example:

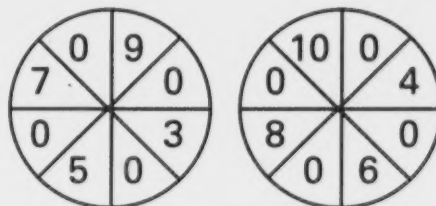


2. Have the students count the sets on each card and push a pipe cleaner through the hole under the correct number sentence.
3. The students can turn the cards over to self-check the answers.

Spin a Zero

Materials: Two spinners per pair of students

Examples:



- Directions:
1. Have the students work in pairs. Give each student a spinner.
 2. Each student will spin his/her spinner and the partners will write an addition sentence using both of their numbers.
 3. Discuss the problems involving zero as an addend.
 4. Have the students repeat the activity several times.

*TEACHER NOTE: Have manipulatives available for problems that involve two nonzero addends.

Catch and Keep

Materials: Construction paper of different colors

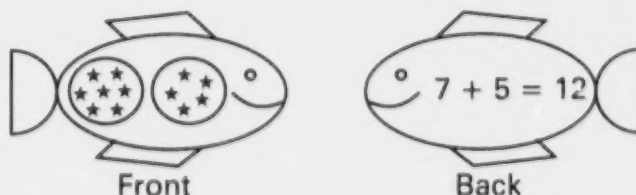
Paper clips

Magnet on a string

Box or fishbowl

- Directions:
1. Cut out several dozen fish from the construction paper.
 2. Give each student several fish. Have the students construct two sets on the front of each fish and write the matching addition number sentence on the back.

Example:



3. Attach a paper clip to the front side of each fish and put all the fish in a box or fishbowl.

4. Tell the students that they are going to play a fishing game called "Catch and Keep." Two or three students may play at the same time.
5. In turn, each student will fish with the magnet and string and pull out a fish. After looking at the sets on the front side of the fish, the student will write the corresponding number sentence on a piece of paper.
6. The student will then turn the fish over and check his/her number sentence. If it is correct, the student will keep the fish.



Problem Solving

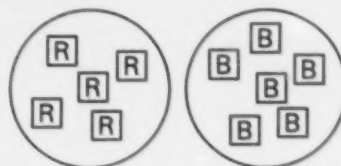
UNIFIX Cubes

Materials: UNIFIX cubes (five different colors, no more than 10 cubes per color)
 UNIFIX Record Sheets (one per student; blackline master follows this activity)
 Crayons

- Directions:**
1. Give each student a pile of UNIFIX cubes and a record sheet.
 2. Say, "Divide your cubes into sets of each color. Then make as many combinations of sets as you can. Use crayons to draw the set combinations on your record sheet and write the number of sentences in the spaces provided."

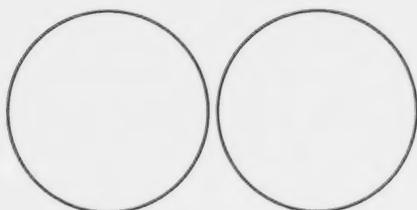
Example: The children could join the red set and the blue set, the green set and the yellow set, the red set and the yellow set, and so forth. For the first set combination, the record sheet would look like this:

$$\underline{5} + \underline{6} = \underline{11}$$

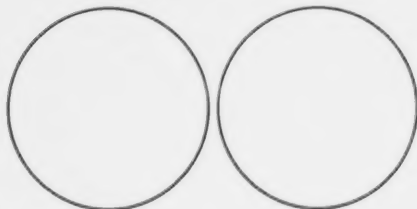




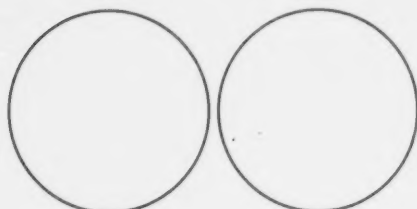
UNIFIX Record Sheet



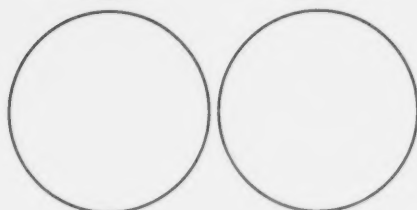
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



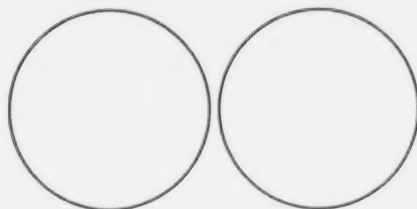
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



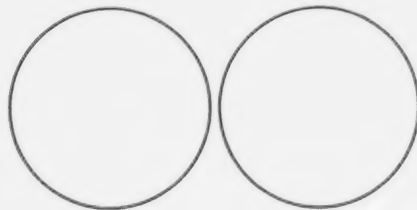
$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

Use with "UNIFIX Cubes" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

Directions: Draw a picture and write a number sentence for each problem.

1. Two boys have the same number of baseballs. One boy has four baseballs. How many baseballs do the boys have altogether? _____

2. There are five girls and six boys in the milk line. Each girl and boy has one carton of milk. How many cartons of milk are there? _____

3. A mother dog has four puppies. A mother owl has three baby owls. A mother cat has six kittens. How many babies will never fly? _____

4. Joe's family has two stereos. Joe has 10 tapes and five records. How many records and tapes does Joe have? _____



*TEACHER NOTE: You may need to read these problems orally to the students. Encourage students to use manipulatives to symbolize objects in the stories.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Fun With a Calculator

Materials: Calculators (one per student)

- Directions:
1. Show the students how to use a calculator for adding. Say, "We are going to add $5 + 0$."
 - a. Have the students record the problem on their paper. (Students need to record the problem first so that they later "remember" what they entered into the calculator.)
 - b. Say, "Press the 5." Pause. "What do you see on the display?" (5)
 - c. Say, "Press the plus sign." Pause. "What do you see?" (5)
 - d. Say, "Press the 0." Pause. "What do you see?" (0)
 - e. Say, "What do you think you will see when you press the equals sign?" Wait for responses. Have students press the equals sign and ask them what is displayed.
 - f. Repeat with other problems involving zero (e.g., $7 + 0$, $0 + 17$).
 2. Have the students discuss what they observed when they added zero to other numbers. Ask, "Did it make a difference if the zero was added first or second?"
 3. Have them make up and record problems involving zero on their own. Ask questions like:
 - a. Do you think what you observed will be true for all numbers?
 - b. Can you find a number that does not work this way? (If a student says there is a counterexample, have the student work the problem with you.)
 - c. What is the smallest number that will work?
 - d. What is the largest number that will work?

SKILL Demonstrating the definition of subtraction

Skill Analysis

- A. Understands the meaning of set
- B. Can count members of sets with 0-20 objects
- C. Understands the concept of removal
- D. Understands the concept of more and less
- E. Can construct a one-to-one matching
- F. Understands the "-" sign and "=" sign



Concept Manipulative Activities

Goldfish

Materials: Pepperidge Farm Goldfish crackers (or certain kinds of cereal)
Blue construction paper

- Directions:**
1. Working with a small group of children, give each child one piece of blue construction paper and 10 Goldfish crackers.
 2. Explain that the paper and crackers represent a lake and fish.
 3. Have the students count their fish to make sure they each have 10. Then have them put their fish on the paper.
 4. Say, "Ten little fish are swimming in a pond. Along comes a shark, who eats two."
 5. Let the children each eat two crackers.
 6. Have the children count how many fish are left. Say, "Ten take away two leaves eight."

7. Continue this process, varying the number of fish taken away, until all the fish are eaten.
8. The activity can be repeated starting with any number through 20.

Student Comparison

Materials: Overhead projector
Blank transparency
Vis-A-Vis pen

- Directions:
1. Call two sets of students to the front of the class (e.g., students wearing red shirts or blouses and students wearing blue shirts or blouses).
 2. Have the rest of the class count the members of each set (e.g., eight in the blue set and five in the red set). Write the numbers on the overhead projector.
 3. Ask, "How can we find out how many more blue shirts or blouses there are than red?" Match a student from the blue set with a student from the red set and have them sit down. Continue until all possible matches have been made. For the example above, after five pairs sit down, there will be three students from the blue set left standing.
 4. Ask, "How can we describe what we have just done using numbers?" After receiving some responses, write $8 - 5 = 3$ on the overhead projector. Read the sentence to the class in the following manner: "The difference between eight and five is three," or, "The difference between eight blue shirts and five red shirts is three blue shirts."
 5. Repeat the activity using different sets, such as students with velcro-fastening shoes and students with tie shoes; those with jeans and dresses; those with belts and no belts; students with yellow shirts and students with white shirts; and so forth.

Cube Trains

Materials: UNIFIX cubes in the following quantities to make trains: one red, two blue, three orange, four green, five yellow, six brown, seven black, eight light blue, nine white

- Directions:**
1. Begin by displaying a brown train and a blue train.
 2. Have the students compare the stacks. Ask:
 - a. How many cubes are in the brown stack? (6)
 - b. How many cubes are in the blue stack? (2)
 - c. What is the difference between the brown stack and the blue stack? (4)
 - d. How did you find out? (Some possible answers: "Counting up." "These matched and these are left." "This side has this many more." "I need four more to make them even.")
 3. Continue the activity using different stacks.
 4. Variation: Have students make graphs using $\frac{3}{8}$ " or centimeter graph paper. (Blackline masters are in the Appendix.) Tell them that each square equals one cube. Have them color in the number of squares for each stack on their graph paper and compare the two lengths.

Finger Play

Materials: Optional: Finger puppets to accompany the following poems

- Directions:**
1. Read the following poems to the class. The students can act out the poems with finger puppets or draw the poem characters on the chalkboard and show the subtraction.

Five Little Snowmen

Five little snowmen, knocking at my door.
 One melts away and then there are four.
 Four little snowmen, playing with me.
 One melts away and then there are three.
 Three little snowmen, playing with you.
 One melts away and then there are two.

Two little snowmen, playing in the sun.
One melts away and then there is one.
One little snowman, when the day is done.
He melts away and then there are none.

Ten Little Candles

Ten little candles on a chocolate cake;
"Wh! Wh!" Now there are eight.
Eight little candles on a candlestick;
"Wh! Wh!" Now there are six.
Six little candles and not one more;
"Wh! Wh!" Now there are four.
Four little candles, red, white, and blue;
"Wh! Wh!" Now there are two.
Two little candles standing in the sun;
"Wh! Wh!" Now there are none.

Twelve Little Rabbits

Twelve little rabbits in a rabbit pen;
Two hopped away, and then there were ten.
Ten little rabbits with ears up straight;
Two hopped away, and then there were eight.
Eight little rabbits doing funny tricks;
Two hopped away, and then there were six.
Six little rabbits eating carrots from a store;
Two hopped away, and then there were four.
Four little rabbits looked for gardens new;
Two hopped away, and then there were two.
Two little rabbits found a new friend;
They hopped away, and that is the end.

Ten Little Chickadees

Ten little chickadees sitting on a line,
One flew away and then there were nine.
Nine little chickadees sitting on a farmer's gate,
One flew away and then there were eight.

Eight little chickadees looking up to heaven,
One flew away and then there were seven.
Seven little chickadees gathering up sticks,
One flew away and then there were six.

Six little chickadees learning how to dive,
One flew away and then there were five.
Five little chickadees sitting at a door,
One flew away and then there were four.

Four little chickadees could not agree,
One flew away and then there were three.
Three little chickadees looking very blue,
One flew away and then there were two.

Two little chickadees sitting in the sun,
One flew away and then there was one.
One little chickadee living all alone,
He flew away and then there were none.

Four Plump Robins

Four plump robins hopping all around,
Picking up bread crumbs off the ground.
One saw a pussy cat up in a tree.
He flew away, and then there were three.

Three plump robins hopping all around,
Picking up bread crumbs off the ground.
One heard a pussy cat say, "Mew, mew, mew."
He flew away, and then there were two.

Two plump robins hopping all around,
Picking up bread crumbs off the ground.
One saw a pussy cat sleeping in the sun.
He flew away, and then there was one.

One plump robin hopping all around,
Picking up bread crumbs off the ground.
He saw a pussy cat starting to run.
He flew away, and then there were none.



Let's Practice

Bear Facts

Materials: Bear counters or other objects
Cards numbered 0–20

- Directions:**
1. Count out six bear counters and above them place the numeral card "6."
 2. Count out nine bear counters and above them place the numeral card "9."
 3. Ask, "Are the sets equivalent?" (No) "What is the difference between the sets?" (3)
 4. Display the numeral card that shows the difference.
 5. Have the students write the subtraction sentence to accompany the problem on a piece of paper. (You may want to model writing a few of these sentences before asking the students to do so independently. Also, go over how to read these sentences with the students (e.g., "The difference between nine and six is three.").
 6. Repeat this activity using other numbers until the children are comfortable with it.

Name Game

Materials: 1" graph paper (blackline master may be found in the Appendix)
Colored construction paper, glue

- Directions:**
1. Cut two rows of the 1" graph paper for each student.
 2. Have the children write their first name on the top line and their last name on the bottom line, one letter per box. Then have them mount their graph paper on a piece of construction paper.

Example:

S	A	M							
G	R	E	E	N	W	A	L	D	

3. Ask:
 - a. How many letters are in each of your names?
 - b. Which of your names is longer?
 - c. How many letters longer is it?
 - d. How many letters shorter is your other name?
 - e. How can you write a subtraction sentence to compare the size of your names? (Example: $9 - 3 = 6$)
4. Variation: Have students compare the total number of letters for both of their names with the total for other students in the class and then write number sentences for each comparison.

Edible Math

Materials: M&M's or other colored candies or cereal

- Directions:
1. Give each child 20 M&M's of assorted colors.
 2. Have the children make sets by colors.
 3. Have them compare the set of red M&M's to the set of dark brown M&M's. Ask, "Which set has more? How many more?" Have the children write the number sentence.
 4. Compare other colors, varying the questioning. (Example: Ask, "How many less?")
 5. Say, "You have 20 M&M's. If you get to eat the green ones, how many are you going to eat? How many will be left? Write the number sentence." Let the children eat the green M&M's.
 6. Repeat the procedure until all the M&M's are eaten.

*TEACHER NOTE: This activity uses both the take away and comparison models. Write subtraction sentences for each model on the board and discuss the interpretation.



Problem Solving

Directions: Use the chart below to answer the following problems. Write the number sentence for each problem.

S
T
O
R
Y

			
Class A	5	9	3
Class B	6	12	5

P
R
O
B
L
E
M
S

- Class B has five  and class A has three .
Class A has how many **fewer**  than class B? $5 - 3 = 2$
- Class B has six  and class A has five .
Class A has how many **fewer**  than class B? _____
- Class B has 12 . They gave 2  away.
How many  are **left**? _____
- Class A has nine . Sam took five  home.
How many  are **left**? _____
- Class A has three . They gave two  away.
How many  are **left**? _____



*TEACHER NOTE: You may need to read these problems orally to the students. Encourage students to use manipulatives to represent the animals.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Adding and subtracting on the number line

Skill Analysis

- A. Can recognize and identify the numbers 0–20
- B. Can count to 20
- C. Knows directions right, left, forward, and backward



Concept Manipulative Activities

Walking Trip

Materials: Walk-on number line

- Directions:
1. Working with a small group, give a student two numbers to hop on the number line. Have the student hop the first number. From that point, he/she should hop the second number. The student should then read the numeral on which he/she has landed.
 2. Addition example: "Hop 6. Hop 2." The student hops 6 and 2. Ask, "Where did you land when you made a hop of 6 and a hop of 2?"
 3. Subtraction example: "Hop 5. Hop back 2." After the student hops, ask, "Where did you land when you hopped 5 and hopped back 2?"
 4. Repeat the activity giving all students an opportunity to hop on the number line with a variety of problems. (Keep the group small enough so that students do not have a long wait before their turn.)

Recording Our Trips

Materials: Walk-on number line
Transparency of number lines (blackline master may be found in the Appendix)
Vis-A-Vis pens
Overhead projector

- Directions:
1. Show a number line transparency on the overhead.
 2. Ask for a volunteer to be the recorder. (You may wish to be the first recorder.)
 3. Repeat the activity "Walking Trip."
 4. As the first student hops on the walk-on number line, the recorder should illustrate the hops on the transparency and write the corresponding number sentence.
 5. Have another student verbalize what the recorder wrote.
 6. Repeat the activity with a new "hopper" and recorder.

Reading Number Lines (Addition and Subtraction)

Materials: Overhead projector
Vis-A-Vis pens
Transparency of number lines (blackline master may be found in the Appendix)

- Directions:
1. Say, "In this activity, we're going to use the number line on the transparency instead of the walk-on number line."
 2. Say, "If we hop 4 and then 1 more, we stop on 5." (Illustrate the hops on the overhead.) Say, "Four plus one equals five."
 3. Ask for a volunteer to write the number sentence.
($4 + 1 = 5$)
 4. Repeat the activity many times using different numbers. Let students take turns making the hops on the transparency number line.
 5. Follow the same procedure for subtraction problems: Say, "Hop to 5 and hop back 3. You end on 2. Five minus three equals two."

6. Ask for a volunteer to write the number sentence.
($5 - 3 = 2$)
7. Repeat the activity using different numbers.

Plus Frog and Minus Toad

Materials: Large number line (numbered from 0 to 20) on tagboard
 Blackline masters "Large Plus Frog and Minus Toad" and
 "Small Plus Frog and Minus Toad," which follow this
 activity
 Tongue depressors
 Tape or stapler
 Crayons

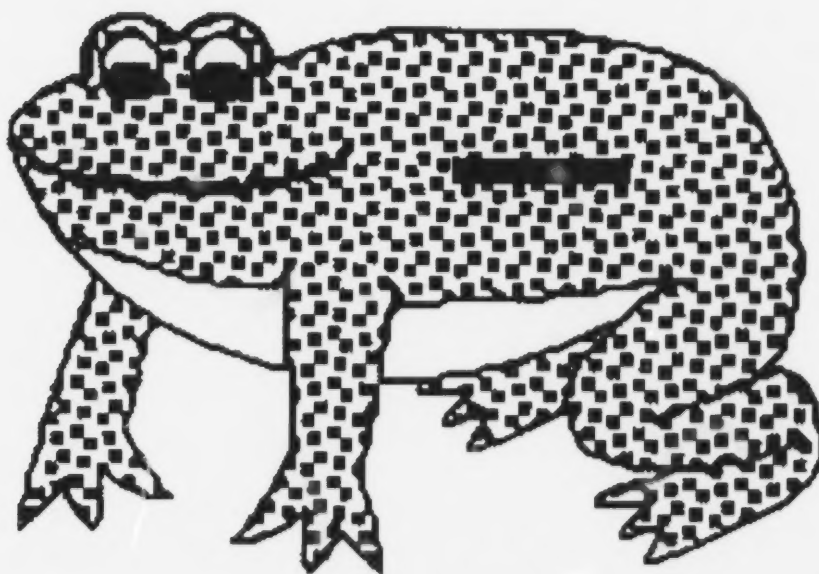
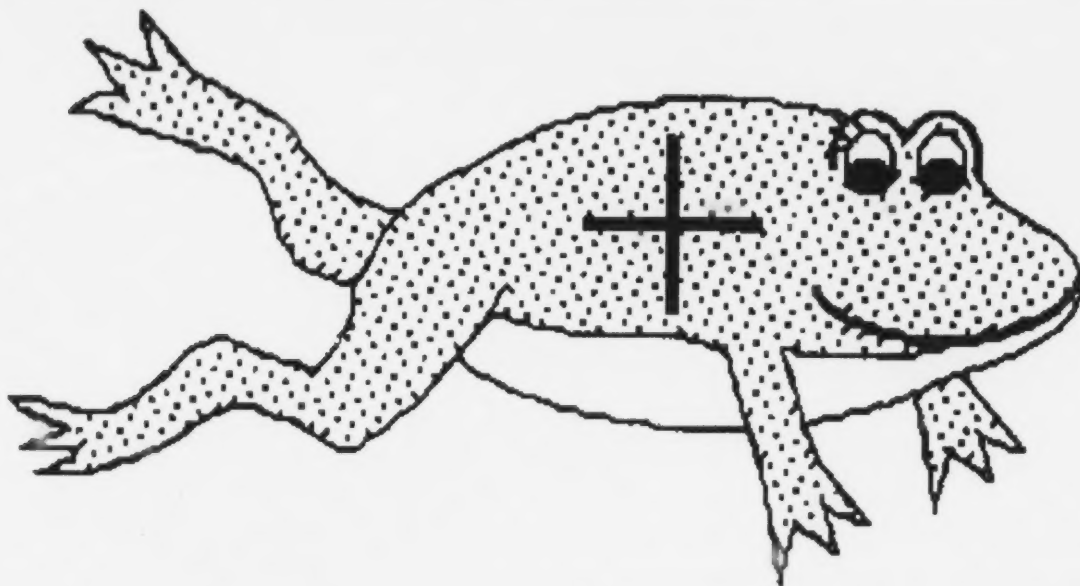
- Directions:**
1. Mount the number line on the chalkboard or bulletin board.
 2. Glue a large Plus Frog and a large Minus Toad on tongue depressors.
 3. Gather the children around the number line and review what they already know about number lines.
 4. Introduce Plus Frog.
 5. Write a simple addition number sentence on the board.
(Example: $4 + 3 = \underline{\quad}$)
 6. Tell the students that Plus Frog is going to work this problem on the number line.
 7. Ask the students what the first number in the addition sentence is.
 8. Have Plus Frog start at 0 and hop to that first number. Draw a curve showing the hop.
 9. Have Plus Frog make the hop for the second number in the addition sentence. Draw a curve showing the second hop.
 10. Tell the class that Plus Frog has now arrived at the answer.
 11. Have someone write the answer to the number sentence.
($4 + 3 = 7$)
 12. Repeat the activity using different numbers and giving each student a turn using Plus Frog.

13. After the class is comfortable with addition and Plus Frog, use the same process for subtraction on the number line using Minus Toad.
14. Write a simple subtraction sentence on the board. (Example: $7 - 2 = \underline{\quad}$)
15. Say that Minus Toad will work the problem.
16. Start Minus Toad on 7 and have the toad hop 2 in the minus direction. Draw a curve showing the hop.
17. Tell the class that Minus Toad has now arrived at the answer. (5)
18. Have someone write the number sentence. ($7 - 2 = 5$)
19. Repeat the activity using different numbers and giving each student a turn using Minus Toad.
20. Pass out copies of Plus Frog and Minus Toad made from the blackline master "Small Plus Frog and Minus Toad."
21. Have the students color them and mount them on their pencils.

*TEACHER NOTE: Plus Frog and Minus Toad are only two of the many "gimmicks" that may be used with students in the beginning stages of teaching about number line addition and subtraction. Other animals, such as roadrunners, kangaroos, and squirrels could also be used. As the students develop the concepts, have them stop using the frog and toad props. Have them "combine" or "take away" the distances on the number line and "solve" the addition and subtraction sentences.



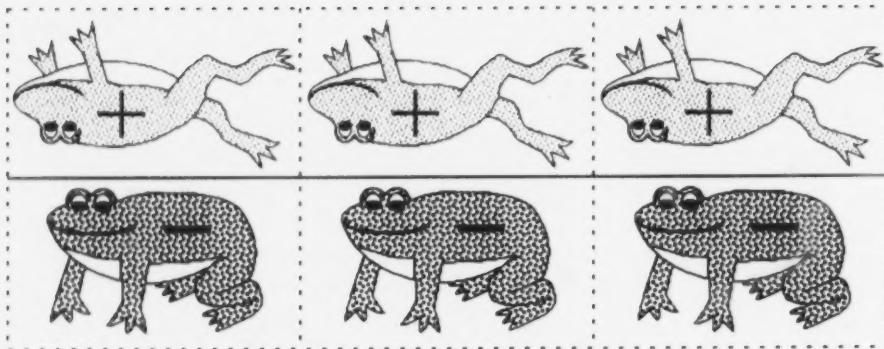
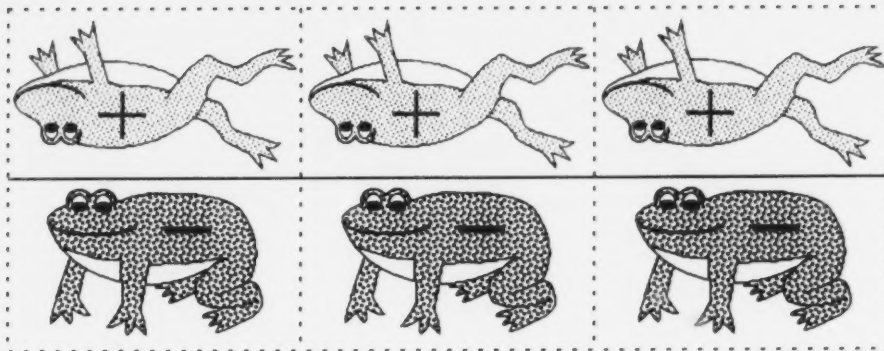
Large Plus Frog and Minus Toad



Use with "Plus Frog and Minus Toad" activity. Color, cut out, and mount each character on a tongue depressor. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Small Plus Frog and Minus Toad



Use with "Plus Frog and Minus Toad" activity. Cut apart on the dotted lines. Give each child a Plus Frog, Minus Toad strip. Have the children fold the strip on the solid line and staple (or tape) it to the top of their pencils. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

50



Let's Practice

Individual Number Lines

Materials: Rulers (one per child)
Markers (paper clips, beans, etc.)

- Directions:**
1. Have a student give the class a subtraction or addition problem.
 2. Instruct the other students to move their markers the appropriate number of spaces on their rulers.
 3. Choose another student to give the next problem.
 4. Once the students are familiar with this activity, extend it by using the metric side of the ruler (cm).
 5. Variations: Have students make up stories to go along with their problems (e.g., "Bunny hopped 12 and then back 7. Where did he land?"). The activity could also be done as a cooperative group activity.

Wire Walkers

Materials: Walk-on number line
Addition and subtraction flash cards

- Directions:**
1. Tell the students that for this activity they should pretend they're at the circus. One student will act as the "wire walker" and another as the "ringmaster." The ringmaster will choose a flash card and tell "the crowd" what act the wire walker will perform. (Example: "Today the wire walker will show us $9 + 10$.") If the wire walker completes his/her stunt, the crowd should clap.
 2. Have the other students work the problem at their desk using number lines.
 3. Ask for volunteers. After the ringmaster reads the problem on the flash card, the wire walker hops the spaces on the number line.

4. The ringmaster then becomes the next walker and the old walker chooses a new ringmaster.

*TEACHER NOTE: It is a good idea to keep a large number line on the wall for students to refer to throughout the year.

Flashy Number Lines

Materials: Addition and subtraction flash cards
Number line handouts (one per student; blackline master may be found in the Appendix)

- Directions:**
1. Have students work with a partner.
 2. Give each student a number line sheet.
 3. Have one student in each group take a flash card, solve the problem written on it by showing the hops on one of the number lines, and write the number sentence beneath that number line.
 4. Have the student's partner check the answer against the one written on the back of the flash card.

*TEACHER NOTE: This can be a center activity.



Problem Solving

STORY

PROBLEMS



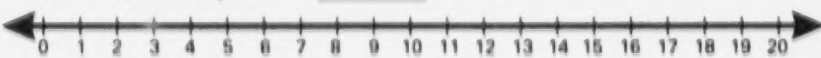
1. Betty Bunny was eating grass. She hopped 6 feet to a new spot. After a while, she hopped back 3 feet. Where did she land? _____



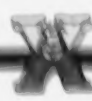
2. One day Andy Kangaroo jumped 7 meters and then 8 meters. Where did Andy end his jumps? _____



3. Mary Robin was eating in a yard one day. She hopped 8 inches. Someone scared her and she hopped back 3 inches. Where did Mary land? _____



4. Betty Bunny hopped 6 yards and landed next to Andy Kangaroo. Betty hopped 6 more yards. How far did Betty hop? _____

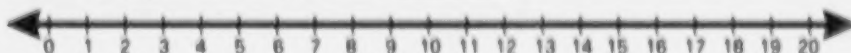


***TEACHER NOTE:** You may need to read these problems orally to the students. Encourage students to use manipulatives to represent the animals.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

MORE STORY PROBLEMS

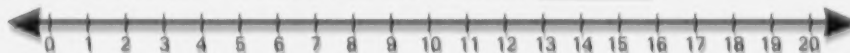
1. Andy Kangaroo was in a contest. He jumped three times in a row. His jumps were 3 yards, 7 yards, and 5 yards. How far did Andy jump altogether? _____



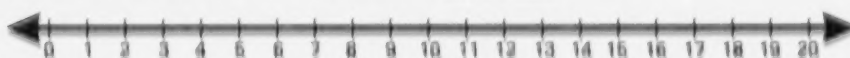
2. Plus Frog hopped 4 meters and landed by the sleeping Betty Bunny. This scared Betty and she jumped 8 meters to where Mary Robin was eating. Mary flew 7 more meters. Where did Mary land? _____



3. Mary Robin hopped forward 6 inches, forward 3 inches, and back 4 inches. Where did she finish? _____



4. Plus Frog jumped 6 feet to Minus Toad, who jumped back 3 feet. Andy Kangaroo then jumped 8 feet forward from that point. Where did Andy finish? _____



*TEACHER NOTE: You may need to read these problems orally to the students. Encourage students to use manipulatives to represent the animals.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Demonstrating a model for addition**Skill Analysis**

- A. Can construct sets of a predetermined amount
- B. Can describe the cardinality of the set
- C. Can join sets and determine the total number of members
- D. Understands the “+” and “=” signs and can use them when recording work
- E. Can transfer the model into a written problem

**Concept Manipulative Activities****Bean Count**

Materials: Lima beans or other counters

- Directions:
1. Give each student 20 counters.
 2. Ask the students to build two sets. (Example: “Build a set of seven and a set of nine.”)
 3. Ask the students to join the sets and find the total.
 4. Write the number sentence $7 + 9 = \underline{\quad}$ on the board and ask the students to explain the connection between the modeling they just completed and this number sentence.
 5. Repeat the activity using other sets and number sentences.

What’s Our Sum?

Materials: Objects to be used as counters

- Directions:
1. Have the students work in groups.
 2. Give each of the students in a group a different number of counters (from 1 to 10).

3. Have the students, in turn, move from person to person in the group putting their set together with the other person's set and stating the sum.
4. The students should keep a record of their work. Example:

I have 4 counters.

Partner		Sentence	Sum
Sally	1	$1 + 4 =$	5
Joe	4	$4 + 4 =$	8
Rachel	7	$7 + 4 =$	11
:	:	:	:

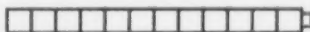
5. The students can share their records as a means of self-checking.

Making Number Sentences

Materials: UNIFIX cubes

Cards or tiles, numbered 0-20 (one set per pair of students)

- Directions:
1. To model this activity, construct two sets of UNIFIX cubes (e.g., one set of seven and one set of five). Place the appropriate number card or tile beneath each set. Then combine the sets and write the number sentence on the board or overhead.

SEPARATE	COMBINED	NUMBER SENTENCE
5 		$5 + 7 = 12$
7 		

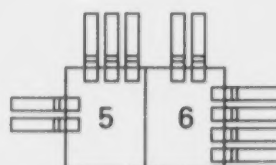
2. Have students work in pairs to practice combining sets using this procedure. One partner should act as the teacher saying how many cubes should be in each set, and the other student should use the UNIFIX cubes to construct the sets, find the sum, and write the number sentence.
3. Partners then reverse roles and repeat the activity.

Clothespin Equations

Materials: 5-1/2" x 8-1/2" cardboard cards that have been laminated or covered with contact paper
 20 clothespins per card
 Vis-A-Vis pens or crayons

- Directions:**
1. Draw a center line on each card as shown below.
 2. Students may work individually, each having his/her own card, or in pairs, sharing a card.
 3. Write two numbers on the board or overhead projector.
 4. Have the students write one of the numbers on each half of their card with a Vis-A-Vis pen or crayon and then attach clothespins to the card to match the numerals they have written.

Example:



5. Select a student to write the number sentence on the board that shows the joining of the sets. The number sentences should be written both vertically and horizontally.
6. Repeat the activity using other numbers.

Pencil Count

Materials: None

- Directions:**
1. Divide the class into groups that are easily organized. For example, if the students' desks are in clusters or rows let those be the groups.
 2. Have all of the students take out the pencils they have in their desks. Each student's pencils will form a set. The students should record the number of pencils in each set for their group on a group recording sheet and then come up with a total from combining the sets together.

Example:

ITEM	JOHN	MARY	MARCIA	TOTAL
<i>PENCILS</i>	3	4	2	9

3. Repeat the activity with other items, such as crayons, markers, books, and so forth.

Cookie Survey

Materials: An Official Cookie Ballot for each child

Example:

Official Cookie Ballot	
Vote For ONE	
Oatmeal	
Chocolate Chip	
Oreo	
Sugar	
Peanut Butter	

3" square Post-its or similar recording cards

- Directions:
1. Have the students vote for their favorite cookie by drawing a picture of it on their Post-it.
 2. Make a visual representation of the results by attaching the Post-its to an Official Cookie Ballot.
 3. Ask questions about the combinations.

Examples:

- a. How many cookies are in the chocolate chip set and the oatmeal set combined?
 - b. If you were to give one cookie each to the people who like oatmeal cookies and the ones who like sugar cookies, how many cookies would you need?
4. Have students make up story problems using data from the Official Cookie Ballot.
 5. Construct a graph of the data using the Post-its.



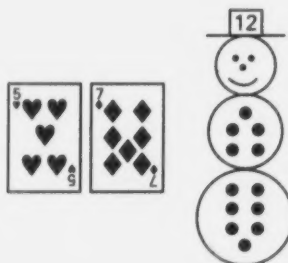
Let's Practice

Snowmen Book

Materials: Six pieces of plain paper per child
Four sets of number cards (numbered 1–10) or a deck of playing cards with face cards removed

Directions: 1. Have each student choose two cards.
2. Have the students draw a snowman on one of their six pieces of paper. Explain that the number cards they have drawn represent the number of “button sets” they should draw on their snowman.

Example:

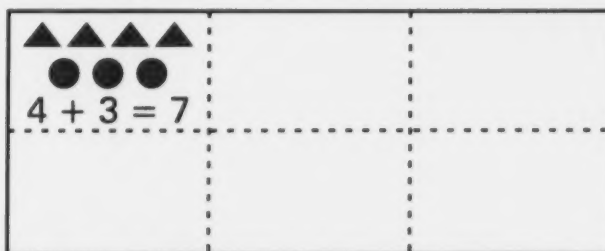


3. The total number of buttons can be written on the snowman's hat.
4. Repeat the activity having the students draw new snowmen and choose new cards.
5. Have the students put their snowmen together in a book.
6. Optional: To integrate this activity with language arts, have the students illustrate a cover for their books and add short stories about each snowman. Some suggestions to include in the stories:
 - a. Name
 - b. Likes
 - c. Dislikes
 - d. What does the snowman see?
 - e. What does the snowman worry about?
 - f. What does the snowman wish for?

Number Sets

Materials: Large newsprint (one sheet per student)
 Counters (UNIFIX cubes, Pattern Blocks, keys, buttons, etc.)
 Stamps and ink pad

- Directions:**
1. Have the students fold their newsprint into thirds and then in half.
 2. The students should work in pairs. Each student should draw two sets per square by tracing around various objects or using the stamps. Each student should then trade papers with his/her partner.



3. The partners should write the number sentence represented by the sets in each square.

Math Problems

Materials: Blackline master "Math Problems," which follows this activity
 Envelopes

- Directions:**
1. Duplicate the blackline master. Cut the sets apart and place them into envelopes so that each envelope has all 12 sets.
 2. Have the students work in pairs. Give each pair an envelope.
 3. Have the partners write down at least six number sentences that they come up with by combining different sets.
 4. Variation: Work with a small group. Choose two sets and have the students write the number sentence.



Math Problems

Use with "Math Problems" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

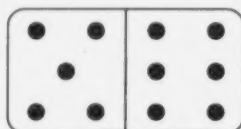
Equation Writing

Materials: "Equation Writing" handouts (one per student; blackline master follows this activity)
 Dominoes or domino cutouts (blackline master may be found in the Appendix)
 Dice (one pair for each student or group of students)
 Lima beans (painted on one side)
 Dot cards (blackline master may be found in the Appendix)

Directions: 1. Set up four centers. Students will record their work at each center on the "Equation Writing" handout.

- a. The Domino Center should include a set of dominoes or domino cutouts placed face down. The students will choose a domino or domino cutout and write the corresponding number sentence on their handout.

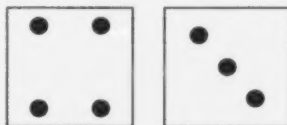
Example:



$$5 + 6 = 11$$

- b. The Dice Center should include a pair of dice for each student or pair of students. The students will roll the dice and write the corresponding number sentence on their handout.

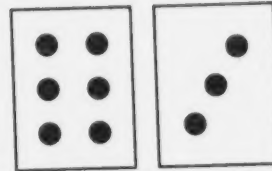
Example:



$$4 + 3 = 7$$

- c. The Bean Center should include a set of 20 beans for each student. The students will take a handful of beans, gently toss them onto the table, and write the addition sentence formed by joining the painted set and the white set on their handout.
- d. The Dot Cards Center should include one set of dot cards placed face down. The students will select two cards and write the corresponding number sentence on their handout.
2. Have students rotate from center to center repeating each activity at least 4 times.

Equation Writing



Topic 1: Modeling Addition and Subtraction



Problem Solving

STORY PROBLEMS

The boat to Paradise Island can hold only 10 people. The captain will take only two families at a time. Seven families want to get to the island. Which families could go in the boat together? (The boat does not have to be full.)



Smith Family



Jones Family



Johnson Family



Roberts Family



Edwards Family



Harris Family



Bates Family

1. The _____ Family and the _____ Family may go. Number sentence: _____
2. The _____ Family and the _____ Family may go. Number sentence: _____
3. The _____ Family and the _____ Family may go. Number sentence: _____
4. The _____ Family and the _____ Family may go. Number sentence: _____
5. The _____ Family and the _____ Family may go. Number sentence: _____

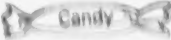


*TEACHER NOTE: Students may need help reading this worksheet. Have the students write the family names of the two sets they wish to combine and then write the corresponding number sentence. Answers will vary since there are more than five possible combinations.

Copyright © 1998 by Larry and Shirley Bradshy. All rights reserved.

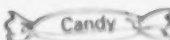
**MORE
STORY
PROBLEMS**

Directions: Draw a picture of each problem.

1. You have some pieces of ,

Give four pieces to Sue. (Draw them in her bag.)

Give eight pieces to Mary. (Draw them in her bag.)

How many pieces of  did you give away? (Write the answer in the rectangle.)



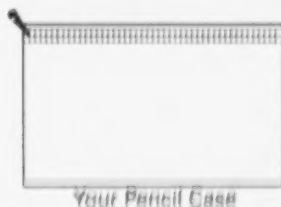
Answer

2. Mother bought 19 's.

She gave 6 to you. (Draw them.)

She gave 9 to your sister. (Draw them.)

How many 's did she give to you and your sister? (Write the answer in the rectangle.)



Answer

TEACHER NOTE: You may need to read the problems orally to some students.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Directions: Draw a picture of each problem.

**MORE
STORY
PROBLEMS**

3. You are making a  with your dad.

He uses 4 scoops of ice cream. (Draw them.)

You use 7 scoops of ice cream. (Draw them.)

You decide to put all of the ice cream together for a Super Sundae. How many scoops of ice cream would be in this Super Sundae? (Write the answer in the Super Sundae Bowl.)



Dad's Dish



Your Dish



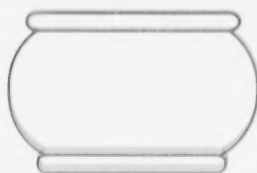
Super Sundae Bowl

4. You and your friends bought 20  at the pet store.

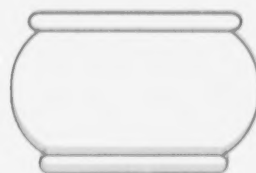
You bought 8 . (Draw them.)

Joe bought 6 . (Draw them.)

How many  did you and Joe buy? (Write the answer in the box.)



Joe's Fish Bowl



Your Fish Bowl

Answer

TEACHER NOTE: You may need to read the problems orally to some students.

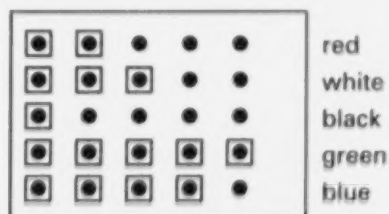
Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Geoboard Combination

Materials: Geoboards (one per student)
UNIFIX cubes (five each of five different colors per geoboard)

Directions: 1. Have the students set up their geoboards with 0-5 like-colored UNIFIX cubes on each of the five rows.

Example:



- The students will mentally combine rows to make as many set combinations as possible and will write down the addition sentences (e.g., $2 + 5 = 7$, $4 + 2 = 6$, etc.).
- Encourage the students to find as many combinations as they can. They should not limit themselves to two addends.

SKILL Demonstrating a model for subtraction

Skill Analysis

- A. Understands the concepts of sets and subsets
- B. Can describe the cardinality of sets
- C. Understands the concepts of less and more
- D. Understands the concept of removal
- E. Understands the “-” and “=” signs



Concept Manipulative Activities

Set Removal

Materials: Counters (UNIFIX cubes, keys, buttons, beans, etc.)

- Directions:**
1. Exhibit a set of eight objects. Remove a subset of three and ask a student to tell how many you removed and how many remain.
 2. Write the equation $8 - 3 = 5$ to illustrate the use of symbols to record this information.
 3. Repeat the activity using a different set and removing a different subset (e.g., $11 - 4 = \underline{\quad}$).
 4. Go over the model with the students, counting out 11, removing 4, and finding the difference (7). Write the equation on the board.
 5. Give each student 20 counters. Give the students more problems and have them build the models and do the record keeping.

Which Set Has More?

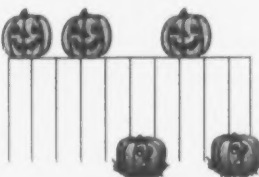
Materials: Counters (UNIFIX cubes, wooden cubes, beans, etc.)

- Directions:**
1. Using the counters, make a set of 10 and a set of 7.
 2. After counting each set aloud, ask, "Which set has more? How many more?" Match the objects in each set one-to-one. Remove the objects that you pair. Identify the number that remains.
 3. Write the corresponding equation on the board ($10 - 7 = 3$). Discuss with the students that the 7 represents the number of objects in the two sets that could be paired. The difference between the two sets, those that could not be paired, is 3.
 4. Give the students another problem (e.g., $12 - 8 = \underline{\quad}$). Have the students build a set of 12 and a set of 8. Ask, "Which set has more? How many more?" Follow the discussion in #3.
 5. Present some more problems and have the students discuss the results. Ask them to record their results. Make sure they understand the connection between the model and the symbols.
 6. Repeat the activity, this time asking, "Which set has less? How many less?"

What Happened?

Materials: Pictures representing set removal

- Directions:**
1. Show the students a picture that could model a subtraction problem.
Example:



2. Ask the students to share story problems they make up about the picture (e.g., "There were five pumpkins

balanced on a fence. Some trick-or-treaters knocked two down. Can you tell how many pumpkins are still sitting on the fence?"').

3. Have the students write a subtraction number sentence to go with one of the stories. Different students should share what they have written.
4. Repeat the activity using other pictures. Students can make their own problem pictures to share with the class or to exchange for problem starters.

Nature Walk

Materials: Plastic or paper bags (one per student)

- Directions:**
1. Take the students out for a nature walk. If this is the class's first outside walk, you may need to establish some rules before leaving the classroom.
 2. Have the students collect leaves, stones, twigs, grass, and similar objects in their bags.
 3. After returning to the classroom, the students should form groups of two and sort out their collections. The partners' collections should not be combined.
 4. The partners should then select an item, such as leaves, and illustrate how many were collected by each of them.

Example:



5. Using these drawings, the students should compare the sets and write a number sentence to show the difference (e.g., $6 - 3 = 3$).
6. Have the students repeat the activity using other items collected (e.g., rocks or sticks).
7. Variation: The students can write number sentences using missing addends.

People Subtraction

Materials: Classroom objects or beans, UNIFIX cubes, etc.

Directions: 1. Give the students a subtraction story problem that uses their names. Allow them to use classroom objects or counters to model the problem.

Example:

"Mary's table has 12 chairs. Steve's table has 5 chairs. Which table has more chairs? How many more chairs does (Mary's) table have?"

2. Record the number sentence on the board after the students have modeled the problem. Have the students write the number sentence on their own paper.
3. Repeat the activity using other story problems. The students should eventually be able to write the corresponding number sentences independently.

**Let's Practice****True or Not True**

Materials: "True or Not True?" handouts (one per pair of students; blackline master follows this activity)

Counters

- Directions:
1. Give a "True or Not True?" handout to each pair of students. Give each student 20 counters.
 2. One student in each pair will choose a problem and use his/her counters to determine if the answer is true or not true.
 3. The other student will check the work. If the answer is NOT TRUE, the correct answer should be written on the handout.
 4. Students should then reverse roles, with the activity continuing until all problems are solved.

**True or Not True?**

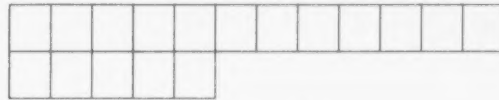
$12 - 6 = 6$	$\begin{array}{r} 9 \\ - 5 \\ \hline 3 \end{array}$	$10 - 4 = 5$	$\begin{array}{r} 5 \\ - 2 \\ \hline 3 \end{array}$
$\begin{array}{r} 9 \\ - 3 \\ \hline 5 \end{array}$	$8 - 6 = 2$	$\begin{array}{r} 11 \\ - 5 \\ \hline 5 \end{array}$	$12 - 3 = 10$
$11 - 3 = 8$	$\begin{array}{r} 9 \\ - 4 \\ \hline 4 \end{array}$	$11 - 6 = 5$	$\begin{array}{r} 11 \\ - 4 \\ \hline 7 \end{array}$
$\begin{array}{r} 9 \\ - 2 \\ \hline 5 \end{array}$	$7 - 5 = 2$	$\begin{array}{r} 6 \\ - 4 \\ \hline 1 \end{array}$	$10 - 6 = 5$
$12 - 5 = 7$	$\begin{array}{r} 13 \\ - 5 \\ \hline 9 \end{array}$	$13 - 7 = 6$	$\begin{array}{r} 14 \\ - 6 \\ \hline 8 \end{array}$
$\begin{array}{r} 10 \\ - 6 \\ \hline 3 \end{array}$	$11 - 3 = 8$	$\begin{array}{r} 12 \\ - 4 \\ \hline 6 \end{array}$	$12 - 5 = 6$

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Grid Subtraction

Materials: Graph paper (one sheet per student; blackline masters may be found in the Appendix)
 Graph paper transparency
 Overhead projector
 Vis-A-Vis pen

- Directions:**
1. Tell the students that you are going to give them some math problems and that they should use their graph paper to make model representations of the problems. (Use the students' names in the problems.)
 2. Go over a problem with the class. Say, "Brad had 12 pencils. Jill had 5 pencils. How many fewer pencils did Jill have?"
 3. On a graph paper transparency, draw a line around 12 squares for Brad and around 5 squares for Jill. Explain that each square represents a pencil.



4. Solutions can be found by removing (crossing out) the squares that can be paired to see what the difference is or by asking, "Five plus what number will give me 12?"
5. Write a number sentence to represent the model. The sentence should be read "Twelve minus five equals seven" because comparison problems are not "take away" problems.
6. Have students use their graph paper to make their own models. Have them cut out their model, glue it onto a piece of plain paper, and write the corresponding number sentence beneath it.
7. Give the class other problems that you or the students make up.

Let's Compare

Materials: Counters (UNIFIX cubes, blocks, Learning Links, etc.)
Cards numbered 0–20

- Directions:**
1. Have the students work in pairs.
 2. Give each student his/her own pile of number cards.
 3. Simultaneously, each student in a pair should draw a number card from his/her pile and, using UNIFIX cubes, make a set with that many members.
 4. The partners should then decide which of their sets is larger.
 5. Have them compare their sets to determine the difference.
 6. Have them write a subtraction number sentence to correspond to their model.
- Example:

								7
								-
								4
								3

7. Repeat the activity with students drawing new cards.

Set Removal

Materials: "Set" handouts (one per student; blackline master follows this activity)

- Directions:**
1. Have the students look at the first set on the handout. Lead the students through a discussion about it (e.g., "What is in the set? How many members are in the set? If you were to remove the footballs, how many objects would remain in the set?").
 2. Say, "Write a number sentence to show what happens when you remove the footballs."
 3. Repeat the discussion for problems 2–4.
 4. For problems 5 and 6, have the students ask each other questions.

5. For several of the problems, ask the students to write a number sentence showing that one subset has been removed. (Example: For #1, say, "Construct a number sentence to show how many footballs remain if you remove the subset of baseballs.") For the same problems, ask, "Can you think of another problem we could work with this set?" Ask, "Is there any way to get **zero** for an answer?"



Set

1.	2.
3.	4.
5.	6.

Use with "Set Removal" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Subtraction Pictures

Materials: None

Directions: 1. Give the students subtraction story problems that are tailored to their school life.

Examples:

- a. In Mrs. Green's room 12 students brought their lunch. In Mr. Book's room 8 students brought their lunch. How many more students brought their lunch in Mrs. Green's room?
 - b. Eighteen students are wearing tennis shoes. Nine are wearing sandals. How many less are wearing sandals?
2. Have the students draw pictures to illustrate the story problems.
 3. Then have them write down number sentences to illustrate the problems (e.g., $12 - 8 = \square$ or $8 + \square = 12$).
 4. Have the students share subtraction story problems they make up.

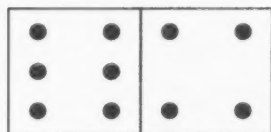
Domino Subtraction

Materials: Dominoes or domino cutouts (blackline master may be found in the Appendix)

Directions: 1. Place the dominoes (or domino cutouts) face down.

2. Have each student, in turn, flip over a domino, draw a picture of it, and use it to write a comparative subtraction problem or a missing addend problem.

Example:



$$6 - 4 = \square \text{ or } 4 + \square = 6$$

3. Suggest to the students that they write the problems both vertically and horizontally. You may need to give examples of both types of problems.
4. This exercise could be done as a small-group activity.



Problem Solving

The Storyteller

Materials: None

- Directions:
1. Tell the students a story problem that involves **take away**. For example, "Mike had 15 Hot Wheel cars. He lost 4 of them in his room. How many cars does Mike still have?"
 2. Ask the students to draw circles on their paper to represent the cars Mike originally had. Then have them cross out four of the circles to represent the 4 cars Mike lost. (Drawing circles instead of pictures saves time and allows more time for mathematics.)
 3. Have students write the corresponding number sentence next to their drawing. Check their numbers sentences for accuracy.
 4. Ask the students to make up their own problems that involve removal of part of a set. For each problem they should draw circles to represent the sets and write the corresponding number sentence.
 5. Have the students take turns telling their stories. The storytellers can check the accuracy of the other students' work on their problems.
 6. Repeat the activity using **comparison** story problems.

*TEACHER NOTE: By allowing the students to be storytellers and by holding them accountable for representations and record keeping, you will help the students relate symbols and verbal language in a meaningful way to the mathematical ideas they are studying.

Fun Survey

Materials: Large tally sheet

- Directions:
1. Have the students develop a survey question, such as "What do you do for fun?"
 2. Then have them select three likely responses for the members of the class and put them on the tally sheet.
- Example:

What do you do for fun?	
Read	
Play Video Games	
Ride Bikes	

3. Instruct the students to survey the class and tally the responses. You may need to explain the use of tally marks.
4. After the survey is completed, discuss the results by comparing the sets. Have the students write the corresponding number sentences.

Take-Away Skits

Materials: None

- Directions:
1. Have the students work in small groups.
 2. Instruct each group to make up a subtraction problem to act out together.
 3. Have each group act out its story problem for the class, and have the other groups write the corresponding subtraction sentence.

Hide-a-Set

Materials: Counters (20 per pair of students)

- Directions:
1. Divide the students into pairs.
 2. Partner 1 should place a set of 10–20 counters on the table. Partner 2 should count the set.
 3. Partner 2 should then close his/her eyes while Partner 1 takes a set of the counters away, hiding them from Partner 2.
 4. Partner 2 should open his/her eyes and write down a subtraction problem to illustrate what took place (e.g., $14 - \underline{\quad} = 6$).
 5. Partner 1 should check the subtraction problem by showing Partner 2 the hidden set of counters and writing the corresponding number in the sentence.
 6. Have the partners change roles and repeat the activity.

Assessment Activities

SKILL Joining sets to model addition

K.1-1

Materials: Objects used when working with sets or objects of interest to the children
Workspace (9" x 12" construction paper, place mat, carpet square, etc.)

- Directions:**
1. Give each child two sets of objects with 0–5 members in each.
 2. Ask each child, in turn, "How many members are there in this set?" (Point to set.) "How many are in this set?" (Point to set.)
 3. Ask, "Can you make a set with all of these objects?" Observe the student to see how he/she combines the sets.
 4. Ask the student, "How many objects do you have altogether?"
 5. Repeat the activity several times with different sets of objects.

SKILL Removing a set to model subtraction K.1-2

Materials: Buttons
Workspace (9" x 12" construction paper, place mat, carpet square, etc.)

- Directions:**
1. Put from 3 to 10 buttons on each child's workspace.
 2. Say, "You each have a set of buttons. How many do you have?" In turn, allow each student to respond.
 3. Say, "I need to sew 3 buttons on my shirt. Can you make a set of the buttons I will need for sewing?" Allow time for the children to separate the sets.
 4. Ask, "How many buttons did you put in my sewing set?" (Allow time for response.) "How many buttons do you have left?" (Allow time for response.)
 5. Repeat the activity by giving each child a different set and varying the number of buttons you need for sewing.

SKILL Joining and separating sets

K.1-3

Materials: Keys

Workspace (9" x 12" construction paper, place mat, carpet square, etc.)

- Directions:
1. Put 4–6 keys on each student's workspace. Have extra keys available.
 2. Say, "In my house I have a key for each door. How many keys do I have in my set?" Allow each child to respond.
 3. Say, "I am going to build a new room onto my house and will need extra keys. Altogether, I will need 8 keys. Can you make your set of keys into a set of 8 keys?" (Allow time for the students to get extra keys and make the new sets.) Check each new set.
 4. Say, "I put all of my keys in my pocket, but there was a hole in it and I lost some keys. Now I have only 4 keys. Can you change your set of 8 keys to a set of 4 keys?" Observe the process used by each child.
 5. Repeat both activities several times so that you can observe each child doing both parts of the assessment.

SKILL Constructing and joining disjoint sets to find the sum

1.1-4

Materials: Counters (UNIFIX cubes, beans, colored pasta, etc.)

- Directions:
1. Give each student a set of 20 counters and a set of written problems (e.g., $2 + 5 = \underline{\quad}$, $7 + 9 = \underline{\quad}$, etc.).
 2. Have the students work the problems using their counters and record their answers on the lines provided.
 3. Circulate among the students, asking them to show you how they are working the problems.

4. After the students have completed the problems, give them each two sets of counters and have them write the corresponding addition sentence. Example:



The student writes $3 + 8 = 11$.

SKILL Demonstrating the definition of subtraction

1.1-5

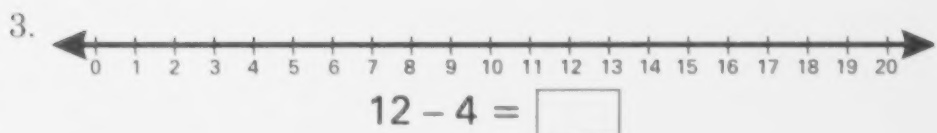
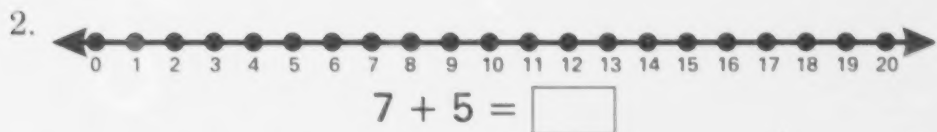
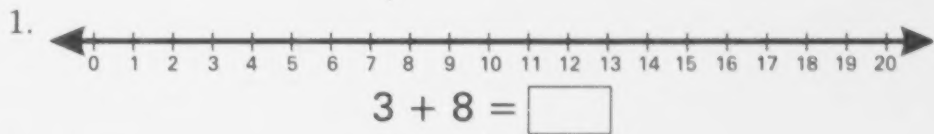
Read these problems orally to the class. Have the students work with manipulatives to solve each problem and then write down the corresponding number sentence. Drawing the objects in the story problem would be helpful, too, but time may be a factor.

1. Toby has six stickers. He gave Susan four stickers.
How many does he have left? _____
Number sentence: _____
2. Bob has eight rocks. Carl has two rocks.
Who has more? _____
How many more? _____
Number sentence: _____
3. Mary had nine buttons on her coat. She lost two of them.
How many buttons does she have left? _____
Number sentence: _____

SKILL Adding and subtracting on the number line

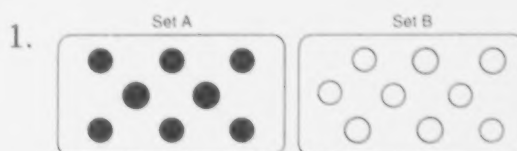
1.1-6

Have the students work the following problems on the number lines. They should show their work and write their answers in the boxes.



SKILL Demonstrating a model for addition 2.1-7

Have the students answer the following questions. You may need to read the questions to the students. They may write the number sentences either vertically or horizontally.



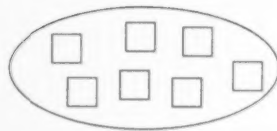
How many black dots are in Set A? _____

How many white dots are in Set B? _____

Write a number sentence showing what the sum would be if the sets were joined. _____

85

2.



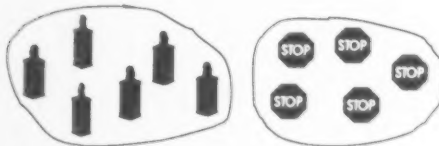
How many squares are in the white set? _____



How many squares are in the black set? _____

Write a number sentence showing what the sum would be if the sets were joined.

3.



How many stop lights are there? _____

How many stop signs are there? _____

Write a number sentence showing what the sum would be if the sets were joined.

SKILL Demonstrating a model for subtraction

2.1-8

Materials: Counters

- Directions:
1. Give each student 30 counters.
 2. Ask the students to use their counters to model the problem "16 take away 7."
 3. Then have each student build a set of 19 and a set of 10. Ask, "How many more objects are in the set of 19 than in the set of 10?"
 4. Ask the students to model the problem "15 minus 7."

5. Have the students write a number sentence to show the following:

There are more teddy bears than rabbits. How many more?



SKILL Solving addition and subtraction problems in both horizontal and vertical form through 20

Skill Analysis

- A. Can count objects through 20
- B. Understands the concepts of addition and subtraction
- C. Recognizes and understands the meaning of the symbols "+," "-", and "="
- D. Knows the value associated with each numeral from 0 to 20

Teaching Note

After students have internalized the concepts of addition and subtraction, they can begin to memorize the basic facts. Drill alone, however, is an inefficient method for helping students learn the facts. Effective strategies for recalling addition and subtraction facts can be learned by all students, and research indicates that students exhibit better retention of the facts when they utilize the strategies. Some suggested strategies are listed below by level of difficulty. Learning these strategies allows students to experience success at an early stage.

Fact Strategies for Addition:

- 1. Identity—use when one addend is 0. Example: $0 + 8 = 8$.
- 2. Counting On—use when one addend is 1, 2, or 3. Example: For $2 + 6$, think 6, 7, 8.
- 3. Doubles—use when both addends are the same. Example: $4 + 4 = 8$.
- 4. Doubles Plus One—use when addends are consecutive numbers. Example: For $5 + 6$, think one more than $5 + 5$.

Fact Strategies for Subtraction:

- 1. Identity—use when 0 is subtracted. Example: $9 - 0 = 9$.

2. Counting Back—use when 1, 2, or 3 is subtracted.
Example: For $9 - 2$, think 9 . . . 8, 7.
3. Sames—use when a number is subtracted from itself.
Example: $9 - 9 = 0$.
4. Fact Families—use when you have a good understanding of the relationship between addition and subtraction.
Example: For $9 - 5$, think $5 + ? = 9$.

Memorizing the problems does not need to be required this year. If memorization is required, it is recommended for adding 0, 1, 2, and doubles. By finding answers with models, students will remember some of the basic facts.



Concept Manipulative Activities

***TEACHER NOTE:** Refer to the concept manipulative activities in Topic 1, "Modeling Addition and Subtraction," if students need more concrete experiences.

Domino Addition and Subtraction

Materials: Dominoes or domino cutouts for each child (blackline master may be found in the Appendix)

- Directions:**
1. Give each child a domino or domino cutout.
 2. Have the child write a horizontal addition sentence for his/her domino and read it aloud.
(Example: For , the child would write " $3 + 5 = 8$.")
 3. Then have the children turn their dominoes vertically, record the vertical addition sentence, and read it aloud.

$$\begin{array}{r}
 \begin{array}{|c|} \hline \bullet \\ \bullet \\ \bullet \\ \hline \bullet \\ \bullet \\ \bullet \\ \hline \end{array} & \begin{array}{r} 3 \\ + 5 \\ \hline 8 \end{array}
 \end{array}$$

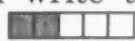
4. Repeat the activity for subtraction.
 - a. Have the children turn their dominoes vertically and count how many dots there are altogether.

- b. Then have them take away the top number to find out how many are left.
- c. Have them rotate their dominoes 180° and write the new number sentence.

	$\begin{array}{r} 8 \\ - 3 \\ \hline 5 \end{array}$		$\begin{array}{r} 8 \\ - 5 \\ \hline 3 \end{array}$
---	---	---	---

UNIFIX Addition and Subtraction

Materials: UNIFIX cubes (two different colors, 10 of each color per child)

- Directions:**
1. Give the class a simple word problem (e.g., "Two cats and three dogs equals how many animals?").
 2. Have the children use UNIFIX cubes to solve the problem.
 3. Ask a volunteer to show his/her cubes and write the number sentence on the board (e.g., $2 + 3 = 5$ ).
 4. Ask if another student can write the sentence in a different way. If the first volunteer wrote the sentence horizontally, look for someone who can show his/her cubes vertically and write a vertical addition sentence:

	$\begin{array}{r} 2 \\ + 3 \\ \hline 5 \end{array}$
---	---

5. Emphasize that the numbers and answers are the same regardless of whether the number sentence is written horizontally or vertically.
6. Repeat the activity using other numbers that total up to 20.
7. Then put two colors of cubes together, take away one of the colors, and have a student write the subtraction sentence.
Example:

	$4 - 1 = 3$
---	-------------

Number Relationships

Materials: Overhead projector
Blank acetate
Vis-A-Vis pen
Attribute Blocks (two shapes)

- Directions:
1. Show a set of 4 blocks of one shape and a set of 8 blocks of another shape on the overhead.
 2. Have a volunteer count the members in each set.
 3. Then ask the child to put the two sets together. Ask, "How many blocks are there altogether?" (12)
 4. Have the child write a number sentence for what he/she just did. ($4 + 8 = 12$ or $8 + 4 = 12$)
 5. Next, have the child to take away the set of 4. Ask, "How many are left?" (8)
 6. Again, have the child write the number sentence. ($12 - 4 = 8$)
 7. Have the student put the set of 12 back together. Ask again, "How many blocks are in our big set?" (12)
 8. Take away the set of 8. Ask, "How many blocks are left?" (4)
 9. Ask the child to write the new number sentence. ($12 - 8 = 4$)
 10. Repeat the activity several times using different numbers and different volunteers.

*TEACHER NOTE: Discuss with the children the concept that addition and subtraction are inverse relationships. Avoid looking at the numbers (e.g., 8, 4, and 12) merely as symbols that can be manipulated in several ways to write related addition and subtraction sentences.



Let's Practice

Bean Tubs

Materials: Lima beans painted red on one side
Containers or tubs (one per student)
Record sheet (one per student)

- Directions:
1. Work with a small group of students.
 2. Place 2–20 beans in each tub, write the total on the tub, and give a tub to each student.
 3. Have the students spill their beans onto their desks and group the beans into two sets: red and white.
 4. Have them count the number of red beans and record the number on their paper. Have them add the number of white beans to obtain the total number of beans and write the number sentence.
 5. To stress the inverse relationship of addition and subtraction, have the students write the total number of beans, subtract the number of white beans, and write the number sentence.
 6. The students can work independently with different tubs once they understand the pattern: $\text{Red} + \text{White} = \text{Total}$, and $\text{Total} - \text{White} = \text{Red}$.
 7. The activity can be expanded to include the pattern $\text{White} + \text{Red} = \text{Total}$ and $\text{Total} - \text{Red} = \text{White}$.
 8. Variation: String wires with two sets of uncooked pasta. You can use different kinds of pasta or different colors. Have the students write addition and subtraction number sentences for each wire following the aforementioned procedure. Have the students reverse the wires to obtain two more number sentences.

*TEACHER NOTE: Stress the concept of inverse relationships throughout these activities.

Universal Game

Materials: Universal Game Board (one per group of students; directions for its construction may be found in the Appendix)
 Game markers
 Addition or subtraction flash cards (including both vertical and horizontal problems)
 Dice

- Directions:
1. Explain that the students will play a game in groups of four.
 2. Put a pile of flash cards on each game board.
 3. Have the first player in each group roll the dice and draw a card.
 4. She/he should read the problem on the card and answer it.
 5. The other players should check the answer.
 6. If it is correct, the player should move the number of spaces shown on the dice.
 7. The game continues with each player taking his/her turn until someone reaches the finish line.

*TEACHER NOTE: Students should be allowed to use manipulatives as they do these practice activities.

Practice Folder

Materials: Paper
 Manila folders (one per student)

- Directions:
1. Write 10 or more problems on the front of each manilla folder.
 2. Cut out a square for each answer.
 3. Have each child choose a folder, put a blank piece of paper inside it, close the folder, and write his/her answers in the open squares.
 4. The children should then take their papers out and check their answers with the answer sheet, which is kept inside the folder.
 5. Have the children exchange folders and repeat the activity.

$2 + 2 =$	
$5 + 3 =$	
4	6
$+ 2$	$- 2$

Ladder Game

Materials: None

Directions: 1. Draw a number of ladders on the chalkboard. Write a different number sentence on each rung.

5 + 7 =	7 + 7 =
6 + 6 =	6 + 8 =
3 + 9 =	3 + 8 =
9 + 2 =	5 + 7 =
4 + 8 =	6 + 6 =
7 + 7 =	9 + 2 =
6 + 8 =	3 + 9 =
3 + 8 =	4 + 8 =

2. Have a child stand close to each ladder. Instruct the children to "climb" to the top by writing down the results at the side of the rungs, starting at the bottom of the ladder.

3. Variation: The ladders can be written on tagboard, laminated, and hung on the wall. Be sure to provide a check card.

Baseball

Materials: Bases (such as chairs, erasers, or beanbags)
Addition and subtraction flash cards

Directions: 1. Place four "bases" on the floor at the front of the classroom, so that they form a baseball diamond.

2. Divide the class into two equally matched teams.

3. Say, "I will be your pitcher today for a game of baseball. You'll all have a turn at bat. I will ask you a math question. If you answer it correctly, you may go to first base. If you do not answer it correctly, you'll be out. The players will advance from base to base with each correct answer. For every run, your team will score a point. When team A makes three outs, it will be team B's turn to bat. At the end of the playing time, the team with the highest score will win."

4. Select a different flash card for each student at bat.

*TEACHER NOTE: Be certain to stop the game at a point at which each team has had an equal number of innings "at bat." You may want to limit each inning to three outs or 5 minutes, whichever comes first.



Problem Solving

Nomograph

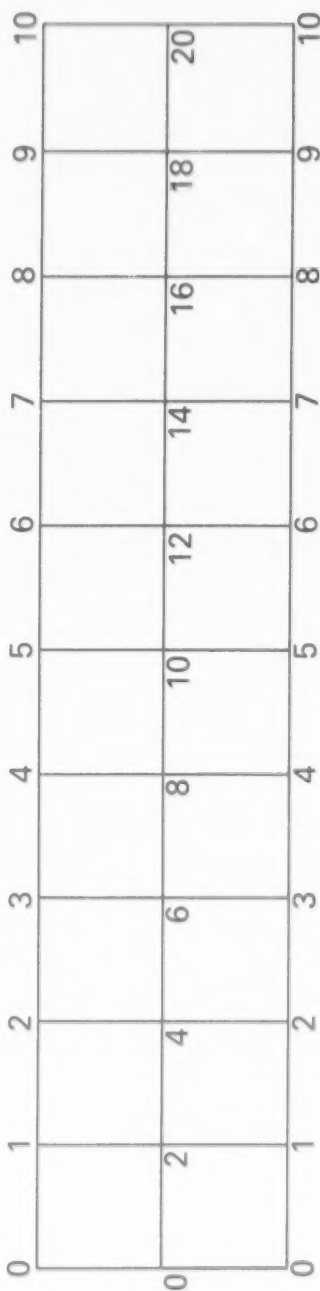
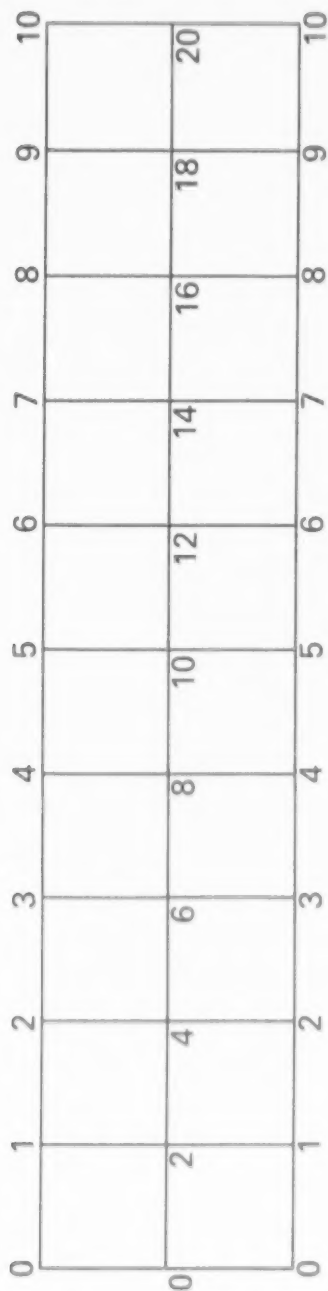
Materials: Nomographs (one per student; blackline master follows this activity)

Straightedge for each student (e.g., ruler or card)

- Directions:**
1. Explain to the students that there is a fun way to add and subtract using a special type of graph. The name of this graph is a "nomograph." Give a nomograph to each student. Say, "The number lines at the top and bottom of the graph represent the numbers to be added."
 2. Ask, "Can anyone figure out how you might use this nomograph to add like numbers, or doubles?" (The sum is found on the vertical line connecting the doubles.)
 3. Ask, "How would you add numbers such as $2 + 4$ or $6 + 4$?" (The sum is found at the center of the diagonal connecting the numbers. Students may want to use a straightedge to connect the numbers.)
 4. Do several problems with even sums by diagonally connecting the addends.
 5. Say, "Try adding $3 + 4$. How can you find the answer?" (The odd numbers are missing on the center line. Students should put the odd numbers in between the even numbers.)
 6. Have the students try other problems involving sums that are odd.
 7. Ask, "Can anyone figure out how to use the nomograph for subtraction?" (Use an example like $10 - 6$. Have the students find 10 in the center and 6 on the top or bottom line. The answer is found on the diagonal line that connects the two numbers.)
 8. Ask such questions as, "What kinds of patterns do you see with subtraction?" "Do you see any patterns when you use the graph to add odd numbers?" "Is this a good system to use for the addition and subtraction facts?"



Nomograph



Use with "Nomograph" activity. Copy the master and cut it apart so each student gets one nomograph. Encourage the students to tell why the graph works. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

**S
T
O
R
Y**

**P
R
O
B
L
E
M
S**

Directions: Write number sentences for the following problems.

1. Write a number between 5 and 11. That is how many dog stickers you have. Write a number between 12 and 17. That is how many more dog stickers Tom gave you. How many dog stickers do you have in all? _____



2. You like to trade your stickers with other people. You are going to trade with Donna. Write a number between 10 and 18. That is how many flower stickers you have. Write a number between 2 and 9. That is how many flower stickers you traded away to Donna. How many flower stickers do you have left? _____



3. You collect all sorts of stickers. Write a number between 1 and 10. That is how many heart stickers you have. Write a number between 13 and 18. That is how many bear stickers you have. How many heart and bear stickers do you have in all? _____



4. One of the attractions at the park is the water slide. Nicole went down 15 times. Alan went down 10 times. How many more times did Nicole go down the slide than Alan? _____



* TEACHER NOTE: Depending on the numbers chosen by each student, the answers will vary. Discuss how each problem was set up and how the students arrived at their answers.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

MATH SENTENCES

Directions: Write a related addition and subtraction number sentence for each picture.

A.



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

B.



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

C.



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

D.



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

E.



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

F.



$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

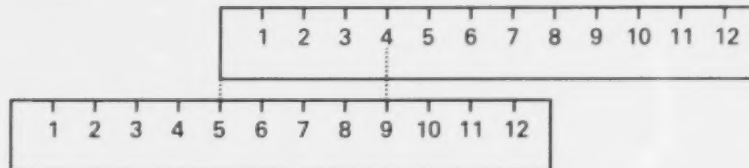
$$\underline{\quad} - \underline{\quad} = \underline{\quad}$$

Adding Rulers

Materials: 12" rulers or paper strips labeled "1-10" (two per child)

- Directions:**
1. Say, "I am going to show you a way to use two rulers to do your adding for you."
 2. Have the students place their rulers one above the other on their desks.
 3. Say that you will be calling the top ruler "ruler A" and the bottom ruler "ruler B."
 4. Say, "Let's add 5 and 4 using these rulers."
 5. Have students place the 0 end of ruler A over the 5 of ruler B.
 6. Then have them find the 4 on ruler A.
 7. Say, "Look directly below it on ruler B. Your answer is 9."

Example:



8. Pose additional problems and have the students work on them individually.

SKILL Describing answers for basic addition and subtraction facts

Skill Analysis

- A. Understands the concepts of addition and subtraction
- B. Recognizes and understands the meaning of the symbols "+," "-", and "="
- C. Has developed strategies for solving addition and subtraction facts



Concept Manipulative Activities

Overhead Addition and Subtraction

Materials: Overhead projector
Blank acetate
Counters (Popsicle sticks, toothpicks, UNIFIX cubes, etc.)
Vis-A-Vis pen

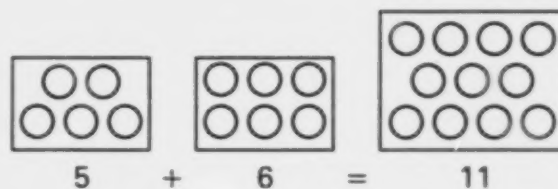
- Directions:**
1. Write a problem (e.g., $5 + 7 = ?$) on the overhead projector.
 2. Using counters at their desks, the students will form the sets to find the answer. Have the students write the complete sentence (e.g., $5 + 7 = 12$) on a piece of paper.
 3. Choose one student to use counters on the overhead to show how he/she solved the problem. The student should write the answer on the overhead with a Vis-A-Vis pen.
 4. Repeat the activity using a subtraction sentence (e.g., $13 - 5 = \square$).
 5. Have students model and complete the problem at their seats. Have one student work at the overhead.
 6. Continue this activity using other addition and subtraction problems.

100

In the Bag

Materials: Counters (colored pasta, beans, buttons, UNIFIX cubes, etc.)
Plastic sandwich bags (one per student)

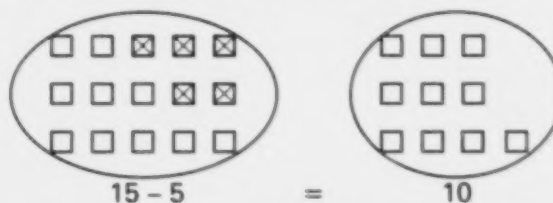
- Directions:**
1. Give each student a plastic bag filled with counters.
 2. Have the students form two sets (e.g., a set of 5 and a set of 6), join them together, and write a number sentence.
Example:



3. Have the students form two new sets (e.g., 12 and 7). Ask them to compare these new sets by telling how many more counters are in one set than the other.
4. Have the students model how they found their answer and write a number sentence. Example: For a set of 12 and a set of 7,

- $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{array}$
 a. If you remove the ones that match,
5 remain. ($12 - 7 = 5$)
or
 $\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{array}$
 b. $7 + \square = 12$. I need 5 more.

5. Have the students form one set of counters (e.g., 15). Ask them to remove 5 counters from this set and write a number sentence for the model. Example:



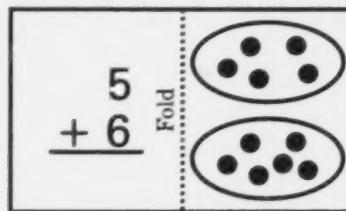
6. Have the students work with all three models in random order. At times, repeat the same model two or more times in a row.

7. Be sure to include the empty set (0) in some of the problems.
8. Encourage students to practice recall of the easy facts.

Dot Flash Cards

Materials: Dot cards (blackline master may be found in the Appendix)
8-1/2" x 11" paper cut in half horizontally
Glue

- Directions:**
1. Give each student a piece of paper. Have the students fold the paper in half, as shown below.
 2. Have each student make a card for an addition fact by writing the fact on the left side of the paper and gluing the corresponding dot cards on the right. Example:



3. Then have the students fold back the half with the dots so that only the fact is showing.
4. When they are finished constructing their cards, the students should pair up and ask each other for the answers to the facts. If a student does not know an answer, he/she may turn the card over and count the dots.

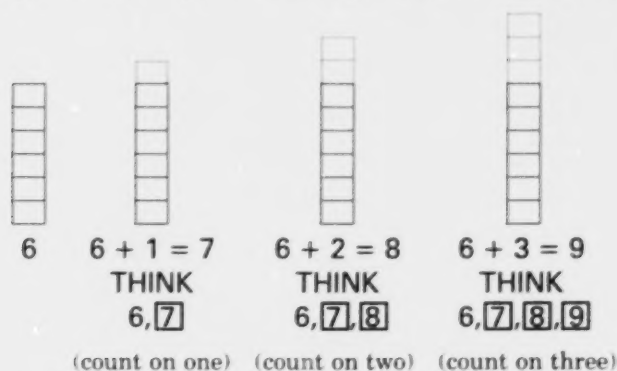


Let's Practice

Counting On—One, Two, Three

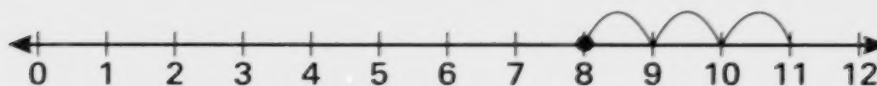
Materials: Counters (UNIFIX cubes, wooden cubes, etc.)
Optional: Number Line handouts (one per student; blackline master may be found in the Appendix)

- Directions:
1. Explain to the students that counting on is an effective strategy for addition when one of the addends is 1, 2, or 3. Addition of 1, 2 or 3 can be "kept track of" mentally without the use of counters such as fingers.
 2. Have the students model the strategy first with counters in order to get a mental picture of the process being used. Walk them through the thought process for the problems $6 + 0 = 6$, $6 + 1 = 7$, $6 + 2 = 8$, and $6 + 3 = 9$. Example:



3. Discuss with the students that each of these models represents one more than the previous model.
4. Because oral practice of counting on will help students to apply this strategy, repeat the activity using different numbers. Examples:
 - a. $9 + 2$
 THINK/SAY 9, 10, 11
 $9 + 2 = 11$
 By naming the larger addend, 9, and then saying 10 and 11, the student can hear himself/herself **adding on two**.
 - b. $3 + 7$
 THINK/SAY 7, 8, 9, 10
 $7 + 3 = 10$
 By saying the larger number, 7, and then saying 8, 9, 10, the student hears himself/herself **adding on three**.
5. Students can practice this strategy with a partner. Each would take turns giving the other an addition problem where one of the addends is 1, 2, or 3. The student would use counting on, aloud, to solve the problem.

6. Variation: Students can use a number line to model this strategy. Example:



For $8 + 3$, the students would start on the largest number and count on three (9, 10, 11).

Contig—Add & Subtract

Materials: Contig Game Sheet (one per group of students; blackline master follows this activity)

Beans or other markers

Three dice per group of students

- Directions:**
1. This game is played in pairs or groups of three or four.
 2. Give each group a Contig Game Sheet, a pile of markers, and three dice.
 3. Each player should roll one die. The person with the highest roll will go first.
 4. That player should roll all three dice and add or subtract the numbers rolled, putting them in any order. Example: For the roll 2, 5, 3, the student could write down $2 + 5 - 3 = 4$, $(5 - 2) + 3 = 6$, $5 - 2 - 3 = 0$, $2 + 5 + 3 = 10$, etc.
 5. The object of the game is to create a problem with an answer that corresponds to an unoccupied square on the Contig Game Sheet.
 6. The student should show the rest of the group one problem and its answer. The other players should check the work. If it is wrong, the person loses his/her turn. If it is correct, the person puts a marker on the square with the answer. That square may not be used again.
 7. Students are given one point for placing a marker on a square plus one additional point for each occupied square that touches that square (above, below, to either side, or diagonally).

8. A time limit may be imposed or the game can continue until the sheet is full. The winner is the person with the most points.
9. Variation: Have teams play against each other. Give each team 15 turns and see who has the most points.

**Contig Game Sheet**

1	2	3	4	5
6	7	8	7	6
9	10	11	12	13
14	15	16	17	18

**Contig Game Sheet**

1	2	3	4	5
6	7	8	7	6
9	10	11	12	13
14	15	16	17	18

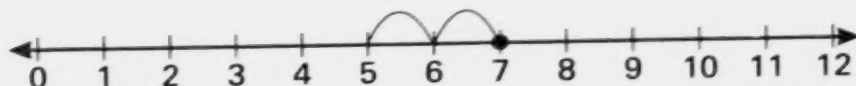
Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Counting Back—One, Two, Three

Materials: Number Line handouts (one per student; blackline master may be found in the Appendix)

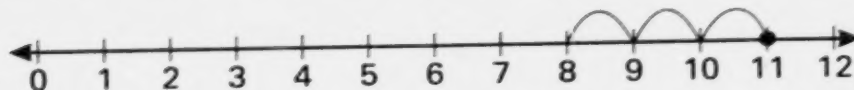
Optional: Counters (wooden cubes, UNIFIX cubes, etc.)

- Directions:**
1. Explain to the students that counting back is an effective strategy for subtraction when the subtrahend is 1, 2, or 3. Subtraction of 1, 2, or 3 can be "kept track of" mentally without the use of counters such as fingers.
 2. Have the students model the strategy first using number lines in order to get a mental picture of the process being used.
 3. Have the students number the first number line 0 – 12 and use this number line to model the subtraction problem "7 – 2." Example:



Begin at 7, count back two (6, 5). So, $7 - 2 = 5$. Explain that the students could do the same thing without the number line, mentally keeping track of the numbers as they count back.

4. Have the students number another number line 0 – 12 and use it to model the problem 11 – 3. Example:

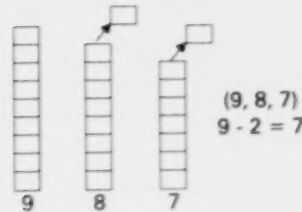


Begin at 11, count back three (10, 9, 8). So, $11 - 3 = 8$. Again, explain that the students can mentally keep track of the numbers as they count back.

5. Have the students work several subtraction problems in which the subtrahend is 1, 2, or 3 without using number lines. They should say the numbers they are counting back aloud.
6. Students can practice this strategy with a partner. Each would take turns giving the other a subtraction problem

that makes use of the counting back strategy. Number lines should be available for students who choose to use them.

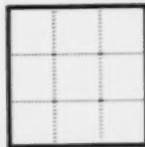
7. Variation: Students can use cubes to model this strategy.
Example:



Doubles

Materials: 9" x 12" newsprint (one sheet per student)
Counters

- Directions: 1. Have students fold their paper into nine squares.
Example:



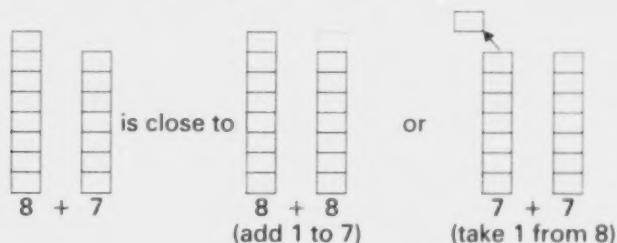
2. Ask the students to open up the paper and write a doubles addition sentence in each square starting with $1 + 1 = 2$ and continuing to $9 + 9 = 18$.
3. If a student cannot recall all nine facts, have him/her use counters to help determine the sum.
4. Variation: Have the students write the subtraction facts associated with doubles (e.g., $18 - 9 = 9$, $16 - 8 = 8$, etc.).

Doubles Plus or Minus One

Materials: Counters (wooden cubes, UNIFIX cubes, etc.) or graph paper
(blackline master may be found in the Appendix)

- Directions: 1. Model the problem $8 + 7 = \square$ using blocks or graph paper squares.

2. Ask, "What doubles fact is this problem close to?" Lead the student to see that it is close to either $8 + 8 = 16$ (only one less) or $7 + 7 = 14$ (only one more).



3. Have students model problems involving doubles plus or minus one and have them state which doubles fact they chose to use. Examples:

	One More Than	One Less Than
$6 + 7 = \underline{\quad}$	$6 + 6 = 12$	$7 + 7 = 14$
$9 + 8 = \underline{\quad}$	$8 + 8 = 16$	$9 + 9 = 18$
$5 + 6 = \underline{\quad}$	$5 + 5 = 10$	$6 + 6 = 12$

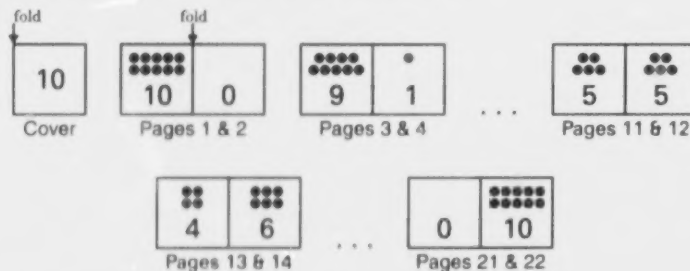
4. Discuss with students when this is an effective strategy to use (when the difference between the addends is one).

Practice Book

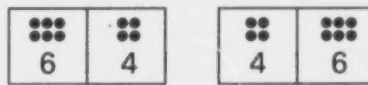
Materials: Construction paper
Adhesive dots, markers, or stamps and ink pad

- Directions:**
1. In this activity students will make booklets showing all of the number combinations for a particular sum.
 2. Cut 8-1/2" x 11" construction paper in half for sums less than 10. Use whole sheets for sums greater than 10.
 3. To make a number book for 10, each student will need six sheets of construction paper. Have the students lay the sheets on top of each other and fold the stack in half making a booklet. They should then write "10" on the cover to name this particular sum book.

4. Beginning on the first two pages and working through consecutive pages, the students should make models of combinations that have a sum of 10.



5. The booklets can be used in a variety of ways.
- Students can use them to practice addition problems with a given sum.
 - Students can use them to see examples of the commutative property. Example:



- Students can use them to quiz one another on problems with missing addends by showing one side and asking what number should be added to that number to reach the given sum.
6. For other books students will need:

Sum	Number of pieces of construction paper
3 and 4	3
5 and 6	4
7 and 8	5
9 and 10	6
11 and 12	7
13 and 14	8
15 and 16	9
17 and 18	10
19 and 20	11

Let's Remember

Materials: Lima beans painted on one side or other counters

Directions:

1. Give the students a certain number of counters (e.g., 10).
2. Have them find and record all of the addition problems for their number of counters. Example: For 10 counters, problems would be



$$5 + 5 = 10$$



$$1 + 9 = 10$$



$$4 + 6 = 10, \text{ etc.}$$

3. Ask the students to put away the counters. Then ask them to recall as many of the problems as they can and write them down (e.g., $10 = 4 + 6$, $10 = 9 + 1$, $10 = 5 + 5$, $10 = 3 + 7$).
4. Explain that if they know the facts for 10, they can use this knowledge to help them solve other problems. For example, $8 + 5$ can be solved by using $8 + 2 = 10$. Since 5 is 3 more than 2, the sum will be 3 more than 10, or 13. This strategy is especially helpful when one of the addends is 7 or 8.

Families

Materials: Counters

- Directions:**
1. Give each student five counters.
 2. Have the students divide the counters into a group of two and a group of three.
 3. Challenge the students to make an addition and a subtraction fact family using these two sets (e.g., $2 + 3 = 5$ and $5 - 3 = 2$). Have the students record both facts on paper.
 4. Ask, "What other number sentences could we have made using these sets?" ($3 + 2 = 5$ and $5 - 2 = 3$) Discuss with the

students that changing the order of the addends does not affect the sum.

5. Constructing inverses will help the students to see the relationship between addition and subtraction. It will also help the students learn subtraction facts. If the students learn all the addition facts, they will also know the subtraction facts (e.g., $7 + 8 = 15$ and $15 - 8 = 7$).
6. Repeat the activity using other fact families (e.g., for 1 and 4, $1 + 4 = 5$ and $5 - 4 = 1$).

Combo

Materials: Counters (UNIFIX cubes, beans, etc.)
Addition or subtraction flash cards

- Directions:**
1. Have each student make 16 squares on a piece of paper.
 2. Have the students write numbers from 0 to 20, if playing addition, or from 0 to 10, if playing subtraction, in each square. The students may use a number more than once.
- Examples:**

Subtraction				Addition			
10	4	7	0	9	18	7	8
8	1	4	3	10	15	2	1
9	5	2	8	14	12	4	16
0	3	6	5	6	15	3	10

3. Ask for a student volunteer to be the caller. Using either the addition or subtraction fact cards (depending on the game being played), the caller should call out facts one at a time. If a student has the number that is the sum or difference, he/she should cover that number. Only one number may be covered at a time.
4. The first student to cover four numbers across, down, or diagonally wins.
5. A record should be kept of the problems stated and the sums or differences so the winning cards can be checked.
6. Students may use counters to help them find solutions to facts they cannot recall from memory.

Triangle Difference

Materials: Eight clothespins
 Tagboard circles
 Counters (beans, UNIFIX cubes, etc.)

Directions: 1. Make a tagboard circle like the one below, writing a minuend in the center and subtrahends on the middle circle. Leave the outer circle blank. Example:



2. Write the differences for the problems on clothespins.
3. Write the correct difference for each problem on the back of the blank areas of the tagboard circle so students can self-correct once the clothespins are in place.
4. Have students solve the problems and attach the "answer" clothespins to the correct blank areas of the circle. Students should use counters if needed.
5. Variation: For "Find the Sum," put one addend in the center, the other addends in the middle circle, and the sums on the clothespins.

Universal Game

Materials: Universal Game Board (one per group of students; directions for its construction may be found in the Appendix)
 Game markers
 Small cards—3" x 2-1/2"
 Counters
 One die numbered 1, 2, 3 per group

- Directions:
1. Explain that the students will play a game in groups of 2 to 4.
 2. Make 25 cards for each game board. Write a subtraction or addition fact on each card. Use both hard and easy

problems. Write the hard facts in blue and the easy facts in red. Let the students know that they can use counters to solve the blue facts. (Easy facts: addends or subtrahends of 0, 1, 2, 3, and doubles.)

3. Place 25 cards in a pile, face down, on each game board and have the students place their markers on START.
4. Instruct the first player to select a card and state the answer to the fact. If the student's answer is correct, the student rolls the die and moves that many spaces. If the student's answer is incorrect, he/she stays at START until his/her next turn.
5. The game is over when the first player reaches the finish line.

Addition Facts Chart

Materials: "Hundred Grid" handouts (blackline master may be found in the Appendix)

Directions: 1. Give each student or pair of students a "Hundred Grid" handout. Have students record the 100 addition facts as follows.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	...					
2	3	4	5	...						
3	4	5	6							
4	5	:		8						
5	:									
6										
7										
8										
9										
10										

- a. Write the addends around the border.
- b. Fill in all sums.

2. The students can use this chart to review the strategies they have learned for addition. The chart is also a visual way for them to represent the facts they know or are learning.

- a. Adding 1: Say, "Look at your chart. What can you tell me about the sum when you add 1?" (The answer is always one more than the number they started with.) Using a crayon and coloring very lightly, students can show the sums for problems they can do easily because the answer is **just one more**.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3									
3	4									
4	5									
5	6									
6	7									
7	8									
8	9									
9	10									
10	11									

- b. Counting On: Ask, "When can you use counting on to help you with the addition facts?" (when adding 1, 2, or 3) Say, "We have already marked the chart for adding 1. Use a different color to shade the sums for problems where you have added 2 or 3."

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5									
5	6									
6	7									
7	8	9	10							
8	9	10	11							
9	10	11	12							
10	11	12	13							

BEST COPY AVAILABLE

115

- c. Doubles: Ask, "Can you locate the sums that are doubles? Do you see any kind of pattern or anything interesting?" (The doubles divide the grid diagonally; you can skip count by twos; the line separates commutative pairs, (such as $3 + 5$ and $5 + 3$.) Encourage students to find as many patterns as they can. Have them shade in this section with another color.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8						
5	6	7	8		10					
6	7	8	9			12				
7	8	9	10				14			
8	9	10	11					16		
9	10	11	12						18	
10	11	12	13							20

- d. Doubles Plus or Minus One: Knowing the addition facts that involve doubles can help students with other facts. For example, if the problem to be solved is $6 + 7$, the student can think "one more than $6 + 6$ " or "one less than $7 + 7$." Ask, "Where do you think these sums will be found on the chart?" (on either side of the doubles diagonal) Students should be encouraged to look for patterns. Have them shade these sums with another color.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9					
5	6	7	8	9	10	11				
6	7	8	9		11	12	13			
7	8	9	10			13	14	15		
8	9	10	11				15	16	17	
9	10	11	12					17	18	19
10	11	12	13						19	20

			6	7
	8			
		10		
6			12	13
7			13	14

$$6 + 7 = 13$$

$$6 + 6 = 12$$

$$7 + 7 = 14$$

116

- e. Adding 10: Adding 10 to the digits 1–9 yields the “teens.” The 10 indicates one 10 in the tens place and the single digit names the number of ones. With the exception of 11 and 12, the ones are part of the name (e.g., 13 is $10 + 3$, 14 is $10 + 4$, etc.). Have the students shade these sums.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9					14
5	6	7	8	9	10	11				15
6	7	8	9		11	12	13			16
7	8	9	10			13	14	15		17
8	9	10	11				15	16	17	18
9	10	11	12					17	18	19
10	11	12	13	14	15	16	17	18	19	20

- f. Adding 9 as One Less Than 10: If a student is given the problem $9 + 6$, he/she can think, “The answer is one less than $10 + 6$. $10 + 6 = 16$. Therefore, $9 + 6 = 15$.” Have the students shade these sums.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9				13	14
5	6	7	8	9	10	11			14	15
6	7	8	9		11	12	13		15	16
7	8	9	10			13	14	15	16	17
8	9	10	11				15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

- g. Sums of 10: It is helpful for students to know the combinations of numbers whose sum is 10. Have the students shade these numbers.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10			13	14
5	6	7	8	9	10	11			14	15
6	7	8	9	10	11	12	13		15	16
7	8	9	10			13	14	15	16	17
8	9	10	11				15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

*TEACHER NOTE: You may want to mention that at this point only five facts remain to be learned. (There are 10 squares that have not been shaded, but the commutative property of addition reduces this number to 5.) There are two strategies that will help with these.

- h. A sum of 10 and How Many More: If a student knows that $7 + 3 = 10$, he/she can figure out that $7 + 4 = 11$ because 4 is one more than the 3 needed to make 10. Likewise, since $8 + 2 = 10$, $8 + 4 = 12$ because 4 is two more than the 2 needed to make 10, and $8 + 5 = 13$ because 5 is three more than the 2 needed to make 10. Have students shade in these sums.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11		12	13	14
6	7	8	9	10	11	12	13		15	16
7	8	9	10	11		13	14	15	16	17
8	9	10	11	12	13		15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20

BEST COPY AVAILABLE

- i. Take One, Add One to Make a Double: If 1 is taken from one addend and added to the other, the sum is not changed. This strategy applies to the two remaining facts as well as to others of the same configuration ($5 + 7$ and $8 + 6$ as well as $1 + 3$, $2 + 4$, $3 + 5$, $4 + 6$). Have students shade in these sums.



$$5 + 7$$

If 1 is taken from
the 7 and added
to the 5, the result
is $6 + 6$.



$$6 + 6$$

Likewise, $8 + 6$ becomes $(8 - 1) + (6 + 1) = 7 + 7 = 14$. Another way to solve these problems is to find the number that comes between the addends (7 comes between 8 and 6) and doubling it.

$$6 + 8$$

$$7 + 7 = 14$$

$$5 + 7$$

$$6 + 6 = 12$$

$$1 + 3$$

$$2 + 2 = 4$$

$$3 + 5$$

$$4 + 4 = 8$$

etc.

+	1	2	3	4	5	6	7	8	9	10
1	2	3	4	5	6	7	8	9	10	11
2	3	4	5	6	7	8	9	10	11	12
3	4	5	6	7	8	9	10	11	12	13
4	5	6	7	8	9	10	11	12	13	14
5	6	7	8	9	10	11	12	13	14	15
6	7	8	9	10	11	12	13	14	15	16
7	8	9	10	11	12	13	14	15	16	17
8	9	10	11	12	13	14	15	16	17	18
9	10	11	12	13	14	15	16	17	18	19
10	11	12	13	14	15	16	17	18	19	20



Problem Solving

Sequence

Materials: Counters

Directions: 1. State a multioperation problem like the example that follows. Have the students use counters to solve the problem.

Example:

Say, "We're going to start with 10. Subtract 5. Now add 1. Subtract 3. Add 7. What's your number?"

2. Have the students record the problem as they use the counters ($10 - 5 + 1 - 3 + 7 = \underline{\quad}$).
3. Give the students other problems to solve mentally.
4. Encourage the students to make up their own brainteasers to try on a partner.

Number Bank

Materials: "Number Bank" handouts (one per pair of students; blackline master follows this activity)
Calculators (one per pair of students)

- Directions:
1. Have students work in pairs. One will be student A and the other student B.
 2. Give each pair of students a Number Bank handout. Have the students take turns solving the addition and subtraction problems in their column. Explain that once a number pair has been used, it cannot be used again.
 3. Students can use calculators to do the computation. However, once they choose a number pair, they must use it. They cannot keep trying combinations on the calculator until they find one that they need. This stipulation will challenge students to use estimation skills.

4. Problems may be solved in consecutive order or random order. Student pairs should agree on the rules before beginning.
5. Variations: (a) Have all answers be the same (e.g., 9); (b) make all problems addition or all subtraction.



Number Bank

Student A

1. $\underline{\quad} + \underline{\quad} = 9$

2. $\underline{\quad} - \underline{\quad} = 8$

3. $\underline{\quad} + \underline{\quad} = 7$

4. $\underline{\quad} - \underline{\quad} = 6$

5. $\underline{\quad} + \underline{\quad} = 5$

6. $\underline{\quad} - \underline{\quad} = 4$

Student B

1. $\underline{\quad} + \underline{\quad} = 9$

2. $\underline{\quad} - \underline{\quad} = 8$

3. $\underline{\quad} + \underline{\quad} = 7$

4. $\underline{\quad} - \underline{\quad} = 6$

5. $\underline{\quad} + \underline{\quad} = 5$

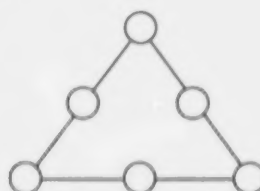
6. $\underline{\quad} - \underline{\quad} = 4$

Use with "Number Bank" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

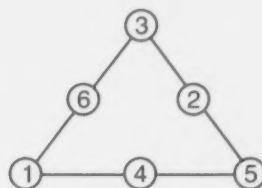
Triangle Problems

Materials: Numbered tiles or numbered squares of paper

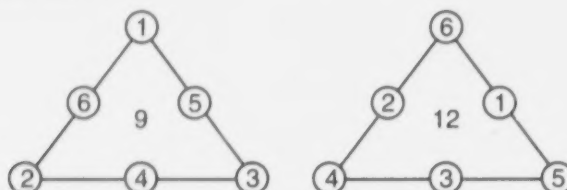
Directions: 1. Have the students draw six circles on a piece of paper in the shape of a triangle. Have them connect the circles with lines as shown below.



2. Give each student six 1" ceramic tiles or squares of paper numbered 1-6. Have the students try to arrange the numbers in such a way that each row of three has a sum of 10. Example:



3. Once students have found a solution using numbered tiles or squares, have them record that solution and then try to find another. Using of tiles or numbered squares encourages students to try different combinations. Using a pencil and having to erase often is discouraging and leads students to think that they are making mistakes rather than experimenting with different number combinations.
4. Variation: Have students also find solutions for sums of 9 and 12. Examples:



Encourage students to look for patterns and strategies when solving these problems (e.g., 9 is the smallest sum and the three smallest numbers are in the corners).

**W
H
A
T

I
S

M
Y

N
U
M
B
E
R
?**

Directions: Solve these problems. Write the mystery number on the line.

1. Add 5 to this number to get 10. _____
2. Subtract 6 from this number to get 2. _____
3. Add 3 to this number to get 12. _____
4. Subtract 2 from this number to get 13. _____
5. Add 7 to this number to get 14. _____
6. Subtract 1 from this number to get 15. _____
7. Add 0 to this number to get 12. _____
8. Subtract 3 from this number to get 17. _____
9. Add 8 to this number to get 16. _____
10. Subtract 5 from this number to get 4. _____



*TEACHER NOTE: Allow students to use counters to help them find the "Mystery Numbers." For an extension, you can have them write a number sentence to accompany each statement (e.g., $5 + \square = 10$ or $10 - 5 = 5$ for problem 1).

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Probability Number Choices

Materials: Counters (beans, UNIFIX cubes, etc.)
 Addition or subtraction flash cards
 Overhead projector, blank transparency, and marker

- Directions:**
1. Have each student make 16 squares on a piece of paper.
 2. Have the students write numbers from 0 to 20, if playing addition, or from 0 to 10, if playing subtraction, in each square. Numbers may be used more than once.

Examples:

Subtraction				Addition			
10	4	7	0	9	18	7	8
8	1	4	3	10	15	2	1
9	5	2	8	14	12	4	16
0	3	6	5	6	15	3	10

3. Ask for a student volunteer to be the caller. Using either the addition or subtraction fact cards (depending on the game being played), the caller should call out facts one at a time. If a student has the number that is the sum or difference, he/she should cover that number. Only one number may be covered at a time. The students can use counters to help them find solutions to facts they cannot recall from memory.
4. As each fact is called, the caller should record the problem and answer on a transparency that can be used for three games. (The transparency will not be shown on the overhead until the end of each game.)

Game 1	Game 2	Game 3
$5 + 4 = 9$		
$3 + 2 = 5$		
$4 + 6 = 10$		
:		

5. The game is over when one student gets four in a row vertically, horizontally, or diagonally.
6. After the game is over, the students should look at the problems on the transparency. Ask them what they notice

about the sums or differences. Ask, "Would anyone like to make a new card before we play the next game?"

7. Mix the fact cards and repeat the game.
8. When the game is over, show the problems on the overhead. Repeat the questioning in #6 above.
9. Have the students play a third game.
10. Ask, "Did any of you change your cards?" (Have the students share why they did/did not change their cards.)
11. Ask, "Do you think you have a better chance to win if you use certain numbers?" (Have the students share the numbers that they think improve their chances.)
12. Discuss with the students that oftentimes **chances** can be improved if you have more information.
13. Ask, "If we had been playing with subtraction (addition) instead of addition (subtraction), do you think there would be numbers that we could use to increase our chances of winning?"
14. Follow the same procedure, having the students play three games using the other operation.
15. Variation: When a fact is given, have the students cover all of the numbers that name the sum or difference (e.g., if a student has three 7s on his/her paper and $3 + 4 = \underline{\quad}$ is called, all three 7s can be covered in one turn). Discuss the choice of numbers on the card.

SKILL Describing missing addends using the inverse relation of addition and subtraction

Skill Analysis

- A. Understands the symbols "+," "-", and "=" as they are used in number sentences
- B. Understands the concepts of addition and subtraction
- C. Understands that there is a relationship between addition and subtraction



Concept Manipulative Activities

Related Sentences

Materials: Counters (UNIFIX cubes, beans, buttons)

- Directions:
1. Give each student some counters.
 2. Ask the students to model the problem $2 + 5 = \square$ and find the answer.
 3. Have a student explain how he/she got the answer. (set of two combined with a set of five)
 4. Ask, "What do you think you will get if a set of five is removed?" (2)
 5. Write the sentence $7 - 5 = \square$ on the chalkboard.
 6. Continue with other problems and the inverses.
- Examples:
- a. A set of seven combined with a set of four is $7 + 4 = 11$.
Remove a set of four and you have $11 - 4 = 7$.

- b. Start with a set of nine. Remove a set of three, and you have $9 - 3 = 6$. Combine a set of six with a set of three and you have $6 + 3 = 9$.
7. Discuss that addition and subtraction "do and undo" each other. The number you have when you start is the number you have after doing a problem and its inverse. (Example: $7 + 4 = 11$, $11 - 4 = 7$. The operations $+4$ and -4 undo each other.)

Make It Balance

Materials: Primary balance scale
Counters (UNIFIX cubes, washers, chips, etc.)

- Directions:
1. Put 5 objects on one side of the balance and 12 on the other. The objects should be uniform in weight.
 2. Tell the students that they are going to use the scale to solve the problem $5 + \underline{\quad} = 12$. Ask, "How can we get the scale to balance?"
 3. Have the students add items to the side with 5 objects until the scale is balanced.
 4. Then have them complete the sentence $5 + \underline{\quad} = 12$.
 5. Repeat the activity with different problems (e.g., $3 + \underline{\quad} = 5$, $2 + \underline{\quad} = 11$, $\underline{\quad} + 4 = 9$).

Same Number

Materials: None

- Directions:
1. Tell the class a hypothetical situation or story such as the following:
"Today is Saturday and I'm going to the park to watch a soccer game. I see the Kickers team has all its players on the field. (Select 11 students to be the Kickers team and have them stand on one side of the room.) But so far the Strikers team has only 7 players. (Select 7 students to form the Strikers team. Have them stand on the other side of the

- room.) How many more players does the Strikers team need to be able to play the game?"
2. Ask the students how they arrived at their answer. There are two ways the students might respond: (a) counting up from 7 or (b) subtracting 7 from 11.
 3. Write these two sentences on the board:
 - a. $7 + \underline{\quad} = 11$. This is the missing addend approach.
 - b. $11 - 7 = \underline{\quad}$. This is the subtraction approach.
 4. This exercise will help the students model missing addends and solve for missing addends by subtracting. Both strategies are useful.
 5. Repeat the activity by describing other real-life situations (e.g., baseball teams, volleyball teams, basketball teams, relay teams, square dance squares).

What's My Fact?

Materials: Counters (UNIFIX cubes, blocks, beans, etc.)

- Directions:**
1. Have the students make a set of six counters and a set of eight counters. Have the students join the two sets. Write the equation $6 + 8 = 14$ on the board.
 2. Call on a student volunteer to model the inverse, or related, subtraction equation and write the subtraction equation underneath the addition equation. Example:

$$6 + 8 = 14$$

$$14 - 6 = 8 \text{ or } 14 - 8 = 6$$
 3. Have the students create other addition problems. Call on volunteers to demonstrate and write the related facts.
 4. Both vertical and horizontal problems should be presented.

In the Bag

Materials: Counters (UNIFIX cubes, blocks, beans, etc.)
Paper bag

- Directions:**
1. Place nine objects on a table at the front of the room for students to observe. Have the children put their heads down on their desks so that they can't see what you are going to do next.
 2. Place five of the objects in a paper bag.
 3. Have the children raise their heads. Say, "There are now four objects on the table. How many are in the bag?"
 4. When a student replies, ask, "How did you find the answer?" Write the sentence $9 = 4 + \underline{\quad}$, or $9 - \underline{\quad} = 4$, or $9 - 4 = \underline{\quad}$ on the board depending on the response. Also write the addition and inverse subtraction sentence ($4 + \underline{\quad} = 9$ and $9 - 4 = \underline{\quad}$ or $5 + \underline{\quad} = 9$ and $9 - 5 = \underline{\quad}$).
 5. Repeat the activity with other number sentences.
 6. Variation: Use beans on the overhead.



Let's Practice

Bag Balance

Materials: Counters of the same size (UNIFIX cubes, washers, etc.)
Paper bags (one per group of students)
Primary balance scales (one per group of students)

- Directions:**
1. Show the students how to model a number sentence on the balance. Example: Write $6 + ? = 13$ on the board and set up a balance that looks like the following at the front of the room.



(You'll need to have 7 counters in the bag. You may need to place an empty bag next to the 13 to get the scale to balance.) Have each student write the sentence $6 + \underline{\quad} = 13$ on a piece of paper.

2. Select several student volunteers to solve the problem using the scale and counters.
3. The students may (a) remove the bag and add counters to the scale, (b) remove 6 counters from each side so the scale will balance, or (c) remove all of the counters, put the bag back on the scale, and add counters to the other side until the scale balances.
4. Have the students write down the true sentence ($6 + \underline{7} = 13$).
5. Discuss the strategies and write related sentences on the board ($6 + \underline{\quad} = 13$, $13 - 6 = \underline{\quad}$, $13 - \underline{\quad} = 6$).
6. Have the students form small groups.
7. Give each group a balance and a paper bag on which you have written a number sentence.
8. Have the students follow the procedure you modeled to solve the number sentences.
9. Rotate the bags among the groups and repeat the procedure.

Universal Operations

Materials: 3" x 5" cards or construction paper cut into strips
 Universal Game Board (one per group of students; directions for its construction may be found in the Appendix)
 Game markers
 Counters for students who want to use them

Directions: 1. Give each student a 3" x 5" card or construction paper strip.
 2. Say, "Write a number sentence with a missing addend on one side of the card (or construction paper) and the answer on the back." Example:

$5 + \square = 12$

side 1

7

back side

3. Divide the class into groups of three to five students.
4. Give each group a Universal Game Board. Have the students in each group mix their cards together and put them in a pile, problem side up, on the board.
5. Each student will, in turn, select a card, write the problem on a piece of paper, and write the answer beside the problem.
6. The student will then turn the card over and check the result. If the answer is correct, the student will move a space on the game board.
7. The game continues until one person reaches the finish or until an allotted amount of time expires.

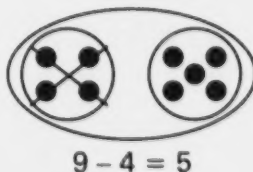
Flash Card Practice—Addition

Materials: Addition flash cards
Counters

Directions: 1. Demonstrate how to write an inverse operation sentence when given an addition problem. Example:

$$\begin{array}{r} 4 \\ + 5 \\ \hline 9 \end{array} \quad \text{subtraction:} \quad \begin{array}{r} 9 \\ - 4 \\ \hline \square \end{array} \quad \text{or} \quad \begin{array}{r} 9 \\ - 5 \\ \hline \square \end{array}$$

2. Write the addition problem on the board. Say, "Explain how this addition problem can be used to help solve this subtraction problem." (Response: If 4 and 5 combined gives 9, then taking 4 away should leave 5.) Model the problem using counters.



3. Divide the students into small groups. Put a pile of flash cards on each group's table, problem side up.
4. Have the students take turns removing the top card, finding the answer using counters if necessary, writing down the problem and answer, writing down an inverse

subtraction problem, and explaining how the addition problem can be used to solve the subtraction problem.
Example:

$$\boxed{\begin{array}{r} 2 \\ + 7 \\ \hline \end{array}} \quad \begin{array}{r} 2 \quad 9 \\ + 7 \quad - 2 \\ \hline 9 \quad 7 \end{array}$$

"If 2 and 7 is 9, then taking 2 away from 9 will leave 7."

5. The students may use counters to demonstrate the process.



Problem Solving

Strategies for Problems

Materials: "Story Problems" handouts (blackline master follows this activity)

- Directions:
1. Distribute a "Story Problems" handout to each student or small group of students.
 2. Read the first problem to the students. Have the students discuss the problem and write an addition sentence **and** a subtraction sentence for the problem.
 3. Discuss the strategies that the students used. Encourage the following strategies: drawing a picture, using manipulatives to make a model, and acting out the problem.
 4. Discuss the interpretation of the following number sentences:
 $12 = \underline{\quad} + 5$ and $12 - 5 = \underline{\quad}$
 Total = Kathy's and Mary's Total less Mary's is Kathy's
 Explain that both sentences can be solved by knowing that $5 + 7 = 12$. Ask, "Why can you solve these problems knowing that $5 + 7 = 12$?"

5. Point out to the students that a subtraction problem can be solved if you know the addition fact.
6. Repeat steps 2-5 for each problem on the handout.
7. Answers: 1a. $5 + \underline{(7)} = 12$, 1b. $12 - 5 = \underline{(7)}$; 2a. $3 + \underline{(5)} = 8$, 2b. $8 - 3 = \underline{(5)}$; 3a. $4 + \underline{(6)} = 10$, 3b. $10 - 4 = \underline{(6)}$; 4a. $6 + \underline{(8)} = 14$, 4b. $14 - 6 = \underline{(8)}$; 5a. $10 + \underline{(5)} = 15$, 5b. $15 - 10 = \underline{(5)}$; 6a. $8 + 10 = \underline{(18)}$, 6b. $\underline{(18)} - 8 = 10$
8. Variation: Have the students write their own problems and exchange them with other students.

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

Directions: Write two sentences for each problem. Make one an addition sentence, like $5 + \underline{\quad} = 7$, and one a subtraction sentence, like $7 - 5 = \underline{\quad}$. Solve each problem.

1. Kathy and Mary brought 12 fish to class. If Mary brought 5 fish, how many did Kathy bring?

a. $\underline{\quad} + \underline{\quad} = \underline{\quad}$ b. $\underline{\quad} - \underline{\quad} = \underline{\quad}$

2. Barb and Fran sold 8 candy bars. If Barb sold 3, how many did Fran sell?

a. $\underline{\quad} + \underline{\quad} = \underline{\quad}$ b. $\underline{\quad} - \underline{\quad} = \underline{\quad}$

3. Dean has 4 marbles. On his birthday, he received more marbles. Now Dean has 10 marbles. How many marbles did he receive for his birthday?

a. $\underline{\quad} + \underline{\quad} = \underline{\quad}$ b. $\underline{\quad} - \underline{\quad} = \underline{\quad}$

4. Marcy baked 14 cookies. She gave some of the cookies to her friends. Now she has 6 cookies left. How many cookies did she give away?

a. $\underline{\quad} + \underline{\quad} = \underline{\quad}$ b. $\underline{\quad} - \underline{\quad} = \underline{\quad}$

5. Tony and Jason are coaching a baseball team. The team has 15 baseballs. If Tony takes 10 baseballs for batting practice, how many does Jason have for fielding practice?

a. $\underline{\quad} + \underline{\quad} = \underline{\quad}$ b. $\underline{\quad} - \underline{\quad} = \underline{\quad}$

6. Devin has some comic books. He reads 8 of them. He has 10 left to read. How many does he have in all?

a. $\underline{\quad} + \underline{\quad} = \underline{\quad}$ b. $\underline{\quad} - \underline{\quad} = \underline{\quad}$



Use with "Strategies for Problems" activity. The students may use counters or other models. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Multiple Missing Addends

Materials: Manipulatives (UNIFIX cubes, beans, etc.)

- Directions:
1. Write the problem $\diamond + \square + 4 = 10$ on the board.
 2. Make a chart like the following beneath the problem.

$\diamond$	+	$\square$	+	4	=	10

3. Have the students find as many answers to the problem as possible. They may use counters if they want to.
 4. Call on volunteers to write the responses on the chart.
- Example:

$\diamond$	+	$\square$	+	4	=	10
0		6		4		10
4		2		4		10
1		5		4		10
3		3		4		10
2		4		4		10
5		1		4		10
6		0		4		10

5. When all of the possibilities have been explored, repeat the activity with another problem. Examples:

$$\begin{array}{rclclcl} \diamond & + & 2 & + & \bigcirc & = & 7 \\ \square & + & 2 & - & \triangle & = & 6 \end{array}$$

SKILL Describing answers for basic addition and subtraction facts

Skill Analysis

- A. Understands the concepts of addition and subtraction
- B. Understands the symbols for addition and subtraction
- C. Can memorize



Concept Manipulative Activities

Domino Addition and Subtraction

Materials: Dominoes or domino cutouts (blackline master may be found in the Appendix)
 Counters for scorekeeping
 Wooden cubes with "+" on three sides and "-" on three sides (one per group of students)

- Directions:**
1. Have the students work with partners or in small groups.
 2. Put a pile of dominoes or domino cutouts face down on each group's playing surface.
 3. Have one student in each group roll the cube to determine the operation and then draw a domino.
 4. Depending on the operation rolled, the student will add the number of dots in each section or subtract the number of dots on one side from the total number of dots. Examples:

+	••••• •••	→ "5 + 3 = 8"
-	••••• •••	→ "8 - 5 = 3" or "8 - 3 = 5"

5. If the answer is correct, the student will take one counter.
6. The game ends when all of the dominoes have been used.
7. The winner is the student with the most counters.

Hand Calculations

Materials: UNIFIX cubes

"Hand Calculations" handout (one per student; blackline master follows this activity)

- Directions:
1. Give each student up to 20 UNIFIX cubes and a copy of the handout.
 2. Have the students join their cubes together to form a "train." Then have them split their train anywhere, holding one set in their left hand and the other set in their right.
 3. The students should count the cubes in their left hand and record the number in the first blank on the handout.
 4. The students should count the cubes in their right hand and record the number in the second blank on the handout.
 5. Have the students complete the first number sentence on the handout and then write a related subtraction number sentence (e.g., for 15 in the left hand and 5 in the right, the student could write $15 + 5 = 20$ and $20 - 15 = 5$).
 6. Have the students put their cubes back together, split them in a different way, and record the new number sentences.
 7. Have them repeat the activity until time is up or all possible combinations have been determined. (Remind the students to use 0!)



Hand Calculations

				
left	+	right	=	sum
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____
_____	+	_____	=	_____

				
total	-	left	=	right
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____
_____	-	_____	=	_____

Use with "Hand Calculations" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Number Cards

Materials: Deck of playing cards with face cards removed (one deck per group)

- Directions:**
1. Groups of two to four students may play. Each student is dealt four cards.
 2. For each group, four other cards are turned face up on the table.
 3. The first player in each group tries to match a card on the table with a card of the same number in his/her hand. If a match can be made, the student puts both cards face down in front of his/her place.
 4. If a match cannot be made, the student may add or subtract two cards on the table to match a card in his/her hand (e.g., a 3 and a 4 on the table can be matched with a 7, or a 10 and a 3 [$10 - 3$] on the table can be matched with a 7).
 5. If no match can be made, the student must put one card from his/her hand face up on the board.
 6. The next player repeats steps 3-5.
 7. When all players are out of cards, four more cards are dealt to each. The game continues until time is up.
 8. At the end of the game, the player with the most matches wins.



Let's Practice

Facts Dictation Records or Tapes

Materials: Commercial tapes or records that dictate addition and subtraction problems at a set pace

- Directions:**
1. Play the tape or record. Have the students write the number sentence for each dictated fact.

140

2. Have the students check their answers with the answer section of the record or tape.

**TEACHER NOTE:* You can make your own addition and subtraction tapes or ask a volunteer parent to do so.

Horse Race

Materials: "Horse Race—Addition" and "Horse Race—Subtraction" handouts (blackline masters follow this activity)

- Directions:**
1. Pass out the handout "Horse Race—Addition," "Horse Race—Subtraction," or both.
 2. Tell the students a rule for Race #1 (e.g., "Add 4"). Have the students write the rule on the handout on the line following the words "RACE #1."
 3. Explain that the students will apply this rule to each number as they make their way around the track (e.g., for "7" on the track, the students will add 4 and write 11 in the box).
 4. Everyone will start at your signal.
 5. Have the students raise their hands as they finish. Tell them their finishing position.
 6. When all of the students have finished, give the correct answers so the students can check their work.
 7. The goal for students is to "beat" their previous finishing positions.



Horse Race—Addition

RACE TRACK

RULE

RACE #1 _____

RACE #2 _____

RACE #3 _____

FINISH POSITION

RACE #1 _____

RACE #2 _____

RACE #3 _____



←3

←2

←1

FINISH

START

Race #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1																														
2																														
3																														
4																														
5																														
6																														
7																														
8																														
9																														
10																														
11																														
12																														
13																														
14																														
15																														
16																														
17																														
18																														
19																														
20																														
21																														
22																														
23																														
24																														
25																														
26																														
27																														
28																														
29																														
30																														

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Horse Race—Subtraction

RACE TRACK

FINISH

START

← 3
← 2
← 1

Race #

RULE

RACE #1 _____

RACE #2 _____

RACE #3 _____

FINISH POSITION

RACE #1 _____

RACE #2 _____

RACE #3 _____

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Problem Solving

Story Problems

Materials: None

- Directions:
1. Have the students write addition or subtraction story problems to share with the class. Each student should furnish the key for his/her problem(s).
Example: I have a bag of candy. I give one friend six pieces and another friend eight pieces. If I have three pieces left, how many did I have altogether?
 2. Extension: Staple the students' story problems together to make a classroom story problem book.

SKILL Describing missing addends using the inverse relation of addition and subtraction

Skill Analysis

- A. Can model addition and subtraction problems with regrouping
- B. Understands the relationship between addition and subtraction
- C. Understands the symbols "+," "-", and "="

*TEACHER NOTE: The students need to have completed regrouping for addition and subtraction for this skill. See 2.3-1 for regrouping skill.

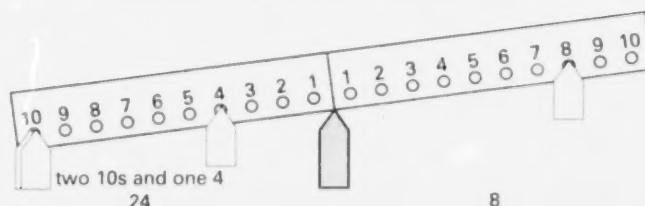


Concept Manipulative Activities

Invicta Balance

Materials: Invicta balance and weights

Directions: 1. Put weights on the Invicta balance to represent 24 on the left side and 8 on the right side. Example:



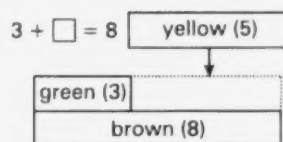
- 2. Ask, "What number is represented on the left side?" (24)
"What number is represented on the right side?" (8)
- 3. Ask, "How can we get the scale to balance?"
- 4. Write the sentence $24 = 8 +$ on the board.

5. Start adding weights to the right side of the scale. Continue until the scale is balanced (e.g., $10 + 6$, or 16, will balance the scale).
6. Show the students that if $24 = 8 + \square$, they can find the answer by subtracting: $24 - 8 = \square$.
7. Repeat steps 1-6 with different numbers.

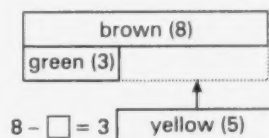
Cuisenaire Rods

Materials: Cuisenaire rods

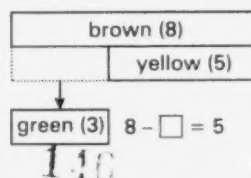
- Directions:
1. Give the students the equation $3 + \square = 8$.
 2. Have each student take a brown rod (8) and a light green rod (3).
 3. Have the students lay the light green rod side by side with the brown rod, as shown below, and then find a rod that will fill the empty space. They should select the yellow rod (5).
Example:



4. By putting the yellow rod in the space, the students demonstrate that $3 + 5 = 8$.
5. Have the students remove the yellow rod. Help them understand that by doing so they are demonstrating that $8 - 5 = 3$. Example:



6. Have the students replace the yellow rod and then remove the light green rod to demonstrate that $8 - 3 = 5$. Example:



7. After providing several experiences of working with the rods, have the students write down the related addition and subtraction equations (e.g., $\square + 5 = 8$ and $8 - 5 = \square$).
8. Variation: Graph paper can be used instead of Cuisenaire rods. For $8 - 5 = \square$, the students would color in eight squares on one row and five on the row below. By counting the blank squares in the second row that are under the eight squares they have colored in, the students can demonstrate that three squares are left.

Boxes

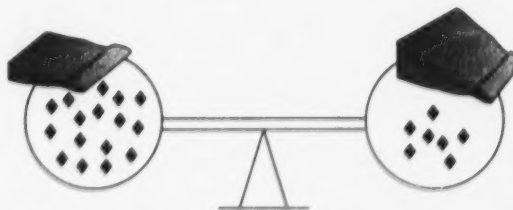
Materials: Two small boxes (one with a lid, one without)
Ten objects small enough to fit into the boxes

- Directions:**
1. Working where the students cannot see what you are doing, put 7 objects in the box without a lid and 3 in the box that has a lid.
 2. Cover the box with the 3 objects.
 3. Show both boxes to the students. Let them count the 7 objects in the uncovered box.
 4. Tell the students that there are 10 objects altogether in both boxes.
 5. Ask them to find how many objects are in the covered box without removing the cover. ($7 + \square = 10$)
 6. When a correct answer is given, ask the student to tell how he/she got that answer. (Possible response: $10 - 7 = \square$.)
 7. Have the class brainstorm the different ways to find a missing addend (e.g., knowing the addition facts, using counters, subtracting the first addend from the total).
 8. Repeat the activity using larger numbers.

Balance Scale

Materials: A primary balance scale
 Counters (UNIFIX cubes, beans, chips, etc.)
 Two bags of the same size (cloth or paper)

- Directions:**
1. Working where the students cannot see what you are doing, put 9 counters in a paper or cloth bag.
 2. Set up the balance scale with the bag of 9 counters and 7 other counters on the right side, and an empty bag (laid flat) and 16 counters on the left side. The scale should balance.



3. Ask the students to write a number sentence that represents this model (e.g., $16 = \quad + 7$).
4. Ask the students how they can find the number of counters in the bag.
5. After several responses are given, "take away" one counter from each side until the right side is holding only the bag of counters (take away 7).
6. Write the sentence $16 - 7 = \quad$ on the board. Explain that since there are 9 counters remaining on the left side, $\quad = 9$.
7. Have a student open the bag and count to see if there are 9 counters inside.
8. Write the two sentences $16 = \quad + 7$ and $16 - 7 = \quad$ on the board, explaining that they are equivalent and that the subtraction undoes the addition.
9. Repeat the activity using larger numbers (e.g., $\quad + 26 = 63$, $47 - \quad = 16$).

Missing UNIFIX Cubes

Materials: Two colors of UNIFIX cubes or other colored counters (beans, sticks, etc.)

- Directions:**
1. Give each student a piece of paper on which you have written the number sentence $9 + \square = 14$.
 2. Using one color of UNIFIX cubes or other counters, each student should place the appropriate number of counters below the numerals that have been given.
 3. Have the students use counters of another color to build up the set of 9 until it is equal to the set of 14. The students should have added 5 counters.
 4. Have the students write the sentence $9 + 5 = 14$.
 5. Variation: Have the students solve this problem by removing 9 from the set of 14 and looking at how many counters remain.
 6. Repeat the activity with other problems.
Examples: $7 + \square = 16$, $13 + \square = 30$

Cover Domino

Materials: Dominoes or domino cutouts (blackline master may be found in the Appendix)

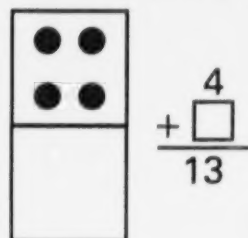
- Directions:**
1. Give each child a domino or domino cutout that has one side covered and a piece of paper on which you have written a number sentence that corresponds to the domino, such as $4 + \square = 13$.



$$4 + \square = 13$$

2. Have the students place dots on the blank side until they get the total indicated in the number sentence.
3. The students should then count the dots and complete the sentence.

4. Repeat the activity using other dominoes.
5. The activity may be done with the dominoes (and number sentences) in horizontal or vertical form.



Let's Practice

Are We Inverses?

Materials: None

- Directions:
1. Use the students to generate missing addend problems. Example: Have all of the girls come to the front of the classroom. Have those girls with blue eyes move to one side of room. Write a missing addend sentence on the board. (If there are 17 girls and 6 have blue eyes, you could write $\quad + 6 = 17$.) Have the girls find the missing number. Then write the inverse sentence on the board (e.g., $11 + \quad = 17$).
 2. Pose other problems for the students to act out.
 3. After a few examples, have the students attempt to write the equation before the situation is displayed.

Universal Addend

Materials: Universal Game Board (one per group of students; directions for its construction may be found in the Appendix)

Game markers

Game cards

Example:

Back	Front
7	$\square + 2 = 9$
12	$15 + \square = 27$

- Directions:
1. Divide the class into groups of two to four students. Give each group a game board and markers. Put a pile of game cards on each board, problem side up.
 2. In turn, each student in a group should draw a card and read the problem aloud. That student should then state the missing addend.
 3. If the student is correct, he/she should move one space on the game board. If the student's answer is incorrect, the student should not advance on the game board. (The students should self-check their answers by turning the cards over.)
 4. The game continues until the first student reaches the finish line.

Go Fish! Game

Materials: Game cards

Example:

$9 + \square = 24$	15
One card	Another card

- Directions:
1. Make a set of game cards. Put missing addend problems on some cards (one per card) and the answers on other cards (again, one per card).

2. Deal seven cards to each player. Have each player, in turn, ask another student for the sentence card or answer card that matches one of his/her cards (e.g., "I have $9 + \quad = 24$. Mary, do you have a 15 card?").
3. If the player questioned has the card, he/she must give it to the requesting player, who then puts the pair on the table and takes another turn.
4. If the player questioned does not have the card, he/she tells the requesting player, "Go fish!" The requesting player then draws a card from the Fish Pond of excess cards. Play continues until one player depletes his/her hand.
5. The students need to listen carefully. For the example above, if Mary did not have a 15 card, the person with the 15 card would know to ask for the $9 + \quad = 24$ card when it is his/her turn.

Let's Check It Out

Materials: None

- Directions:
1. Discuss with students the application of inverse equations in solving missing addend problems.
 2. Put inverse problems (vertically) on the board and highlight the subtraction procedure. Example:

$$\begin{array}{r} 9 \\ + \square \\ \hline 15 \end{array} \longrightarrow \begin{array}{r} 15 \\ - 9 \\ \hline 6 \end{array} \longrightarrow \begin{array}{r} 9 \\ + \boxed{6} \\ \hline 15 \end{array}$$

3. Give students other missing addend problems. Again, show how inverse equations can be used to solve them. Examples:

$$\begin{array}{r} 8 \\ + \square \\ \hline 12 \end{array} \quad \begin{array}{r} \square \\ + 3 \\ \hline 11 \end{array} \quad \begin{array}{r} 6 \\ + \square \\ \hline 13 \end{array} \quad \begin{array}{r} \square \\ + 4 \\ \hline 9 \end{array}$$

4. Give the students other missing addend problems. Have them write and solve the subtraction sentences.



Problem Solving

Tell Me a Story

Materials: 9" x 12" pieces of tagboard
Magazines
Scissors
Glue

- Directions:
1. Have students use pictures from magazines to make one or more picture cards. Each card should have at least 10 pictures of a certain set (e.g., dogs, toys, people, cars, or fruit). A picture may have more than one member of the set.
 2. Have each student choose two cards and create a story problem that involves joining the sets and then "undoing" the action. Example:
There are nine dogs and five cats playing in the park. How many animals are there altogether? If the dogs chased the cats away, how many animals are left?
 3. Have the students exchange their stories. Have them record the number sentence and inverse equation for the story they have been given and check their answers with the story's author.

Are You Checking Up on Me?

Materials: None

- Directions:
1. Provide students with addition and subtraction problems and their "checks." Make several of the problems incorrect. Examples:

$\begin{array}{r} 37 \\ +59 \\ \hline 86 \end{array}$	Check:	$\begin{array}{r} 86 \\ -59 \\ \hline 37 \end{array}$	Check:	$\begin{array}{r} 42 \\ -19 \\ \hline 24 \end{array}$	Check:	$\begin{array}{r} 24 \\ +19 \\ \hline 42 \end{array}$	Check:	$\begin{array}{r} 68 \\ +19 \\ \hline 87 \end{array}$	Check:	$\begin{array}{r} 87 \\ -19 \\ \hline 68 \end{array}$
---	--------	---	--------	---	--------	---	--------	---	--------	---

2. Have the students find the incorrect problems. Have them correct those problems as well as their checks.
3. Instruct the students to generate their own set of problems and checks, some of which are incorrect. Have them exchange problems and correct the ones they have been given.

Are They Correct?

Materials: None

- Directions:
1. Provide students with missing addend problems and an answer for each. Make some of the answers incorrect.
 2. Have the students find the incorrect answers. They should correct the problems by providing the correct answer.
 3. Examples:

$$\begin{array}{r}
 \square \\
 + 9 \\
 \hline
 14
 \end{array}
 \quad
 \begin{array}{r}
 13 \\
 + \square \\
 \hline
 27
 \end{array}
 \quad
 \begin{array}{r}
 6 \\
 + \square \\
 \hline
 8
 \end{array}
 \quad
 \begin{array}{r}
 \square \\
 + 2 \\
 \hline
 21
 \end{array}
 \quad
 \begin{array}{r}
 \square \\
 + 16 \\
 \hline
 33
 \end{array}
 \quad
 \begin{array}{r}
 2 \\
 + \square \\
 \hline
 6
 \end{array}$$

Answers: (3) (4) (3) (19) (13) (5)

4. Have the students generate their own problems with some incorrect answers. Have them exchange problems and correct the ones they've been given.

Calculator Check

Materials: Calculator (one per pair of students)
Overhead projector
Blank acetate

Directions: 1. Put the following problems on the overhead.

$$\begin{array}{r} \square \\ + 12 \\ \hline 27 \end{array} \quad \begin{array}{r} 6 \\ + \square \\ \hline 9 \end{array} \quad \begin{array}{r} \square \\ + 22 \\ \hline 41 \end{array} \quad \begin{array}{r} \square \\ + 3 \\ \hline 16 \end{array} \quad \begin{array}{r} 6 \\ + \square \\ \hline 14 \end{array} \quad \begin{array}{r} 12 \\ + \square \\ \hline 38 \end{array} \quad \begin{array}{r} 6 \\ + \square \\ \hline 15 \end{array}$$

$$6 + \square = 17 \quad 7 + \square = 14 \quad \square + 12 = 26$$

$$3 + \square = 7 \quad 15 + \square = 41 \quad \square + 1 = 19$$

$$10 + \square = 18 \quad \square + 6 = 6 \quad \square + 25 = 53$$

2. Have the students work in pairs with the partners taking turns working the problems and checking the answers with the calculator.
3. The student working the problem should give an answer. The student using the calculator should check the answer by plugging in the number his/her partner gave. Example: For $3 + \square = 7$, if the student working the problem gives the answer 4, the other student would add $3 + 4$ on the calculator to see if the answer is correct.
4. If the answer is correct, the students should exchange roles. If the answer is incorrect, the student working the problem should give another possible answer.

More Missing

Materials: Counters if students wish to use them

- Directions:
1. Give the students a problem with two missing addends (e.g., $6 + \square + \Delta = 14$).
 2. Have the students find at least three different ways the problem could be solved. For the example above, they might write:

$\square$	Δ
2	6
1	7
4	4

3. Repeat the activity with other problems. Examples:

$$\square + \Delta + 4 = 14$$

$$8 + \square + \Delta = 20$$

$$9 - \square + \Delta = 8$$

$$\square + \Delta - 4 = 9$$

156

STORY PROBLEMS

Directions: Write a missing addend equation for each word problem and then solve the problem by writing the inverse sentence. Example: Jim and Mary have 10 bags of apples with a total of 68 apples. If Mary had 28 apples, how many did Jim have? $68 = 28 + \square$; $68 - 28 = \square$; 40 apples.

1. Nick can invite 14 boys to his ninth birthday party. So far he has invited Monty, Ben, and Grant. How many more boys can Nick invite?
2. Becky's supply list said she needed 16 sheets of notebook paper. She looked all over the house but found only 9 sheets. She also found 7 pencils. How much more paper does she need?
3. Cameron started the school year with 24 crayons. By November 15 he had 16 crayons left. How many crayons would he need to replace the missing ones?
4. Angela and Jan are going to buy a toy car for 18¢. Angela has 10¢. If Angela and Jan have 19¢ together, how much money does Jan have?



Assessment Activities

SKILL Solving addition and subtraction problems in both horizontal and vertical form through 20

1.2-1

Have the students write the answers for these problems. They may use counters or a number line to help them.

1.
$$\begin{array}{r} 8 \\ + 2 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 9 \\ + 5 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 9 \\ + 5 \\ \hline \end{array}$$

4. $12 - 7 = \underline{\quad}$

5. $6 + 3 = \underline{\quad}$

6. $18 - 9 = \underline{\quad}$

7. $6 + 2 = \underline{\quad}$

8.
$$\begin{array}{r} 11 \\ - 3 \\ \hline \end{array}$$

SKILL Describing answers for basic addition and subtraction facts

2.2-2

Have the students write the answers for these problems. They may use counters.

1.
$$\begin{array}{r} 7 \\ + 8 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 6 \\ + 7 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 8 \\ + 9 \\ \hline \end{array}$$

4. $9 + 6 = \underline{\quad}$

5. $7 + 5 = \underline{\quad}$

6. $9 + 7 = \underline{\quad}$

7.
$$\begin{array}{r} 15 \\ - 6 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 14 \\ - 8 \\ \hline \end{array}$$

9.
$$\begin{array}{r} 17 \\ - 9 \\ \hline \end{array}$$

10. $16 - 7 = \underline{\quad}$

11. $14 - 5 = \underline{\quad}$

12. $15 - 9 = \underline{\quad}$

SKILL Describing missing addends using the inverse relation of addition and subtraction

2.2-3

Materials: Counters (UNIFIX cubes, beans, bottle caps, etc.)
Cards containing missing addend problems (four per card)

- Directions: 1. Give each student 20 counters and an activity card.
2. Each activity card should have four problems. Examples:

$$2 + \square = 11$$

$$7 + \square = 15$$

$$\square + 6 = 13$$

$$8 - \square = 4$$

$$15 - \square = 7$$

$$\square - 6 = 5$$

3. Say, "Show me how to do this addition problem" (point to $\square + 6 = 13$).
4. Say, "Show me how this subtraction problem" (point to $8 - \square = 4$).
5. Continue until all students have explained how to do a problem.

SKILL Describe answers for basic addition and subtraction facts

3.2-4

Have the students solve the following problems.

1. $6 + 7 = \underline{\quad}$

2. $8 + 3 = \underline{\quad}$

3.
$$\begin{array}{r} 9 \\ + 8 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 5 \\ + 6 \\ \hline \end{array}$$

5. $12 - 5 = \underline{\quad}$

6. $16 - 9 = \underline{\quad}$

7.
$$\begin{array}{r} 13 \\ - 5 \\ \hline \end{array}$$

8.
$$\begin{array}{r} 17 \\ - 9 \\ \hline \end{array}$$

159

SKILL Describing missing addends using
the inverse relation of addition
and subtraction

3.2-5

Have the students solve the following problems.

1. $\square + 3 = 12$

2.
$$\begin{array}{r} 22 \\ + \square \\ \hline 66 \end{array}$$

3. $92 = 47 + \square$

4.
$$\begin{array}{r} \square \\ + 72 \\ \hline 73 \end{array}$$

5. $\square - 9 = 16$

6. $26 - \square = 14$

7. $39 - 17 = \square$

8. $57 - \square = 28$

SKILL Regrouping to solve addition and subtraction problems through 99

Skill Analysis

- A. Understands the concepts of addition and subtraction
- B. Can find the answer to basic addition and subtraction facts
- C. Understands the concept of place value
- D. Can model two-digit numbers
- E. Understands that an equation written horizontally can be transformed into a vertical equation without changing the result

***TEACHER NOTE:** The amount of time and types of experiences needed by students for mastery of this skill will vary. Research suggests that once students have a grasp of the concept of regrouping, frequent but short reviews are necessary. Using short reviews will help prevent burnout on the part of both teachers and students and at the same time will help ensure a higher rate of success for students. Base 10 materials are used in many of the activities in this objective. Like any other manipulatives, they can be classified by degree of "concreteness" or "abstractness." Materials that can be taken apart or divided to show the unit pieces (UNIFIX cubes, 10 loose beans in a Ziploc bag, etc.) are more concrete than those that are set representations (Base Ten Blocks, Chip Trading chips, etc.). For introductory activities it is best to use the most concrete materials. The following are examples of a variety of base 10 materials. It is not necessary to have all of these materials, but some assortment will be needed for the activities in this objective.

Loose beans and bags or cups to hold groups of 10

Popsicle sticks

Straws

Stir sticks

UNIFIX cubes

Tongue depressors and beans to make bean sticks

Cuisenaire rods (orange white)

Graph paper

Base Ten Blocks

Chip Trading chips

} rubber banded together to
form groups of 10



Concept Manipulative Activities

How Many?

Materials: Straws
Place value mat
Example:

Tens	Ones

Paper or cloth bag

- Directions:
1. Put between 20 and 90 straws in the bag.
 2. Have one student grab as many straws as possible with one hand and place them on the table.
 3. Have the other students guess the number of straws on the table.
 4. Choose another student (or students) to count the straws, grouping them by tens and ones and placing them on the place value mat.

Example: For 28 straws, the mat would look like this:

Tens	Ones
2	8

5. The student whose guess was closest to the actual number is the next person to draw straws from the bag.

Sums

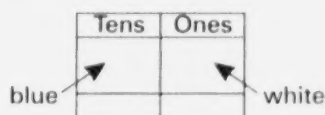
Materials: Base 10 materials
Two dice per pair of students

- Directions:
1. Have the students work in pairs.
 2. Explain to the students that with this activity the rule is, "If you have 10 units, you have to put them together to make a group of 10."
 3. Put the base 10 materials in a place where they will be easily accessible to the students.
 4. Work through one example with the students as a demonstration.
 5. Have a student roll the dice and select that many objects. If the numbers on the dice add up to more than 10, the student needs to make a group of 10 and a group of the remaining objects. For rolls of 10 or more, the student would put the group of 10 in a pile to the left and the ones in a pile to the right.
 6. Have the student say the total he/she has rolled at the end of his/her turn.
 7. Then have the student's partner roll the dice. Instruct the student to add the amount he/she rolled to the total his/her partner just rolled, regrouping when necessary, and state the new total.
 8. Have the student pairs work independently. Play continues until time is called or a total of 100 is reached. Play can then start again.

No Regrouping Addition

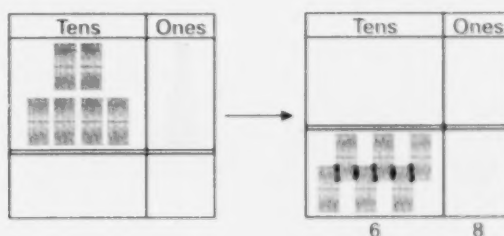
Materials: Place value mats (one per student), color coded so that the tens and ones are different colors

Example:



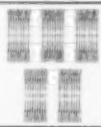
Base 10 materials

- Directions:**
1. On their place value mats, students will build two numbers using loose objects for the ones and groups of 10 for the tens (e.g., 10 beans in a cup or 10 UNIFIX cubes hooked together). They will then add the numbers together.
 2. Say, "Show me 23 on your mat." Allow the students enough time to build the model for this number.
 3. Say, "Now, below the 23, build 45." Allow the students time to build this model.
 4. Ask the students to add the ones by combining them and bringing them down to the ones answer box. Ask the students to name the number of ones they now have. (8)
 5. Repeat step 4 for the tens.



6. Ask, "What is the sum of 23 and 45?" If a student gives "6 tens and 8 ones" as the answer, encourage the student to rename it as "68."
7. After students demonstrate an understanding of addition without regrouping using manipulatives, introduce the written symbols that they will use to keep a record of their work. Say, "Build 33 on your mat. Write the numeral for 33 on a piece of paper. Now build 26 on your mat. Write the numeral for 26 under the 33 on your paper. Write the

symbol for addition in front of the 26." Demonstrate each step on the board or overhead before the students write on their papers.

Tens	Ones
	

$$\begin{array}{r} 33 \\ + 26 \\ \hline \end{array}$$

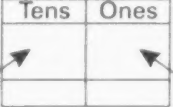
8. Have the students combine the ones, move them down to the ones answer box, and write the numeral that names the number on their paper. Demonstrate.
9. Repeat this same procedure for the tens. Demonstrate.
10. Read the answer. (59)
11. Repeat the activity using other numbers that do not require regrouping.

Subtraction Without Regrouping

Materials: Base 10 materials

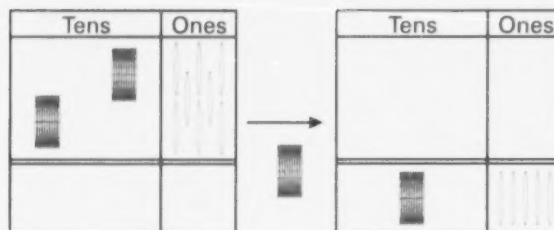
Place value mats (one per student), color coded so that the tens and ones are different colors

Example:

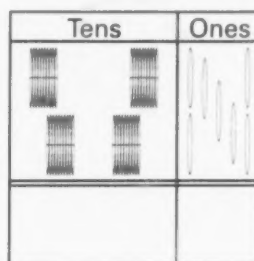
Tens	Ones
	

blue white

- Directions:
1. Say, "On your place value mat, build a model for 28." Allow sufficient time for students to build the model.
 2. Say, "We are going to take away 13. How many ones will we take away?" (3) "Remove these from your mat and place the remaining ones in the ones answer box. How many ones remain?" (5)
 3. Say, "If we are subtracting 13, how many tens will we take away?" (1) "Remove 1 ten from your mat and place the remaining tens in the tens answer box. How many tens remain?" (1)



4. Give each student an opportunity to name his/her final answer. (15)
5. After students demonstrate understanding using manipulatives alone, introduce the use of symbols to record their work.
6. Say, "Build 47 on your mat. Write the numeral for 47 on a piece of paper. We are going to subtract 23 from 47. Write the numeral for 23 under the 47 on your paper. Write the symbol for subtraction." Demonstrate each step on the board or overhead before the students write on their papers.



$$\begin{array}{r} 47 \\ - 23 \\ \hline 24 \end{array}$$

7. Have the students remove 3 ones, move the remaining ones down to the ones answer box, and write the numeral that names the number on their paper. Demonstrate.
8. Repeat this same procedure for the tens.
9. Give each student the opportunity to name the answer. (24)
10. Repeat the activity using other numbers that do not require regrouping.

Regroup

Materials: Base 10 materials

- Directions:
1. Ask the students to build a model of 2 tens and 14 ones.
 2. Ask, "Do you have enough ones to make a group of 10?" (yes) Say, "We are going to have a rule: Whenever you **can** make a group of 10, you have to." Let the students make a group of 10 from their 14 ones.
 3. Ask, "How many ones do you have left?" (4) "How many tens do you have now?" (3) "Can anyone say the number for this model?" (34)
 4. Repeat the activity using other examples, such as 3 tens and 15 ones, 1 ten and 11 ones, 6 tens and 19 ones, etc. Also give the students numbers where regrouping is not necessary, such as 4 tens and 8 ones, 5 tens and 2 ones, etc.
 5. After the students have had sufficient experiences with manipulatives alone to understand this regrouping process, they should begin to record what they are doing on paper. Example: For 3 tens and 13 ones they would write:



6. Variation: Have the students work in pairs. One student should tell his/her partner what model to build. After the partner models the example, the students should work together to combine groups of 10 when possible. They should then switch roles and continue the activity.

Regrouping—Addition I

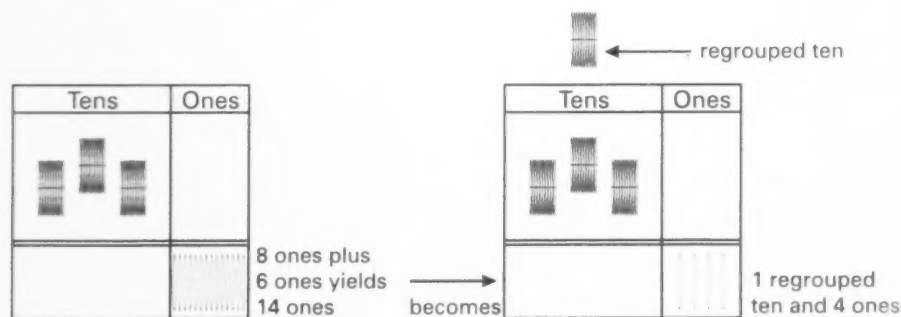
Materials: Place value mats (one per student)
Base 10 materials

Directions: 1. Say, "Build the number 18 on your place value mat. Below that model, build the number 26."

Tens	Ones

Tell the students to add the ones together and move them to the ones answer box. Ask, "How many ones do you have?" (14)

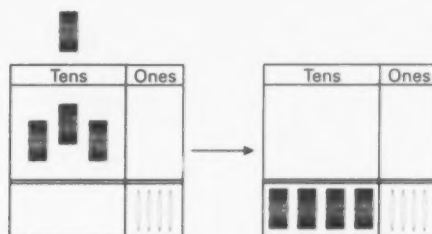
- Ask, "Does anyone know what we do when we have more than 10 ones?" Accept any response similar to, "We trade 10 ones for a ten." If the students do not know what to do, see Concept Manipulative Activity "How Many" for this objective. Ask, "Do we have enough ones to make a ten?" (yes)
- Tell students to make the trade. Have the students place the regrouped ten above the tens in the model. (This is done to model what the students will be doing later on with symbols.) Example:



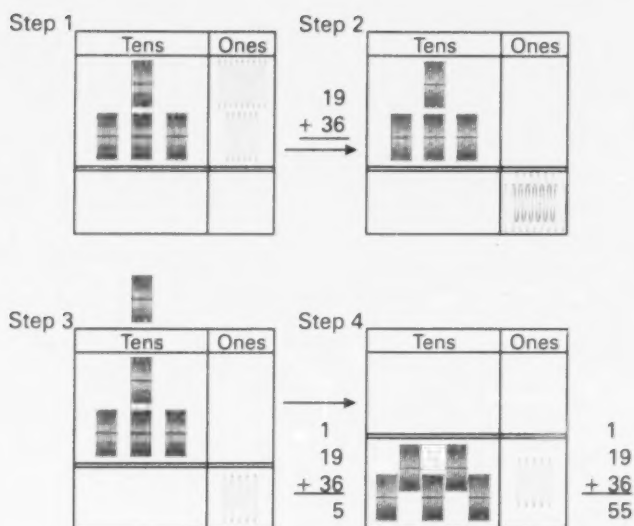
- Ask, "How many tens did you make?" (1) "How many ones remain?" (4)

168

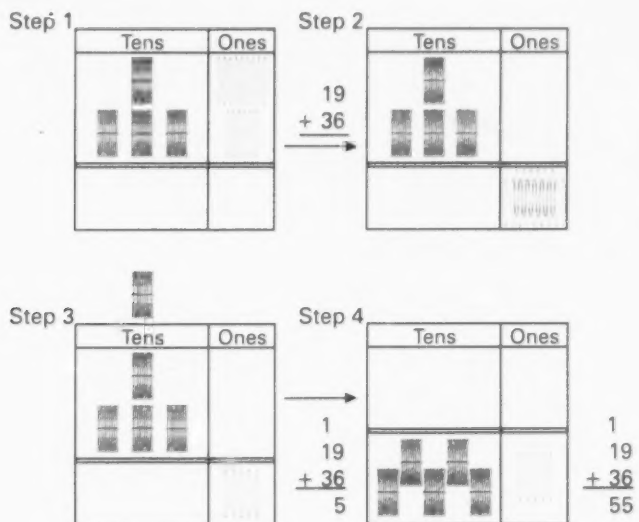
5. Say, "We started with 1 ten and 2 tens. How many tens is that?" (3) "Now you have one more ten because of the trading you did. How many tens do you have altogether?" (4)
6. Have the students move all of the tens to the tens answer box.



7. Repeat the activity several times with two-digit numbers that require regrouping (e.g., $17 + 36$, $25 + 48$, $19 + 37$, etc.).
8. Students should be given the opportunity to work many problems with just manipulatives. After they are comfortable with the concept of regrouping for addition, students need to learn how to record their work. Ask the students to build 19 and to build 36 below it. Ask the students to write the number names on their paper and to write the symbol for addition. Demonstrate this on the board or overhead. (See example below.)
9. Have the students combine the ones. Ask, "How many ones do you have?" (15) "Are there enough ones to trade for a ten?" (yes) "Make the trade. How many ones are left?" (5) Say, "Now, I will show you how to record what we have done so far." (See steps 1-3 of example.)



10. Ask, "How many tens did you have to begin with?" (4: 1 ten in 19 and 3 tens in 36) "How many tens do you have altogether?" (5) "Where did the extra ten come from?" (The extra ten came from regrouping the ones.) See step 4 of the example.



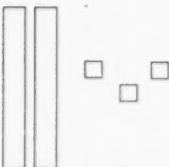
11. Continue working with the students until they understand the record keeping well enough to continue on their own.

***TEACHER NOTE:** It is important to let students work with the physical model and the written symbols at the same time. Doing so allows them to develop strategies and to see the relationship between the concrete and the abstract. Students should not be given independent seatwork or homework assignments until they have demonstrated an understanding of addition with regrouping and the ability to work on their own.

Tens to Ones (Regroup)

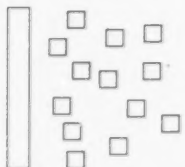
Materials: Base 10 materials

- Directions:
1. Ask the students to model the number 23. Allow sufficient time for them to model the number.
 2. Say, "Instead of grouping ones into sets of 10, we are going to take one of our tens and trade it for 10 ones."
 3. Ask, "How many tens do you have?" (2) "If you take one of your tens and trade it for 10 ones, how many tens will you have left?" (1)
 4. Tell the children to make the trade. Ask, "How many ones do you have now?" (13)
 5. Repeat the activity using other numbers, such as 56, 48, 13, etc.
 6. After the students have had sufficient experience with manipulatives alone, they should be taught how to record regrouping from tens to ones. Example: For 23 they would write:



2 tens and 3 ones
or $20 + 3$

becomes =



1 ten and 13 ones
or $10 + 13$

= 23

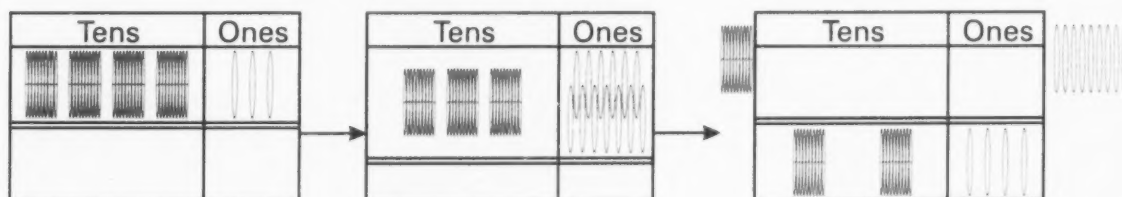
7. Variation: Have the students work in pairs. One student should tell his/her partner what model to build. After the partner models the example given, the students should work together to change a group of 10 to a group of ones and record what they have done. They should then switch roles and continue the activity.

*TEACHER NOTE: This activity is an important prerequisite to subtraction problems that require regrouping.

Regrouping—Subtraction

Materials: Place value mats
Base 10 materials

- Directions:**
1. Say, "Build the number 43 on your place value mat. We are going to take away the number 19. Does anyone know what to do when we don't have enough ones?" Accept any response similar to, "We can trade a ten for 10 ones." Have the students make the trade. (If the students do not know what to do, refer to Concept Manipulative Activity "Tens to Ones" for this objective.)
 2. After the students have done the regrouping, have them take away 9 ones. Ask, "How many ones do you have left?" (4)
 3. Ask, "How many tens did you have in the number 43?" (4) "How many tens do you have after trading a ten for 10 ones?" (3) "How many tens are you going to take away when you subtract the number 19?" (1) "How many tens do you have left?" (2)
 4. Have the students bring down the tens and ones in their models and state the answer. (24) Example:

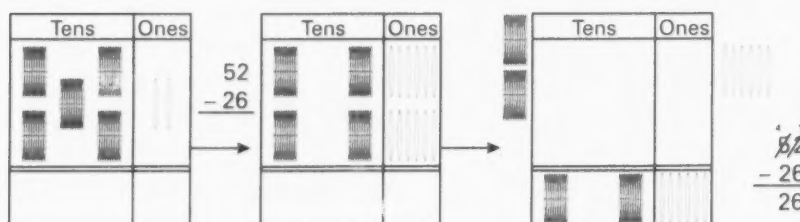


5. Repeat the activity several times with two-digit numbers that require regrouping, such as $64 - 27$, $51 - 38$, and $93 - 75$.
6. Students should be given the opportunity to work many problems with manipulatives alone. After they feel comfortable with the concept of regrouping for subtraction, students need to learn how to record **their work**.
7. Have the students model 52 and write the numeral on their paper. Tell them they are going to take away 26. Have them

172

write that numeral under the 52 on their paper and write the symbol for subtraction. Demonstrate this on the board or overhead.

8. Have the students trade a ten for 10 ones, as they did when working with manipulatives only. Say, "Now I will show you how to record your regrouping on your paper. How many tens did you start with?" (5) "After regrouping, how many tens do you have?" (4) "Draw a line through the 5 and write a 4 on top to show that there are now 4 tens. How many ones did you start with?" (2) "After regrouping, how many ones do you have?" (12) "Draw a line through the 2 and write a 12 on top to show that there are now 12 ones." Students should mark out the 2 and write 12 to emphasize the change. Merely placing a 1 in front of the 2 can be very confusing.
9. Now the students can subtract, following the same procedure that they followed earlier. As they subtract each part, the ones first and then the tens, have them write their results below the problem. Ask, "What is your answer?" (26)



10. Continue working with the students until they understand the record keeping well enough to continue on their own.

***TEACHER NOTE:** It is important to let students work with the physical model and the written symbols at the same time. Doing so allows them to develop strategies and to see the relationship between the concrete and the abstract. Students should not be given independent seatwork or homework assignments until they have demonstrated the understanding of subtraction with regrouping and the ability to work on their own.



Let's Practice

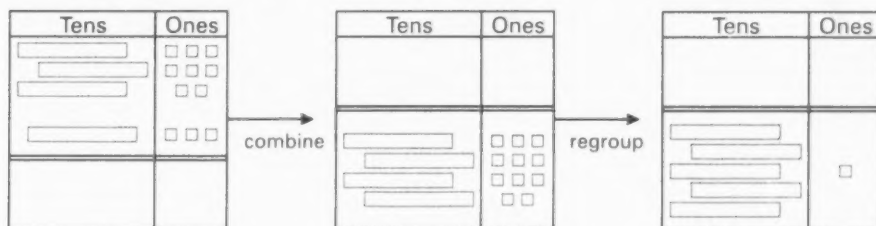
Trading

Materials: Base 10 materials
Place value mat (one per student or pair of students)

- Directions:**
1. Give each student or pair of students at least 10 groups of ten and 20 units.
 2. Have the students display the number 32 on their mat using the base 10 materials.
 3. Have them display the number 25 below the 32 on their mat.

Tens	Ones

4. Say, "Combine the ones and the tens and say the answer."
(57)
5. After the students demonstrate understanding of this concept, have them begin to record the problems on paper.
6. Explain that if the unit pieces make a set of 10 when the two numbers are added, this set of 10 is traded for a group of 10.
Example: $38 + 13$



7. Next, show the students how to model subtraction without regrouping by taking away unit pieces for the ones being

subtracted and taking away groupings of 10 for the tens being subtracted. For example, when subtracting 12 from 26, the students should begin by modeling 26. Then they would remove 2 unit pieces and 1 group of 10, leaving 4 unit pieces and 1 group of 10. The answer would be 14. Go over some examples with the class.

8. Finally, show the students how to model subtraction with regrouping. For this process, a group of 10 is traded for 10 unit pieces and the above process is repeated. Go over some examples with the class.

Regrouping Master

Materials: Base 10 materials
"Place Value Work" handouts (one per student; blackline master follows this activity)

- Directions:**
1. Give each student a "Place Value Work" handout and base 10 materials.
 2. Write four addition problems on the board. Ask the students to copy the problems on their worksheet.
 3. Have them use the base 10 materials to model the problems. After modeling each problem, the students should record their work on the worksheet.
 4. Circulate and work with the students, assessing their understanding through questioning.
 5. Give them four more addition problems.
 6. Repeat the activity using subtraction problems.



Place Value Work

Tens	Ones
+	

Tens	Ones
+	

Tens	Ones
+	

Tens	Ones
+	

Tens	Ones
+	

Tens	Ones
+	

Tens	Ones
+	

Tens	Ones
+	

Tens	Ones
-	

Tens	Ones
-	

Tens	Ones
-	

Tens	Ones
-	

Tens	Ones
-	

Tens	Ones
-	

Tens	Ones
-	

Tens	Ones
-	

Use with "Regrouping Master" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Solve-a-Problem

Materials: 3" x 5" cards, cut in half (one small card per student)
Base 10 materials

- Directions:
1. Give each student a card. Have them write any two-digit number on it. Combine the cards to make a deck.
 2. Instruct the students to work in pairs.
 3. One student in each pair will take two cards and model the addition of these numbers using base 10 materials. The partner will record the work. Then the partners will reverse roles and repeat the activity.
 4. Next, have the partners take turns drawing one card and adding it to the sum they had before. They can model the problem first, then record their work. Have them continue the activity until someone reaches 99. At that point, you may want them to start the activity again.
 5. Variation: Have each student start with 99. (The students should model this number with base 10 materials.) Then have the students, in turn, draw a card and subtract that number from 99. The partner will record the work. Then the partners will reverse roles. The game ends when one student reaches 0.

Problem Exchange

Materials: Base 10 materials

- Directions:
1. Have the students work in pairs. Each student should write down a problem, either two-digit addition without regrouping, two-digit addition with regrouping, two-digit subtraction without regrouping, or two-digit subtraction with regrouping. The addition problems should have sums less than 100.
 2. Each student should solve his/her problem using base 10 materials and then record his/her work.
 3. Students should exchange problems with their partners, solve the new problems using base 10 materials, and record

their work. The partners should compare answers. If the answers do not match, the students should rework the problem together.



Problem Solving

***TEACHER NOTE:** Since story problems are so readily accessible in commercial textbooks and resource books, they have not been included in this objective. Instead, a variety of problem-solving activities that include estimation and creative solutions have been included to challenge the students to use higher levels of thinking.

Random Numbers

Materials: Dice (two per group)

- Directions:**
1. Divide the class into groups of two to four students.
 2. The students in each group will take turns rolling the dice. The first student to roll the dice will add the two numbers rolled and record the sum. The next student will roll the dice and add the sum to the previous total. The activity continues until a certain total is achieved (e.g., 50) or time is called. Example:

First Roll			$6 + 5$	11	1st sum
Second Roll			$3 + 4$	$+ 7$	2nd sum
				18	Total
Third Roll			$1 + 5$	$+ 6$	
				24	Total

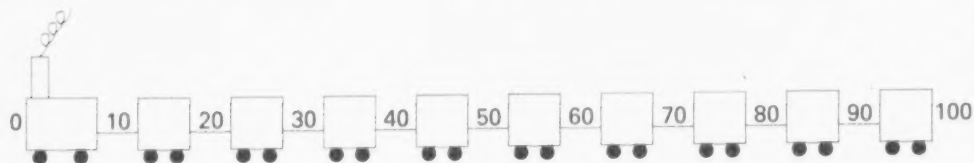
3. Variation: Each student rolls the dice five times and adds all the numbers rolled. The student with the total closest to 50 gets to choose the next total and the number of rolls to be taken.

**ESTIMATION
CODE**

Directions: Estimate the answer for each problem. Between which two numbers on the train do you think the answer will fall? Write the corresponding letter on the correct car to break the code for each train.



$72 - 20$ (G)	$56 + 25$ (S)	$25 + 10$ (D)
$94 - 18$ (E)	$9 + 7$ (O)	$39 + 59$ (S)
$89 - 26$ (U)	$17 + 6$ (O)	$25 - 17$ (G)



$36 + 49$ (Z)	$16 - 8$ (M)	$99 - 47$ (W)
$17 + 17$ (H)	$82 - 14$ (H)	$8 + 5$ (A)
$67 - 46$ (T)	$50 + 25$ (I)	



$15 - 6$ (W)	$93 - 7$ (N)	$82 - 14$ (D)
$43 + 28$ (O)	$92 - 47$ (E)	$15 + 7$ (')
$16 + 15$ (R)	$7 + 7$ (E)	$96 - 4$ (E)



*TEACHER NOTE: After students have estimated the answers and written in all the letters, have them solve the problems. They can check their estimation skills by seeing how many letters they must change.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Shopping

Materials: Newspaper or magazine ads for individual items under \$1.00
 Glue
 Base 10 materials or play money

- Directions:**
1. Have each student select two ads.
 2. Then have the students glue their ads to a piece of paper, add the cost of the items advertised, and write the problem to the side. They may use base 10 materials or play money to model the problem.
 3. Variation: Tell the students they have only a given amount of money (e.g., 75¢) to spend when they go shopping. Ask them what they could buy from the pile of advertisements and how much money they would have left.
 4. Variation: Challenge the students by saying they must buy a certain number of items (e.g., three) and that they have only 99¢ to spend. Ask them what items they could buy.

Calculator Square Off

Materials: "Calculator Square Off" handout (one per group of students; blackline master follows this activity)
 Crayons
 Calculators (one per group of students)

- Directions**
1. Each group of students forms two teams or put students in pairs. Each team chooses a different colored crayon for marking its squares.
 2. Team A selects two of the numbers from the Number Box on the handout and adds or subtracts the numbers. If the answer is in the Answer Box on the handout, the team uses its crayon to mark that square.
 3. Team B takes its turn.
 4. The first team to make a path across the board vertically, horizontally, or diagonally wins.

*TEACHER NOTE: This activity is designed to strengthen estimation skills. However, calculators should be allowed for the actual computations.



Calculator Square Off

Number Box:

7		34		99
81	14	23	58	45
		72	64	

Answer Box:

65	48	98	37	21
92	11	65	88	14
20	86	78	27	54
24	67	30	57	95
19	6	41	7	13

Use with "Calculator Square Off" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Regrouping to solve addition and subtraction problems through 999

Skill Analysis

- A. Understands the concepts of addition and subtraction
- B. Can find the answer to basic addition and subtraction facts
- C. Understands the concept of place value (ones, tens, and hundreds)
- D. Can model numbers in expanded notation
- E. Understands that an equation written vertically can be transformed into a horizontal equation without changing the result
- F. Can regroup for addition and subtraction with tens and ones

*TEACHER NOTE: The amount of time and types of experiences needed by students for mastery of this objective will vary. Research suggests that frequent but short reviews are necessary for students to prevent burnout and attain a higher success rate.



Concept Manipulative Activities

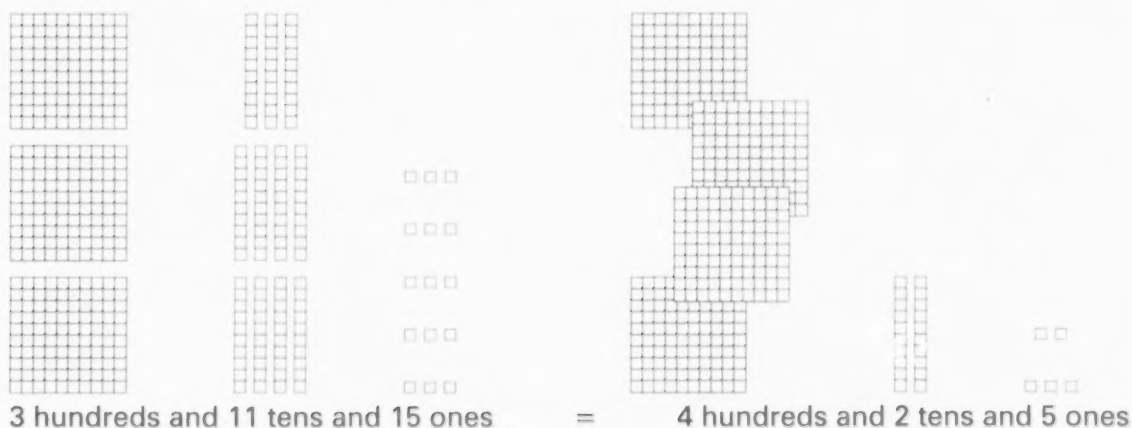
Making Hundreds

Materials: Manipulatives that can be grouped into sets of hundreds, tens, and ones (UNIFIX cubes, beans and cups or Ziploc bags, Popsicle sticks, bean sticks, etc.)

“Base 10 Patterns” handout (one per student; blackline master may be found in the Appendix) or Base Ten Stamps

Directions: 1. Have the students work in small groups.
2. Ask the students to build a model of 3 hundreds, 14 tens, and 6 ones.

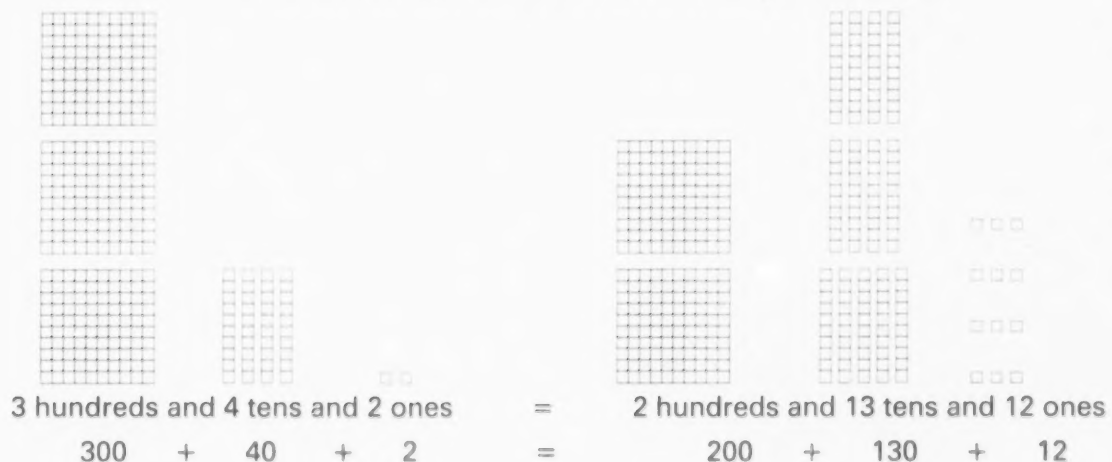
3. Say, "Do you have enough ones to make a group of 10?" (no) Tell the students, "We are going to have a rule: Whenever you have a set of 10, they must be grouped and traded."
4. Ask, "Can you make a group of 10 tens from the 14 tens?" (yes) "How many tens do you have left?" (4) "How many hundreds do you now have?" (4) "Can anyone say the number name for this model?" (446)
5. Repeat the activity using examples where regrouping is necessary in two places (e.g., 2 hundreds, 13 tens, and 15 ones; 1 hundred, 17 tens, and 10 ones; 3 hundreds, 14 tens, and 18 ones). Also give students numbers where regrouping is necessary in only one place (e.g., 4 hundreds, 8 tens, and 15 ones; 2 hundreds, 16 tens, and 7 ones) and where regrouping is not necessary at all (e.g., 5 hundreds, 7 tens, and 1 one; 3 hundreds, 0 tens, and 6 ones; 2 hundreds, 9 tens, and 0 ones).
6. Variation: Have students take turns telling a partner what model to build.
7. After students have had sufficient experiences with the manipulatives to understand this regrouping process, they should begin to record what they are doing. Students can cut apart the "Base 10 Patterns" handout for a model or can use Base Ten Stamps. Example:



Hundreds to Ones

Materials: Manipulatives that can be grouped into sets of hundreds, tens, and ones (UNIFIX cubes, beans and cups or Ziploc bags, Popsicle sticks, bean sticks, etc.)
 “Base 10 Patterns” handout (one per student; blackline master may be found in the Appendix)

- Directions:**
1. Have the students work in pairs.
 2. Ask the students to model the number 317.
 3. Say, “Instead of grouping ones into sets of 10 and tens into sets of hundreds, we are going to take one of our hundreds and trade it for 10 tens. Then we will take one of the tens and trade it for 10 ones.” Have the students make the trades.
 4. Ask, “After trading, how many hundreds do you have?” (2) “How many tens do you have?” (10) “How many ones do you have?” (17)
 5. Repeat this activity using other numbers, such as 102, 459, and 680.
 6. After students have had sufficient experience with manipulatives alone, they should be taught how to record regrouping from hundreds to tens and tens to ones. Students can cut apart the “Base 10 Patterns” handout for a model or can use Base Ten Stamps. Example:



7. Variation: Have the students take turns telling their partner what model to build. After the partner models the example and records the response, roles should be reversed.

Regrouping Addition to Hundreds

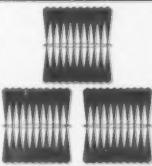
Materials: Place value mats (one per student)

Example:

Hundreds	Tens	Ones

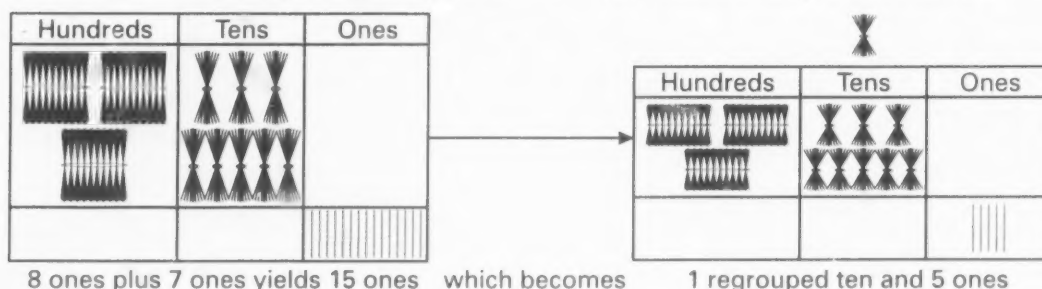
Manipulatives that can be grouped into sets of hundreds, tens, and ones (bean sticks, Popsicle sticks, coffee stirrers, Base Ten Blocks, etc.)

- Directions: 1. Say, "Build the number 137 on your place value mat. Below the 137, build the number 241. We are going to add these numbers."

Hundreds	Tens	Ones
		
Answer Box →		

2. Tell the students to add the ones together and move them down to the ones answer box. Ask, "How many ones do you have?" (8) "Do you have enough to make a trade?" (no)
3. Tell the students to add the tens together and move them down to the tens answer box. Ask, "How many tens do you have?" (7) "Do you have enough to make a trade?" (no)
4. Tell the students to add the hundreds together and move them down to the hundreds answer box. Ask, "How many hundreds do you have?" (3)
5. Ask, "What is your final answer?" (378)

6. Repeat the activity several times with three-digit numbers that do not require regrouping.
7. Ask, "Can someone tell me what we do when we have more than 10 ones (or 10 tens)?" Accept any response similar to, "We trade 10 ones for 1 ten (or 10 tens for 1 hundred)."
8. Say, "Build the number 238 on your place value mat. Below the 238, build the number 157. We are going to add these numbers."
9. Tell the students to add the ones as before, moving them to the ones answer box. Ask, "How many ones do you have?" (15) "Do you have enough to make a trade?" (yes)
10. Tell the students to make the trade and place the regrouped ten **above** the tens on their place value mat. Ask, "How many tens did you make?" (1) "How many ones remain?" (5)



8 ones plus 7 ones yields 15 ones which becomes 1 regrouped ten and 5 ones

11. Ask, "How many tens do you have now, including the regrouped ten?" (9) Say, "Place your tens in the tens answer box. Do you have enough to make a trade?" (no)
12. Tell the students to add the hundreds as before. Ask, "How many hundreds do you have?" (3) "What is your final answer?" (395)
13. Students should be allowed to work many problems using only the manipulatives. When they have had sufficient practice, assist them in recording their work. Have them build a number. Then ask the same types of questions as earlier in the activity while recording their responses beside an example on the overhead or chalkboard.

For the previous example you would write:

$$\begin{array}{r} 238 \\ + 157 \\ \hline 395 \end{array}$$

14. Repeat the activity with numbers requiring regrouping from tens to hundreds (e.g., $375 + 283 = 658$), and with numbers requiring regrouping from both ones to tens **and** tens to hundreds (e.g., $469 + 378 = 847$).

Subtraction Modeling

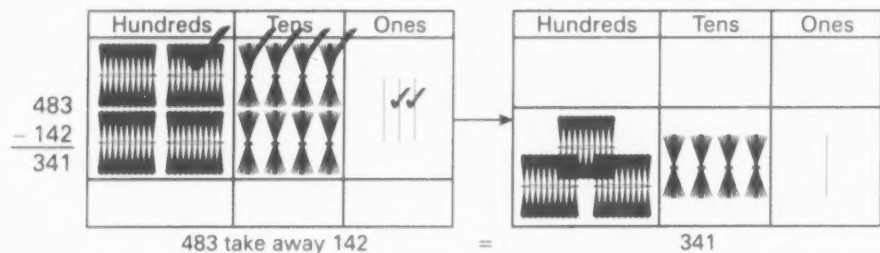
Materials: Place value mats (one per student)

Example:

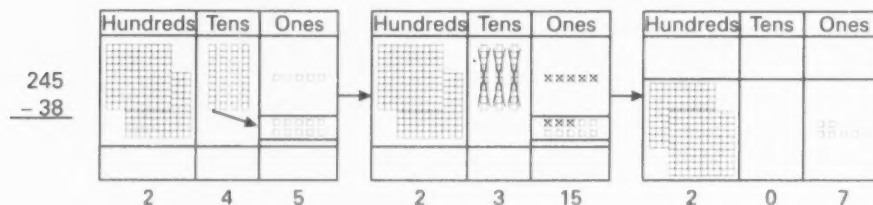
Hundreds	Tens	Ones

Manipulatives that can be grouped into sets of hundreds, tens, and ones (rods and grids; Base Ten Blocks; paper squares; ten strips and hundred squares; beans, bean sticks, and bean rafts; UNIFIX cubes; etc.)

- Directions:
1. Review place value for ones and tens. Then discuss the hundreds place and demonstrate that the model for 100 is 10 tens. (For example, show that 10 squares make 1 ten strip and 10 ten strips make 1 hundred square.)
 2. To model subtraction, begin with subtraction sentences that do not require regrouping. Model the problems, taking away from each place. For example, for $483 - 142$:



3. Progress to problems that require regrouping in the ones place. Provide practice with many problems. Example:

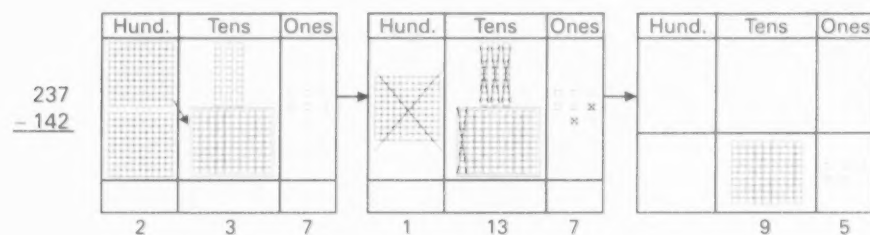


“How do we take away 8 ones?” (Regroup 1 ten into 10 ones.) Have the students model this. Say, “Now we can take away the 8 ones and 3 tens.” Ask, “What’s the difference?” (2 hundreds, 0 tens, and 7 ones, or 207)

4. Show the students that 245 (or 2 hundreds, 4 tens, and 5 ones) was written as 2 hundreds, 3 tens, and 15 ones, or that:

$$\begin{array}{r} 245 \\ - 38 \\ \hline \end{array} \text{ can be written as } \begin{array}{r} ^3 ^{15} \\ 245 \\ - 38 \\ \hline 207 \end{array}$$

5. Next, work on problems that need regrouping in the tens place. Example:

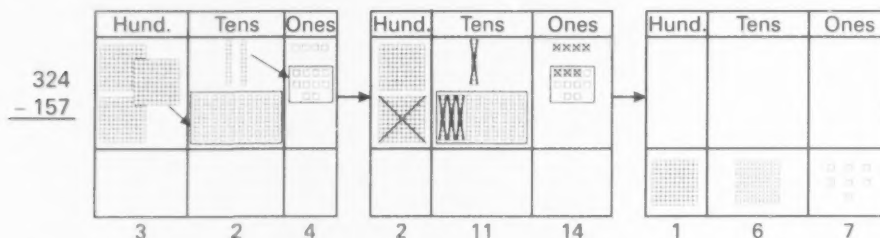


- Say, “Start with the ones place. Can we take 2 away from 7?” (yes, 5)
- Next, have the students look at the tens place. Ask, “Can we take 4 away from 3?” (no) Have the students regroup 1 hundred as 10 tens so that there are 13 tens.
- Say, “Take away the 4 tens from the 13 tens. There are 9 tens left.”
- Have the students take away the 1 hundred from the 1 hundred that is left. Ask, “What’s the difference?” (0 hundreds, 9 tens, and 5 ones, or 95)

6. Show the students that 237 (or 2 hundreds, 3 tens, and 7 ones) was written as 1 hundred, 13 tens, and 7 ones, or that:

$$\begin{array}{r} 237 \\ - 142 \\ \hline \end{array} \quad \text{can be written as} \quad \begin{array}{r} \overset{1}{\cancel{2}} \overset{13}{3} 7 \\ - 142 \\ \hline 95 \end{array}$$

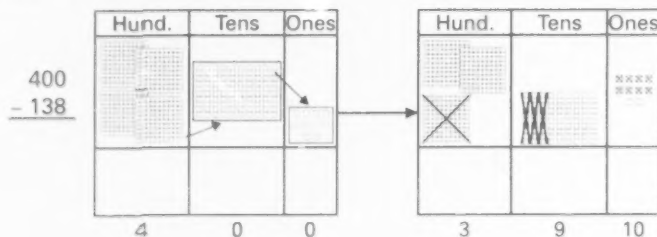
7. Have the students practice modeling problems with regrouping in the tens place.
8. The final step is to work on problems that require regrouping in either the ones place, the tens place, or both places. Below is an example of regrouping for both tens and ones.



This can be written as follows:

$$\begin{array}{r} 324 \\ - 157 \\ \hline \end{array} \rightarrow \begin{array}{r} \overset{1}{\cancel{3}} \overset{14}{2} 4 \\ - 157 \\ \hline 7 \end{array} \rightarrow \begin{array}{r} \overset{2}{\cancel{3}} \overset{11}{\cancel{2}} \overset{14}{4} \\ - 157 \\ \hline 167 \end{array}$$

9. Question the students as before, taking them slowly through each step in the regrouping process.
10. Zero can sometimes create difficulty for students in subtraction. Have students model problems with zeroes, like $400 - 138$. Example:



This can be written as follows:

$$\begin{array}{r} 400 \\ - 138 \\ \hline \end{array} \rightarrow \begin{array}{r} \overset{3}{\cancel{4}} \overset{10}{0} 0 \\ - 138 \\ \hline 262 \end{array}$$

11. Question the students as before, taking them through each step.
12. Let the students practice regrouping with the models and using the written notations. Give them problems that need regrouping with ones, tens, or both. Examples:

$\begin{array}{r} 426 \\ - 317 \\ \hline \end{array}$	$\begin{array}{r} 126 \\ - 73 \\ \hline \end{array}$	$\begin{array}{r} 324 \\ - 165 \\ \hline \end{array}$	$\begin{array}{r} 406 \\ - 214 \\ \hline \end{array}$	$\begin{array}{r} 500 \\ - 218 \\ \hline \end{array}$
---	--	---	---	---



Let's Practice

Up and Down

Materials: Index cards
Base 10 or place value manipulatives

- Directions:
1. Write three-digit addition and subtraction problems horizontally on index cards. Include problems that require regrouping and problems that don't. Give each student one index card.
 2. Allow the students to use manipulative models when solving the problems.
 3. Have each student write his/her problem vertically on the chalkboard and then solve it.

Problem Puzzles

Materials: Construction paper cut into holiday or theme shapes
Whole pieces of construction paper
Place value or base 10 manipulatives

- Directions:
1. Have the students trace and cut out 10 of the same paper shapes.
 2. Say, "Write an addition or subtraction problem, with or without grouping, on the top half of each of your paper shapes and write the answer on the bottom half. Then cut the shapes in half."

3. Have the students mix their puzzle pieces and trade them with a classmate, who should then put each shape back together.

**TEACHER NOTE:* Stress the need to cut the pieces exactly in half so that whoever is solving the problems will have to find the answer, as opposed to just fitting the puzzle pieces together.

Books

Materials: Construction paper

- Directions:
1. Have the students make a construction-paper book of six to eight pages.
 2. On each page, have them write a rebus addition or subtraction problem. Some problems should include regrouping and some should not. Example:

176  in a .

98  flew away.

How many  are left?

3. Have the students exchange books and solve the problems on a separate piece of paper.
4. Variation: Have the students use props to act out the problems in their book.

Beanbag Toss

Materials: Poster board
Eraser or beanbag
Base 10 or place value manipulatives

- Directions:
1. On a piece of poster board, make a chart that includes three-digit addition and subtraction problems, some that require regrouping and some that don't.

Example:

$\begin{array}{r} 352 \\ - 121 \\ \hline \end{array}$	$\begin{array}{r} 800 \\ + 121 \\ \hline \end{array}$	$\begin{array}{r} 300 \\ - 200 \\ \hline \end{array}$
$\begin{array}{r} 628 \\ + 157 \\ \hline \end{array}$	$\begin{array}{r} 324 \\ - 112 \\ \hline \end{array}$	$\begin{array}{r} 135 \\ + 423 \\ \hline \end{array}$
$\begin{array}{r} 843 \\ - 589 \\ \hline \end{array}$	$\begin{array}{r} 521 \\ + 370 \\ \hline \end{array}$	$\begin{array}{r} 655 \\ - 205 \\ \hline \end{array}$

- Put the poster board on the floor. Have students toss a beanbag onto the poster board and answer the problem the beanbag lands on. They may use manipulatives if necessary.
- The students score a point for each correct answer. If the beanbag lands on a problem that has already been answered, the student loses his/her turn. The student with the most points when all problems have been solved wins.

Dice

Materials: Place value manipulatives

Three cubes marked in the following manner: a yellow cube with sides numbered 1, 2, 3, 3, 4, 4; a green cube numbered 1, 2, 3, 3, 4, 4; and a blue cube numbered 5, 5, 6, 7, 8, 9

- Directions:
- Tell students that the yellow cube will be the hundreds cube, the green cube will be the tens cube, and the blue cube will be the ones cube.
 - Divide the class into groups of three or four students.
 - The students in each group will, in turn, roll the three cubes, write down the three-digit number rolled, roll the cubes again, and find the sum of the two numbers.
 - Students can use place value manipulatives to find the sum.
 - When it is their turn to roll the dice again, the students will roll the three cubes and add the new three-digit number to their previous sum.

6. The first player whose sum exceeds 900 is the winner.
7. Variation: Have the students write the number 999 on a piece of paper. With each of their rolls, the students subtract the three-digit number they rolled from their total. Play continues until one player's answer is less than 100 or subtraction is not possible with whole numbers (e.g., $196 - 243$).

Adding Different Ways

Materials: Place value manipulatives

Directions: 1. Write the following chart on the board and have the students copy it onto a piece of paper.

432	432	432	432
263	263	263	263
149	149	149	149

2. Instruct the students to cut the numbers apart. Explain that they will use them as addends to find as many different sums as possible. They may use only two addends per problem.
3. Ask, "How many different problems can you make?" (6: $432 + 432$, $432 + 263$, $432 + 149$, $263 + 263$, $263 + 149$, $149 + 149$)
4. Have the students find the six sums.
5. Variation: Repeat the activity, but this time use **four** different numbers as possible addends. Ask, "How many different problems can you make?" (24)

Out to Lunch

Materials: Magazines that include pictures of food
Manipulatives

- Directions:**
1. Have the students cut out pictures of food that could be eaten for a particular meal.
 2. Working together as a class, come up with a price (under \$5.00) for each item and write it on the picture.
 3. Have the students select three of the food items and add up the cost.
 4. Variation: Give the students a certain amount of money to spend, say \$5.00. Ask them how much money they would have left if they bought one of the items, two of the items, three of the items, a full lunch, etc.

*TEACHER NOTE: If the students have difficulty with the decimal point, write the numbers as cents (e.g., write \$2.37 as 237¢).

**Problem Solving****Nursery Rhyme Fun**

Materials: Place value manipulatives

- Directions:**
1. Have students solve the following problems:
 - a. Little Bo Peep has 160 white sheep. She also has 137 black sheep. How many sheep does she have in all?
 - b. The Little Old Woman Who Lives in a Shoe has 123 girls and 289 boys. How many children does she have in all?
 2. Students can use place value manipulatives to solve the problems.
 3. Variation: Have the students write their own story problems involving a favorite nursery rhyme for other classmates to solve.

**S
P
O
R
T
S

T
I
M
E**

Directions: The table at the left shows how many of various sports items were sold last year at a local sporting goods store. Use the table to answer each question.

Item	Number Sold
	716
	190
	397
	242
	356
	258
	840

1. How many more soccer balls were sold than football helmets? _____
2. How many more skateboards were sold than in-line skates? _____
3. How many targets, baseballs, and in-line skates were sold? _____
4. How many more baseballs were sold than skateboards? _____
5. How many footballs and targets were sold? _____
6. How many soccer balls and football helmets were sold? _____



*TEACHER NOTE: Students should be allowed to use manipulative if needed.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Regrouping to solve addition and subtraction problems through 9,999

***TEACHER NOTE:** Computation errors often occur because students do not have place values properly aligned. Using either lined paper turned sideways or graph paper will help students keep numbers properly aligned.

5	6	3	or	5	6	3
2	8	6		2	8	6

Skill Analysis

- A. Understands the concepts of addition and subtraction
- B. Knows addition and subtraction facts
- C. Understands the concept of place value
- D. Understands how to regroup tens and hundreds
- E. Can estimate sums and differences



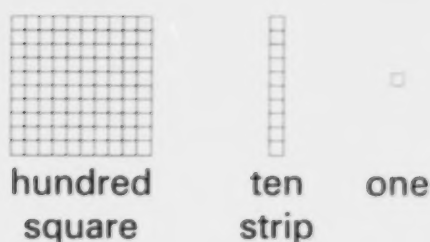
Concept Manipulative Activities

Graph Paper Models to 999—Addition

Materials: Graph paper (blackline masters may be found in the Appendix)

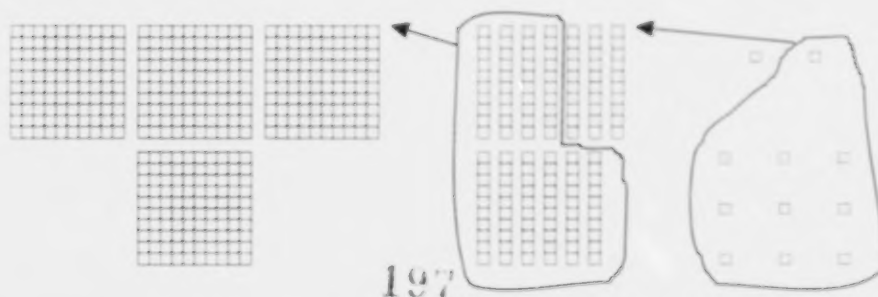
Envelopes

Directions: 1. For this skill, a 10 x 10 grid will be referred to as a “hundred square,” a 1 x 10 grid strip will be referred to as a “ten strip,” and 1 x 1 squares will be referred to as “ones.”



Give each student several pieces of graph paper. Have the students cut out 10 hundred squares, 20 ten strips, and 20 ones to be kept in an envelope and used when modeling addition and subtraction problems.

2. Instruct the students to work with a partner. Ask one student in each pair to model 372 and the other to model 169.
3. Say, "Now I would like you to add your numbers together. What place should we start with?" (ones) Allow time for the partners to add their ones together. Ask, "How many ones did you get when you joined them?" (11)
4. Ask, "Can anyone tell me what we do when we have more than 10 ones in the ones place?" (Trade them for a ten.) Have the students make this trade. Ask, "How many ones do you have after the trade?" (1)
5. Repeat the same type of questioning and discussion with the tens and hundreds places. When all joining and trading are completed, restate the problem for the students: $372 + 169 = 541$.



6. Repeat the activity using other numbers. Choose some problems that require regrouping and some that do not.
7. As students become comfortable with the process, have them start to keep a written record of their work. Students

should begin by modeling and writing at the same time so that they can see the close relationship between the physical model and the symbols used to represent the model. As trades are made, the new values are immediately written down. Students should be able to verbalize this connection to you. Example:

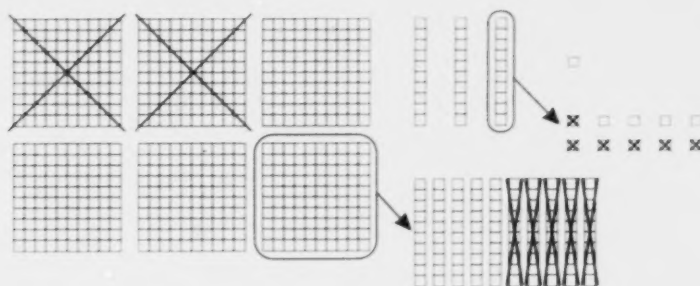
$ \begin{array}{r} \textcircled{1} \textcircled{1} \\ 3 \ 7 \ 2 \\ + 1 \ 6 \ 9 \\ \hline 5 \ \textcircled{4} \ \textcircled{1} \end{array} $	These numbers represent the trades and should be written down as soon as the trade or regrouping is done. These numbers represent the amount left after the trade and should be written down as soon as the regrouping for that place is completed.
--	---

Graph Paper Models to 999—Subtraction

Materials: Envelopes containing 10 hundred squares, 20 ten strips, and 20 ones

- Directions:**
1. Have the students work with a partner. Ask the students to model 631. Check to see that they have modeled it correctly.
 2. Say, "I would like you to take away 256 from the number you have modeled. What place should you start with?" (ones) Ask, "Do you have enough ones in your model to take away 6 ones?" (no) "What can you do so that you will have enough ones in the ones place?" (Trade 1 ten for 10 ones.) Have the students make the trade.
 3. Ask, "How many tens do you have after the trade?" (2) "How many ones do you have?" (11) "Do you have enough ones to take away 6?" (yes) "How many ones remain after you remove 6?" (5)

4. Repeat the same type of questioning and discussion with the tens and hundreds places. When all trading and subtraction are completed, restate the problem for the students: $631 - 256 = 375$.



5. Repeat the activity using other numbers. Choose some problems that require regrouping and some that do not.
6. As students become comfortable with the process, have them start to keep a written record of their work. Students should begin by modeling and writing at the same time so that they can see the close relationship between the physical model and the symbols used to represent the model. As trades are made, numbers are immediately crossed out and the new values are written down. Students should be able to verbalize this connection to you. Example:

Trading 1 hundred for 10 tens will result in 12 tens when the 10 tens are added to the 2 tens that were already there.

$$\begin{array}{r} \textcircled{5} \textcircled{2} \\ 6 \textcircled{3} \textcircled{1} \\ - 2 \textcircled{5} \textcircled{6} \\ \hline 5 \end{array}$$

By looking at the model, students can see that there are 11 ones because 1 ten was traded for 10 ones.

Place Value—Regrouping for Addition

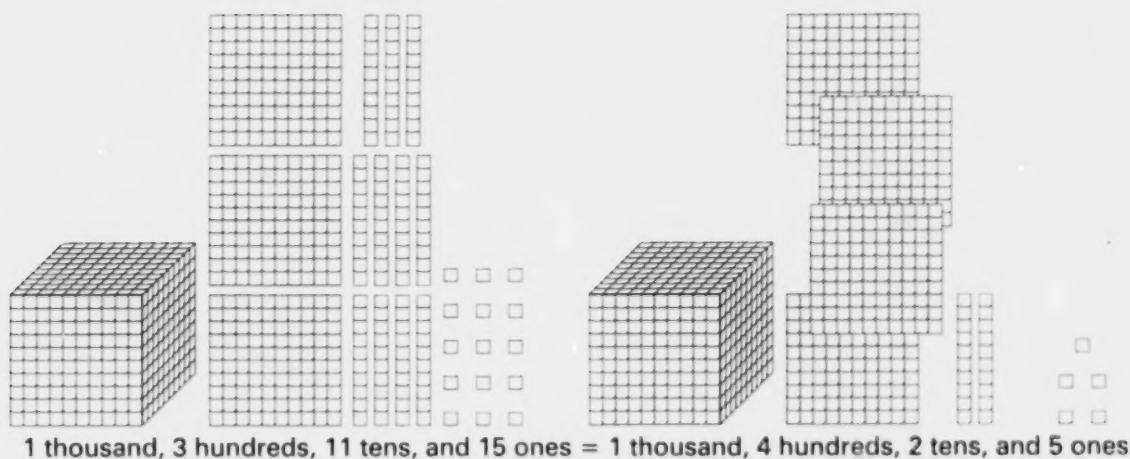
Materials: Manipulatives that can be grouped into sets of thousands, hundreds, tens, and ones

“Base 10 Patterns” handout (one per student; blackline master may be found in the Appendix) or Base Ten Stamps

- Directions:**
1. Have the students work in small groups.
 2. Instruct the students to build a model of 2 thousands, 3 hundreds, 14 tens, and 6 ones.

190

3. Ask, "Do you have enough ones to make a group of 10?" (no)
4. Say, "We are going to have a rule: Whenever you do have a set of 10, they must be grouped and traded."
5. Ask, "Can you make a group of 10 tens from the 14 tens?" (yes) "How many tens do you have left?" (4) "How many hundreds do you now have?" (4)
6. Ask, "Can you make a group of 10 hundreds?" (no)
7. Ask, "Can anyone say the number name for this model?" (2,446)
8. Repeat the activity using other examples (e.g., 5 thousands, 12 hundreds, 3 tens, and 15 ones; 2 thousands, 1 hundred, 17 tens, and 10 ones; 1 thousand, 3 hundreds, 14 tens, and 18 ones).
9. After students have had sufficient experiences with the manipulatives to understand this regrouping process, they should begin to record what they are doing. Students can cut apart the "Base 10 Patterns" handout for a model or can use Base Ten Stamps. Example:

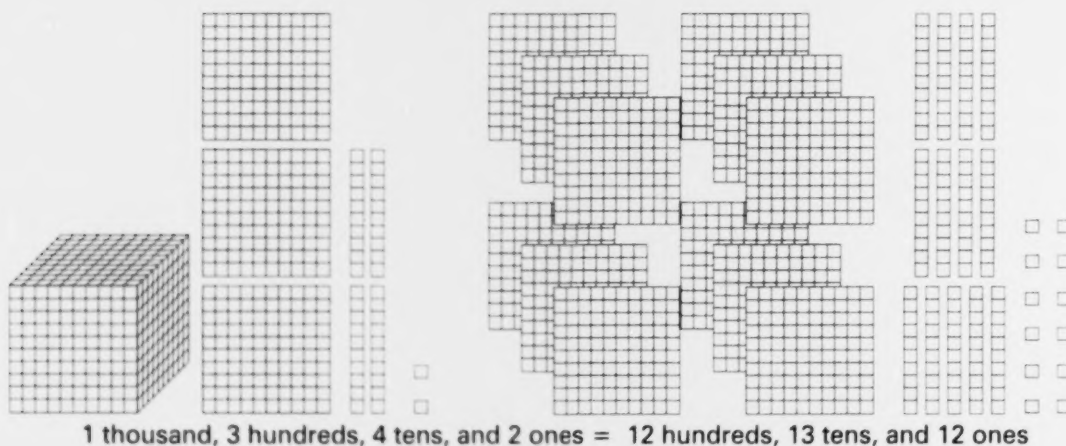


10. Variation: Have the students take turns telling a partner what model to build.

Place Value—Regrouping for Subtraction

Materials: Manipulatives that can be grouped into sets of thousands, hundreds, tens, and ones
 “Base 10 Patterns” handout (one per student; blackline master may be found in the Appendix) or Base Ten Stamps

- Directions:**
1. Have the students work in pairs.
 2. Say, “I’d like you to model the number 1,234.”
 3. Say, “In this activity, instead of grouping ones into sets of 10, tens into sets of 100, and hundreds into sets of 1,000, we are going to take one of our thousands and trade it for 10 hundreds.” Allow time for the students to make the trade.
 4. Ask, “After trading, how many thousands do you have?” (0) “How many hundreds do you have?” (12)
 5. Say, “Now we are going to trade one of our tens for 10 ones. After the trade, how many tens do you have?” (2) “How many ones do you have?” (14)
 6. Repeat this activity of trading down for different places using other numbers, such as 2,102; 4,591; 1,680; etc.
 7. After students have had sufficient experience with manipulatives alone, they should be taught how to record regrouping from thousands to hundreds, hundreds to tens, and tens to ones. Students can cut apart the “Base 10 Patterns” handout for a model or can use Base Ten Stamps.
- Example:**



201

8. Variation: Have the students take turns telling a partner what model to build. After the partner models the example and records the response, roles should be reversed.

Regrouping Addition With Record Keeping

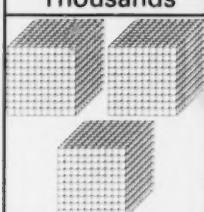
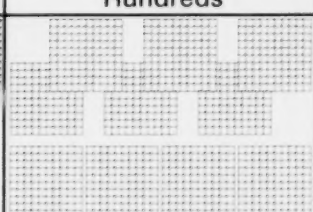
Materials: Place value mats (one per student)

Example:

Thousands	Hundreds	Tens	Ones

Manipulatives that can be grouped into sets of thousands, hundreds, tens, and ones

- Directions: 1. Say, "Build the number 2,637 on your place value mat. Below the 2,637, build the number 1,434. We are going to add these numbers." Example:

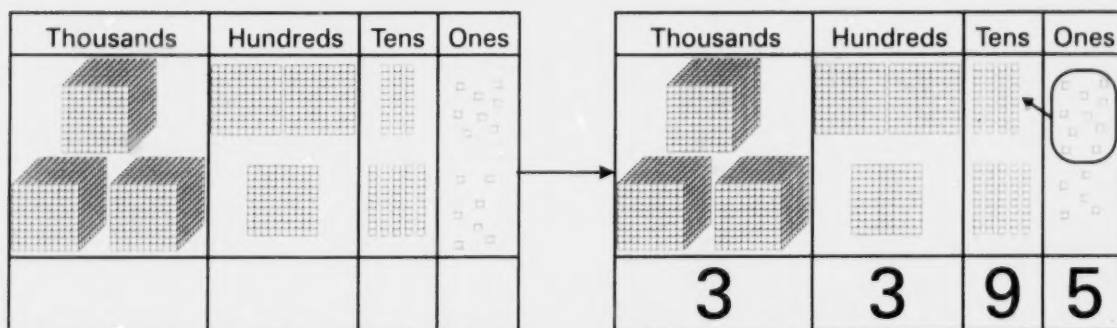
Thousands	Hundreds	Tens	Ones
			
Answer Box →			

- Tell the students to add the ones together and move them down to the ones answer box. Ask, "How many ones do you have?" (11) "Do you have enough ones to make a trade?" (yes) Allow time for the students to make the trade.
- Tell the students to add the tens together and move them down to the tens answer box. Ask, "How many tens do you have?" (7) "Do you have enough to make a trade?" (no)
- Tell the students to add the hundreds together and move them down to the hundreds answer box. Ask, "How many hundreds do you have?" (10) "Do you have enough to make

202

a trade?" (yes) Allow time for the students to make this trade.

5. Ask, "After trading, how many hundreds do you have?" (0)
"What did you trade your 10 hundreds for?" (1 thousand)
6. Ask, "How many thousands do you now have?" (4)
7. Ask someone in the class to write the answer on the board.
(4,071)
8. Repeat the activity several times with other four-digit numbers. Include some problems that do not require any regrouping.
9. Ask, "Can someone tell me what we do when we have more than 10 ones (or 10 tens)?" Accept any response similar to, "We trade 10 ones for 1 ten (or 10 tens for 1 hundred)."
10. Say, "Now I'd like you to build the number 1,238 on your place value mat. Below the 1,238, build the number 2,157. We are going to add these numbers."
11. Have the students add the ones as before. Ask, "How many ones do you have?" (15) "Do you have enough to make a trade?" (yes)
12. Tell the students to make the trade and place the regrouped ten **above** the tens in the model. Ask, "How many tens did you make?" (1) "How many ones remain?" (5) Example:



8 ones plus 7 ones yields 15 ones, which becomes 1 regrouped ten and 5 ones

13. Ask, "How many tens do you now have, including the regrouped ten?" (9) Have the students place the tens in the tens answer box. Ask, "Do you have enough tens to make a trade?" (no)

2 3

14. Have the students add the hundreds as before. Ask, "How many hundreds do you have?" (3)
15. Have the students add the thousands and ask one of the students to write the final answer on the board (3,395). Ask another student to read the answer.
16. Students should be allowed to work several problems using only the manipulatives. When they have had sufficient practice, assist them in recording their work. Have them build two numbers. Then ask the same types of questions as earlier in the activity while recording their responses on the overhead or chalkboard. For the previous example, you would write:

$$\begin{array}{r} 1,238 \\ + 2,157 \\ \hline 3,395 \end{array}$$

17. Repeat the activity with problems requiring regrouping from tens to hundreds (e.g., $1,375 + 2,283 = 3,658$, and with problems requiring regrouping from both tens to hundreds **and** hundreds to thousands (e.g., $4,693 + 4,762 = 9,455$).

Subtraction Modeling

Materials: Place value mats (one per student)

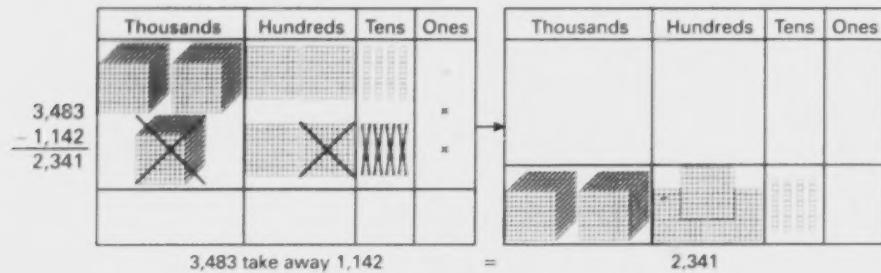
Example:

Thousands	Hundreds	Tens	Ones

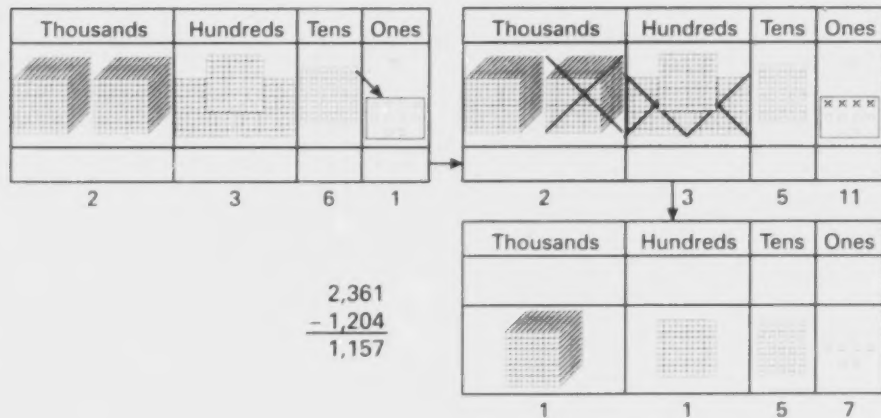
Manipulatives that can be grouped into sets of thousands, hundreds, tens, and ones

- Directions: 1. Review place value for ones, tens, and hundreds. Then discuss the thousands place and demonstrate that the model for thousands is 10 hundreds. (For example, 10 strips of 10 make one square of 100 and 10 squares make the 1,000 cube.)

2. To model subtraction, begin with subtraction sentences that do not require regrouping. Model the problems, taking away from each place. For example, $3,483 - 1,142$:

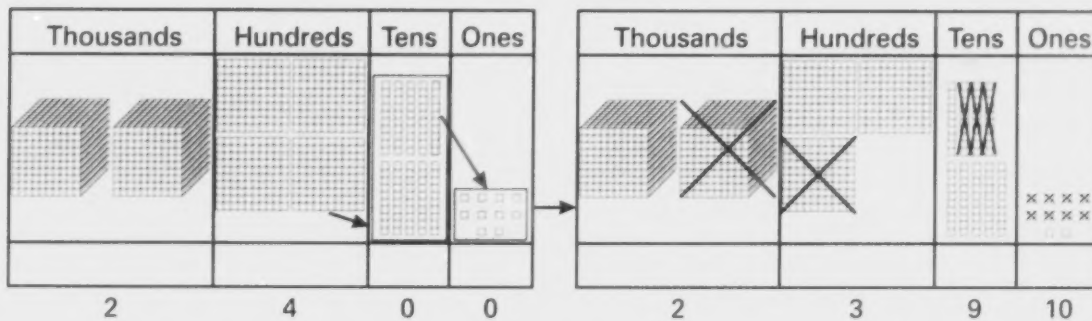


3. Progress to problems that require regrouping. For example, $2,361 - 1,204$:



4. Provide practice with many problems that do and do not require regrouping.

5. Because zero can sometimes create difficulty for students in subtraction, have students model problems with zeroes, like $2,400 - 1,138$. Example:



This can be written as follows:

$$\begin{array}{r}
 2,400 \\
 - 1,138 \\
 \hline
 \end{array}
 \rightarrow
 \begin{array}{r}
 2,400 \\
 - 1,138 \\
 \hline
 \end{array}
 \rightarrow
 \begin{array}{r}
 2,400 \\
 - 1,138 \\
 \hline
 1,262
 \end{array}$$

6. Review a problem using the model as the basis for record keeping. Explain the connection between the “actions” with manipulatives and the symbols used to record the work.
7. Let the students practice regrouping with the models and using the written notations. Give them problems like:

$$\begin{array}{r}
 3,426 \\
 - 2,317 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 4,126 \\
 - 1,073 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 3,324 \\
 - 2,165 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 4,406 \\
 - 1,214 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 6,500 \\
 - 3,218 \\
 \hline
 \end{array}$$

Addition Place Value Mat

Materials: Markers (wooden cubes, UNIFIX cubes, chips, etc.)
 Place value mat (one per student)
 Cards or tiles numbered 0–9

- Directions:
1. Give each student a place value mat or have each student make one.
 2. Ask the students to place markers in the appropriate places on their mats to represent the number 1,243.

3. Next, ask them to place markers in the appropriate places below the first set to represent 2,352. Example:

	Thousands	Hundreds	Tens	Ones
1,243	•	• •	• • • •	• • •
2,352	• •	• • •	• • • •	• •

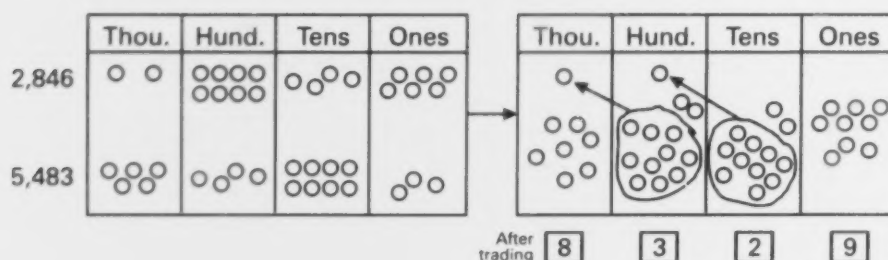
4. Ask, "If we joined the two sets of markers, how many markers would be in the combined set?" Have the students place the corresponding number cards below each section of the mat. Example:

	Thousands	Hundreds	Tens	Ones
Join sets together	• • •	• • • •	• • • • • • • •	• • • •
	3	5	9	5

5. Ask questions about joining the sets.
- "Did you have enough markers in any place to make a trade?" (no)
 - "How many markers would you need to have before you could make a trade?" (10)
6. Ask one of the students to read the answer.
7. Repeat the activity using the numbers 2,846 and 5,483.
8. Ask, "Do you have enough markers in any place to make a trade?" (Yes, in the tens place and the hundreds place.)
9. Have the students make the appropriate trades and place the correct number cards below each section of the mat.

207

Example:



10. Ask one of the students to read the answer.
11. Repeat the activity using some problems that do not require regrouping and some that do.

Subtraction Place Value Mat

Materials: Markers (cubes, chips, etc.)
 Place value mat (one per student)
 Cards or tiles numbered 0-9

- Directions:
1. Give each student a place value mat or have each student make one.
 2. Ask the students to place markers in the appropriate places on their mats to represent the number 2,567. Example:

Thou.	Hund.	Tens	Ones
2	5	6	7

3. Write the number 1,234 on the chalkboard or on a card and tell the students that you want them to take this amount away from the 2,567 they have modeled.
4. Beginning with the ones place and moving on to the tens, hundreds, and thousands, ask questions similar to the following:
 - a. "Can you remove 4 ones?" (yes)
 - b. "What about 3 tens?" (yes)

- c. "Do you have enough hundreds to remove 2 hundreds?" (yes)
 - d. "Are there enough thousands to take away 1 thousand?" (yes)
5. Ask the students to place the corresponding number cards under each section of the mat. Then ask one of the students to read the answer. Example:

Thou.	Hund.	Tens	Ones
			

$$2,567 - 1,234 = \boxed{1} \quad \boxed{3} \quad \boxed{3} \quad \boxed{3}$$

6. Ask the students to clear their mats and model 3,254. Check to see that they have modeled it correctly.
7. Write the number 1,426 on the chalkboard or a card and tell the students that this time you want them to remove this amount from the 3,254 they have modeled.
8. Beginning with the ones place ask, "Can you remove 6 ones?" (no) "What will you need to do so that you can remove 6 ones?" (One ten will have to be traded for 10 ones.) Have the students make the trade. Then ask, "After making the trade, how many ones do you have?" (14) "Now can you take away 6 ones?" (yes) Have the students remove the ones.
9. Ask, "How many tens do you have?" (4, since 1 ten was traded) "Can you take away 2 tens?" (yes) Have students remove the 2 tens.
10. Ask, "How many hundreds do you have?" (2) "Can you take away 4 hundreds?" (no) "What can you do so that you have enough hundreds to remove 4?" (You can trade 1 thousand for 10 hundreds.) Have the students make the trade. Then ask, "How many hundreds do you have after making the trade?" (12) Have the students remove the 4 hundreds. Ask, "How many hundreds are left?" (8)
11. Ask, "After you remove 1 thousand from the two that are left, how many remain?" (1)

12. Have the students place the appropriate number cards under each section of the mat. Ask one of the students to read the answer. Example:

Thou.	Hund.	Tens	Ones
Trade for 10 hundreds 		Trade for 10 ones 	
3	2	5	4

$- 1,426 =$

Thou.	Hund.	Tens	Ones
1	8	2	8

13. Repeat the activity using some problems that require regrouping and some that do not.



Let's Practice

Dice

Materials: Place value manipulatives

Four cubes marked in the following manner: a red cube with sides numbered 1, 1, 1, 2, 2, 2; a yellow cube numbered 1, 2, 3, 3, 4, 4; a green cube numbered 1, 2, 3, 3, 4, 4; and a blue cube numbered 5, 5, 6, 7, 8, 9

- Directions:
1. Tell the students that the red cube will be the thousands cube, the yellow cube will be the hundreds cube, the green cube will be the tens cube, and the blue cube will be the ones cube.
 2. Divide the class into groups of three or four students.
 3. The students in each group will, in turn, roll the four cubes, one at a time, write down the four-digit number rolled, roll the cubes again, and find the sum of the two numbers.
 4. Some students may need to use place value manipulatives to find the sum.

5. When it is their turn to roll the dice again, the students will roll the four cubes and add the new four-digit number to their previous sum.
6. The first player whose sum exceeds 9,000 is the winner.
7. Variation: Have the students write the number 9,999 on a piece of paper. With each of their rolls, the students will subtract the four-digit number they rolled from their total. Play continues until one player's answer is less than 1,000 or the subtraction is not possible with whole numbers (e.g., $1962 - 2437$).

Shopping Spree

Materials: Magazines that include pictures of things students would like to buy
Manipulatives
Glue

- Directions:
1. Have the students cut out pictures of things they would like to buy.
 2. Working together as a class, come up with a price (under \$10) for each item and write it on the picture.
 3. Tell the students that they have \$99.99 to spend.
 4. Have them glue pictures of the items they "buy" to a piece of paper and keep a running tally of the amount of money they have spent.
 5. See which student can get closest to \$99.99.
 6. Variation: Ask the students how much of the \$99.99 they would have left if they bought one of the items, two of the items, three of the items, etc.

Front-End Rounding

Materials: None

- Directions:
1. Write the problem $5,673 + 346$ on the chalkboard or overhead projector. Tell the students that you do not want

them to add the numbers, but you want them to tell you what they think the answer will be close to. Record their responses.

2. Tell the students that you are going to teach them a strategy that will help them to make a quick estimate of what the answer will be **reasonably** close to. Let them know that other strategies exist that may get them closer to the actual answer, but, for now, you are going to be showing them a quick method known as "front-end rounding."
3. Refer to the problem on the board. Ask the students to identify the numeral at the **front end** of each number. (5 and 3)
4. Tell the students that in front-end rounding, you use only the front number and forget about the other numbers or pretend that they are not there. So:

$$\begin{array}{r} 5,673 \\ + 346 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 5,000 \\ + 300 \\ \hline \end{array}$$

5. Ask the students if they can quickly give you an answer to the new problem. Explain to the students that although this is not the **exact** answer, in many situations it would be close enough. A close **educated guess** is an **estimate**. This type of estimate is called front-end rounding because you look only at the front end of the number and all other places go to zero (thus, the term "rounding").
6. Repeat the activity using other examples, such as:

$$\begin{array}{r} 3,419 \\ + 5,812 \\ \hline \end{array} \rightarrow \begin{array}{r} 3,000 \\ + 5,000 \\ \hline \end{array} \quad \begin{array}{r} 2,703 \\ - 1,209 \\ \hline \end{array} \rightarrow \begin{array}{r} 2,000 \\ - 1,000 \\ \hline \end{array} \quad \begin{array}{r} 9,134 \\ - 7,299 \\ \hline \end{array} \rightarrow \begin{array}{r} 9,000 \\ - 7,000 \\ \hline \end{array}$$

7. After the students give you estimated answers for a number of problems, ask them to work the original problems to find the **exact** answers.
8. Then have them compare the exact answers to the estimated answers. Ask, "Do you think the estimated answer you get with front-end rounding will be larger sometimes and smaller sometimes, always larger, or always smaller than the exact answer?" Have them make up problems of their own to test their hypothesis.

9. Have a group discussion to arrive at a consensus. (When adding, the estimate from front end rounding is always smaller than the exact answer unless the numbers are unchanged by the rounding, in which case the answer is the same. When subtracting, the estimate can be higher or lower, depending on the numbers used. If students think that the estimate will always be high with subtraction, challenge them to find an example where the estimate is low.)
10. Ask the students to brainstorm times when estimating an answer would be helpful. (Examples: estimating the bill at the grocery store; before using the calculator to get a sense of what the answer should be close to; when taking a test to see what answers are possible choices, etc.)

***TEACHER NOTE:** Front-end rounding differs from what is traditionally thought of as rounding in that, with front-end rounding, you are rounding numbers to different places. (For example, in the problem $3,479 + 857$, the 3,479 is rounded to 3000 [the thousands place], while 857 is rounded to 800 [the hundreds place].) This method resembles expanded notation, and the numbers can be joined together easily. In a traditional rounding problem, both numbers are rounded to the same place. (For example, if 3,479 and 857 were both rounded to the hundreds place, the problem would become $3,400 + 800$.) While this method yields a closer estimate, it is not as quick. The situation and the degree of closeness desired dictate the method used.

Color Cards

Materials: Four different colored sets of cards, one color for each place value—ones, tens, hundreds, thousands (each group will need 10 cards of each color)

Die marked with three plus signs and three minus signs (one per group)

Directions: 1. Divide the class into groups of two to four students. Give each group 10 cards of each set stacked in piles, face down. The cards should be numbered as follows:

ones—0, 1, 2, 3, 4, 5, 6, 7, 8, 9

tens—0, 10, 20, 30, 40, 50, 60, 70, 80, 90

hundreds—0, 100, 200, 300, 400, 500, 600, 700, 800, 900

thousands—0, 1,000, 2,000, 3,000, 4,000, 5,000, 6,000, 7,000, 8,000, 9,000

2. Explain to the students that each color of card represents a different place value, and write the colors and their corresponding place values on the board.
3. Explain that each student in the group will, in turn, draw two cards from each pile. The student will then arrange the cards to form two numbers. These numbers will be displayed in expanded form (e.g., 2,000, 300, 40, 0 and 8,000, 500, 0, 6).
4. Each student will then roll the die to see what operation he/she will perform on the two numbers. The student will write the numbers and operation on a piece of paper and work the problem. For the example above, the student would write:

$$\begin{array}{r} 8,506 \\ - 2,340 \\ \hline 6,166 \end{array} \quad \text{or} \quad \begin{array}{r} 2,340 \\ + 8,506 \\ \hline 10,846 \end{array}$$

5. The student can then give his/her paper to another member in the group for checking or can check his/her own work with a calculator.

Almanac Find

Materials: An almanac

- Directions:**
1. The almanac is full of data of all kinds. Problems can be made by looking at the data (e.g., problems comparing the populations of two towns; problems determining differences in game attendance).
 2. Find data the students would find interesting and make up the problems involving those data.
 3. Have the students work the problems.
 4. Give the almanac to a student to take home and make a problem from data that he/she finds interesting. Have the student share the problem with the class.



Problem Solving

Digit Switch

Materials: Tiles or squares numbered 1–9 (one set per student)

Directions: 1. Instruct the students to arrange their cards or tiles to make three 3-digit numbers that, when added together, have a sum that is as close to 1,000 as they can find. Examples:

123	135	976
456	246	158
<u>+ 789</u>	<u>+ 789</u>	<u>+ 234</u>
1,368	1,170	1,368

2. Encourage the students to try several different solutions.
3. Variations:
 - a. Have the students find the sum closest to 1,000 without going over.
 - b. Have the students find the sum closest to 2,000, 3,000, etc.
 - c. Have the students find the numbers whose sum is exactly 999.
 - d. Have the students subtract 3-digit numbers from 4-digit numbers, looking for the pair whose difference is the smallest possible.
 - e. Have the students find two 3-digit numbers whose sum is the largest possible 4-digit number.

Grocery Shopping

Materials: Weekly grocery store ads

- Directions:**
1. Have the students work in pairs.
 2. One student will make a grocery list of five (or more) items and their prices, as indicated in the ads.
 3. The partner will calculate the total cost. The student making up the list will check the arithmetic.

4. When they are finished, the students will exchange roles and repeat the activity.
5. For additional practice, challenge the students to come up with the highest or lowest totals for a given number of items.
6. Variation: Using the menu from a fast food restaurant, have the students make up an order and calculate the cost. The partners check should arithmetic, as before.

Textbook Story Problems

Materials: Any mathematics textbook that has story problems

- Directions:**
1. The following small-group activities will provide problem-solving practice:
 - a. Have each student read a story problem to the group and say which operation should be used to solve it. The student should then share with the group what led him/her to choose that operation. The problems should not be solved. The intent is to determine which operations can be used to solve various story problems.
 - b. Find problems that have too much information. Have the students discuss ways of determining which information is pertinent and which is not.
 - c. Find problems that do not have enough information. Discuss what additional information is needed before the problem can be solved.
 - d. Ask the groups to write their own story problems and then have the groups exchange problems. The problems can be critiqued and/or solved after they are exchanged.

Travel Agents

Materials: U.S. maps or atlas (one per student)

- Directions:**
1. Tell the students to pretend that they have a free pass to travel the United States by bus and that the pass is good for 1 month.
 2. Have them plot the route they would like to take on a map and make a list of the sites they would like to see.

3. Have the students figure out the distances between the places they want to visit, keeping track of the miles from one destination to another and the total miles traveled.

SKILL Regrouping to solve addition and subtraction problems through 99,999

Skill Analysis

- A. Understands the concepts of addition and subtraction
- B. Understands the place value of each digit in a five-digit number
- C. Understands the concept of regrouping
- D. Recalls basic addition and subtraction facts
- E. Understands inverse relations
- F. Can estimate sums and differences



Concept Manipulative Activities

Pocket Regrouping

Materials: "Base 10 Patterns" (blackline master may be found in the Appendix) or Base Ten Blocks

Four pockets made from tagboard, labeled "ones," "tens," "hundreds," and "thousands"

Example:

Thousands	Hundreds	Tens	Ones

Directions: 1. Set the labeled pockets on the chalkboard ledge (or another convenient place).

2. Write the following four-digit subtraction problem on the chalkboard:

$$\begin{array}{r} 2,635 \\ 218 \overline{) 1,569} \end{array}$$

3. Say, "Before we begin this problem, estimate what you think the answer might be. What strategy did you use?"
4. Place the appropriate grid amounts, cut from the base 10 patterns, in each of the four pockets to correspond to the minuend (i.e., 2,635).
5. Start the subtraction process by pointing to the 9 in the ones place in the subtrahend and asking, "Do we have enough ones in our ones place to take 9 away?" (no) "What can we do?" (Take some from the tens place.)
6. Once the students have identified what needs to be done, take 1 ten strip from the tens pocket, and add 10 ones squares to the ones pocket. Then have a student remove (subtract) 9 squares from the ones pocket.
7. Go to the tens place (pocket) and ask, "How many tens do we have left here?" (2) "How many tens do we need to take away (subtract)?" (6) "How can we do that?" (Regroup from the hundreds.)
8. Have a student remove 1 hundred square from the hundreds pocket and add 10 ten strips to the tens pocket. The student should then subtract (remove) 6 tens from the tens pocket, which should leave 6 tens there.
9. Proceed to the hundreds place (pocket) and ask, "How many hundreds do we have left here?" (5) Have a student come up and count them, if necessary. Then ask, "Can we subtract the hundreds place or do we need to regroup?"
10. Have a student come up and subtract (remove) the hundreds (all 5).
11. Move to the thousands place (pocket) and ask, "What do we do with the thousands place?" (Subtract 1 from 2.) Have a student come up and subtract (remove) 1 thousand.
12. Finally, check the pockets to see what is left in them. (The answer should be 1,066.) Ask, "Is this close to your estimate?" Have a student come to the board and work the problem on the board to see if the answers correspond.
13. Variation: Work the problem on the board at the same time the students are working the problem with the pockets.

14. Variation: Use the base 10 materials to give the students practice with four-digit addition.

What's Your Strategy?

Materials: Base Ten Blocks

- Directions:
1. Ask the students what strategies they use to make adding numbers easier.
 2. Allow students time to share some of their strategies, possibly listing them on the board.
 3. Students may or may not come up with the "Adding Tens and Ones Separately" strategy and the "Break Up Addition" strategy. These, however, are valuable strategies to teach using Base Ten Blocks. Model these strategies for students as well as allowing students to model or "teach" the strategies they have described.
 4. To model "Adding Tens and Ones Separately," give students two-digit numbers to add and build using Base Ten Blocks. Use numbers such as $29 + 33$. Tell students to add the tens, then add the ones. Then have them add those sums together. Example:

$$\begin{array}{r}
 29 \longrightarrow \\
 + 33 \longrightarrow \\
 \hline
 \end{array}
 \begin{array}{c}
 \text{Base Ten Blocks} \\
 \text{representing } 29 + 33 \\
 \text{as } 50 + 12
 \end{array}
 = 62$$

5. Allow students to practice this strategy using Base Ten Blocks and then begin applying it mentally.
6. Decide whether students can learn other strategies, too, in the same day or if this activity needs to be spread over several days. When students are ready to learn the "Break Up Addition" strategy, explain it as follows.
7. When adding two numbers, break up one of the numbers and add the parts separately to the other number.

Examples:

$$53 + 34 \dots 53 + 30 = 83, 83 + 4 = 87$$

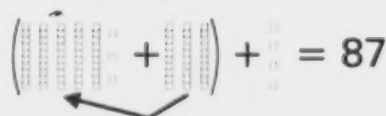
$$356 + 31 \dots 356 + 30 = 386, 386 + 1 = 387$$

$$629 + 230 \dots 629 + 200 = 829, 829 + 30 = 859$$

(It doesn't matter which number is broken up.)

8. Model the above problems with Base Ten Blocks. Example:

$$53 + 34 \dots 53 + 30 = 83, 83 + 4 = 87$$



9. Expand the strategy to two-column addition. Explain that you break-up the tens and ones digits for each number you add. Example:

33	33, ...	→		33
13	43, 46, ...	→		$33 + 10 = 43, 43 + 3 = 46$
21	66, 67, ...	→		$46 + 20 = 66, 66 + 1 = 67$
<u>+ 12</u>	77, 79	→		$67 + 10 = 77, 77 + 2 = 79$

10. Have students practice this strategy with Base Ten Blocks as long as they need to conceptualize it. Then offer practice opportunities with both strategies.

Partial Sums—Another Way

Materials: Base Ten Blocks

- Directions:
1. This is an alternative algorithm for students who have difficulty with the more conventional algorithm. It does not **have** to be taught to all students, but it can be taught to students who might benefit from its use.
 2. Say, "We are going to go over another way to add a column of numbers. Start in the ones column and add numbers until you get a double-digit sum. Write that double-digit sum into the problem and continue adding. Do the same for the other columns."

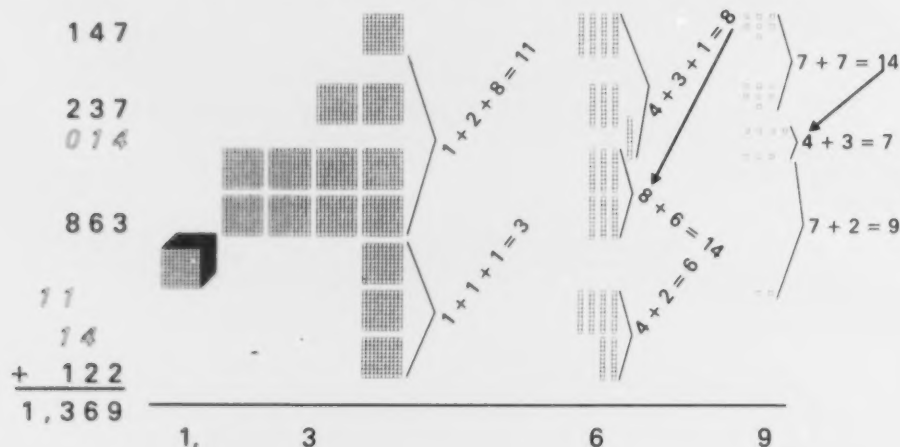
226

Example:

$$\begin{array}{r}
 147 \\
 237 \\
 014 \\
 863 \\
 11 \\
 14 \\
 + 122 \\
 \hline
 1,369
 \end{array}$$

$\left. \begin{array}{l} 147 \\ 237 \end{array} \right\} 7 + 7 = 14$
 $\left. \begin{array}{l} 014 \\ 863 \end{array} \right\} 4 + 3 = 7$
 $\left. \begin{array}{l} 11 \\ 14 \end{array} \right\} 7 + 2 = 9$

3. Model this process of adding with Base Ten Blocks.



4. Have students practice the strategy with Base Ten Blocks. Then provide opportunities for them to practice with problems.

Circle Digits—Another Way

Materials: None

- Directions:
1. This is an alternative algorithm for students who have difficulty with the more conventional algorithm. It does not **have** to be taught to all students, but it can be taught to students who might benefit from its use.
 2. Explain the procedure as follows: To add a column of numbers, start in the ones column and add numbers until you get a double-digit sum. At that point, circle the last

addend used to obtain the double-digit sum. Then add the number in the ones place of the double-digit sum to the following number.

3. Continue the process until you finish the column. Go back and count the number of circles. Put the number of circles above the tens column.
4. Continue the process in each column until the problem is finished. Be sure to add in any numbers you've written at the top of the columns. Example:

There is one digit circled in the ones column, so a "1" is put above the tens place.

$$\begin{array}{r}
 1 \quad 1 \quad 1 \\
 1 \quad 4 \quad 7 \\
 2 \quad 3 \quad \textcircled{7} \\
 \textcircled{8} \quad \textcircled{6} \quad 3 \\
 + \quad 1 \quad 2 \quad 2 \\
 \hline
 1,369
 \end{array}
 \quad
 \begin{array}{l}
 7 + 7 = 14 \\
 4 + 3 = 7 \\
 7 + 2 = 9
 \end{array}$$



Let's Practice

As Near As Possible

Materials: Dice (one die per group)
Calculators for each student playing

- Directions:
1. The goal for this game of two to six players is to come as close to zero as possible.
 2. All players start at 100 and enter that number into the calculator. (If calculators are not used, have students use paper/pencil.)
 3. Players take turns rolling one die.
 4. As each player rolls the die, he/she may choose to use the number itself or the number that is 10 times that number. For example, when 6 is rolled, it may be recorded as 6 or as 60.

223

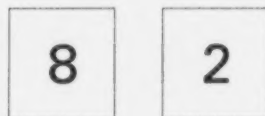
6. For subsequent turns, the players subtract the number rolled from the previous difference.
7. The game continues until each player has had seven turns or until a player gets a negative answer.
8. The person closest to zero without getting a negative number is the winner.
9. Variation: After playing the game once, students write down the strategy they developed. They then use their strategy during the next game to see if it works. As they are playing, they may revise their strategy to make it better. After the game, the students exchange strategies. For the next game, have the students play using another player's strategy.

Hit the Target Number

Materials: Deck of cards (with tens and picture cards removed) or a homemade deck of 36 cards numbered 1–9 (four cards with each number)

- Directions:**
1. Two to five students may play.
 2. Shuffle the cards. Deal each player four cards, face down. Place the rest of the deck in a stack, face down, at the center of the playing area.
 3. To begin the game, have a player turn up two cards from the deck. The first card will represent the tens place and the second card will represent the ones place for that round's **target number**. For example, if an 8 and a 2 are drawn, the target number will be 82.

Round 1



4. The target number is the number all players will try to reach during the round.

224

5. Have each of the players turn up their four cards. Explain that they are to arrange them into two 2-digit numbers that, when subtracted, will have a difference that is as close to the target number as possible.
6. To find his/her score for the round, each player subtracts his/her answer from the target number or subtracts the target number from his/her answer.
7. Suppose a player has 1, 3, 9, 6. By "juggling" the numbers around and using some estimation strategy, the student tries to get a difference as close to 82 as possible. The best combination would be $96 - 13 = 83$. The score for this player would be the difference obtained by subtracting the target number from his/her total. Thus, $83 - 82 = 1$.
8. Have students continue the game by going on to round 2. Instruct a student to turn up two new cards for the new target number. Players may either keep their four cards from the previous round or ask to be dealt four new ones. Used cards are put into a discard pile and shuffled to be used when needed. (Variation: Students may choose to keep some of their previous cards and have the number of cards of their choice dealt after a new target number is turned over.)
9. Each student adds his/her round 2 score to his/her round 1 score.
10. Play continues for five or six rounds. The player with the **lowest** score wins.

Calculator Tic-Tac-Toe

Materials: Calculators (one per game)
 "Calculator Tic-Tac-Toe" handout (one per game; blackline master follows this activity)
 Two colors of UNIFIX cubes for use as game markers

Directions: 1. Two students or two teams play. Supply one "Calculator Tic-Tac-Toe" handout for each game.

2. Each player, in turn, will select a student picks a number from the left column and a number from the right column

of the handout by mentally estimating which numbers will have a sum that is close to the number the student wants to cover on the game board. (A time limit for this step of possibly a minute might need to be established in order to expedite the game.)

3. The student then uses a calculator to add the numbers. If the student's sum is within 25 of the number he/she wants to cover, the student may cover that number with his/her color of UNIFIX cubes.
4. The game is over when a student covers three numbers in a line, vertically, horizontally, or diagonally.

*TEACHER NOTE: It is important to emphasize that the students *first* estimate and then use a calculator, so it doesn't take as many tries to get three in a row.



Calculator Tic-Tac-Toe

Left Column Right Column

655	305
572	254
407	170
206	97
121	54

150	500	900	800	650
550	700	550	500	200
300	500	450	400	300
450	650	250	550	400
650	150	750	850	350

Use with "Calculator Tic-Tac-Toe" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

227

Addition or Subtraction Board Game

Materials: None

- Directions:
1. This game is played in groups of five, six, or seven students.
 2. One student goes to the chalkboard and writes a five-digit numeral.
 3. A second student goes to the chalkboard and writes another five-digit numeral.
 4. A third student designates whether the numbers are to be added or subtracted.
 5. A fourth student works the problem at the board while the rest of the students work it at their desks.
 6. The students check their answers against the sum or difference written on the board.



Problem Solving

Digit Finder

- Materials: 1" tiles numbered 0-9 (two sets per student)
 1" graph paper (one sheet per student; blackline master may be found in the Appendix)

- Directions: 1. Have students write the following problems on their sheet of graph paper, one number per square. Be sure that they outline the boxes where numbers are missing.

$$\begin{array}{r} 527 \\ + 3\Box4 \\ \hline \Box71 \end{array}$$

$$\begin{array}{r} 1,4\Box4 \\ + 6,\Box79 \\ \hline \Box,203 \end{array}$$

$$\begin{array}{r} 8\Box \\ \Box6 \\ + 43 \\ \hline 206 \end{array}$$

$$\begin{array}{r} 731 \\ - 40\Box \\ \hline 3\Box6 \end{array}$$

$$\begin{array}{r} 86\Box \\ 514 \\ + \Box7 \\ \hline \Box,413 \end{array}$$

$$\begin{array}{r} 764 \\ - 38\Box \\ \hline 3\Box9 \end{array}$$

$$\begin{array}{r} 5,60\Box \\ - 4,\Box98 \\ \hline \Box,309 \end{array}$$

$$\begin{array}{r} 51\Box \\ \Box6 \\ + \Box43 \\ \hline 8\Box6 \end{array}$$

$$\begin{array}{r} 8\Box,235 \\ - 1,63\Box \\ \hline \Box9,5\Box8 \end{array}$$

$$\begin{array}{r} 532 \\ 5\Box7 \\ + \Box2\Box \\ \hline 1,\Box90 \end{array}$$

$$\begin{array}{r} 52,532 \\ - \Box,81\Box \\ \hline 48,\Box16 \end{array}$$

$$\begin{array}{r} 8\Box,2\Box3 \\ - 1,531 \\ \hline 78,\Box62 \end{array}$$

2. Say, "Use your number tiles to fill in the blank boxes. Keep trying tiles until you come up with the numbers that make the problems correct.
3. After the students have found the correct numbers, they should write the numbers in the boxes.

Operational Signs

Materials: Calculators (one per pair of students)
 Tiles numbered 0–9 plus several tiles marked with a plus, minus, or equals sign (one set per pair of students)

- Directions:**
1. Have each pair of students make a row of seven to nine numbers (e.g., 3925128).
 2. One student in each pair should, out of view of his/her partner, put operational tiles in the row of numbers and then put in place the tiles representing the answer (e.g., $39 + 25 + 128 = 192$).
 3. The student should then remove the "+" and "-" tiles and show the problem to his/her partner (e.g., $39\ 25\ 128 = 192$).
 4. The partner should use a calculator to try out different ideas for where to place "+" or "-" tiles.
 5. Variation: Students may want to use only addition first, then only subtraction, and then as a challenge use both "+" and "-" tiles. Example:

$$6\ 3\ 5\ 5\ 2\ 4\ 7\ 7 = 543$$

$$635 - 52 - 47 + 7 = 543$$

***TEACHER NOTE:** This is a good activity for practicing estimation strategies, especially when students are calculating which computational signs and combination of numbers to use. They need to mentally estimate before using the calculator.

Calculator Challenge

Materials: One calculator for each student

- Directions:**
1. Students may work with a partner or independently.

2. The students should choose a beginning and ending number (e.g., 16 and 112). Zero cannot be used in this activity.
3. Then, in three steps, using just addition and subtraction, the students should find a way to get from the beginning number to the ending number. Example: For 16 and 112, the students could write:
 $16 + 110 = 126$ $126 - 4 = 122$ $122 - 10 = 112$
4. Each student should find three ways to solve the problem and record each different solution. For the above example, another solution would be:
 $16 + 80 = 96$ $96 + 10 = 106$ $106 + 6 = 112$

Data Detectives

Materials: Almanac or *Guinness Book of World Records* (one per group of students)

- Directions:**
1. Have students work in small groups or with a partner.
 2. Each group should explore their resource book for data in area(s) of interest.
 3. The groups should then share their "interesting data" with one another.
 4. As a class, the students should choose one of the sets of "interesting data," record the data, and generate questions about that information. Example: Students could record the distance between Earth and the other planets and then write down questions such as:
 - a. How many miles would you travel if you went from Earth to Jupiter and back?
 - b. How far would you go if you traveled from Venus to Mars to Pluto?
 5. Have the students return to their groups to discuss the questions and derive answers.
 6. As a class, the students can then share and compare their answers.

SKILL Regrouping to solve addition and subtraction problems through 9,999,999

Skill Analysis

- A. Knows basic addition and subtraction facts
- B. Understands the concepts of addition and subtraction
- C. Understands place value
- D. Can add and subtract five-digit numbers with and without regrouping



Concept Manipulative Activities

Chip Trading

Materials: Chip Trading Kit (one per student)
Four colored dice (yellow, blue, green, and red; one set per group of students)

- Directions:**
1. Explain to the students that the yellow chips represent ones, the blue chips represent tens, the green represent hundreds, and the red represents thousands. Write the colors and their place values on the board for students to refer to.
 2. Place four yellow chips in the ones column of a Chip Trading board. Ask students the place value of the column and how much four chips represent.
 3. Continue adding yellow chips until there are 10 chips in the ones column. Demonstrate how to trade chips using the "trade 10" rule. (10 yellow chips = 1 blue chip.)
 4. Ask, "How many chips do I have now?" (1 ten, 0 ones = 10)

232

5. Next, roll the yellow die or draw a 3" x 5" yellow card (contained in the chip Trading Kit) and add the corresponding number of yellow chips. Continue to do so, making any necessary trades.
6. Ask students what you should do if you get 10 blue chips or 10 green chips.
7. Expand the chip trading process by rolling two dice, yellow and blue, and demonstrating how to add larger numbers by making trades.
8. Tell the students that they are now going to play a game in groups of two or three. Give each group ample chips of each color and a set of colored dice. Each student will need a Chip Trading board.
9. Have the students begin the game by rolling two dice, yellow and blue, and placing the corresponding chips on the board.
10. Next, each student will, in turn, roll both dice, and add the amount rolled to his/her Chip Trading board, and make any appropriate trades. Play continues in that manner until one of the students gets a red chip. That student is the winner.
11. The game can be expanded to rolling three dice (yellow, blue, and green). Students can also record the numbers rolled and the sums obtained, maintaining a running total.
12. To practice subtraction, the groups can begin with nine blue chips and nine yellow chips (99). Depending upon the level of the group, students can roll either two dice or only the yellow die. The number rolled determines how many chips must be removed from the board.
13. Each student will, in turn, roll the dice (or die) and remove the appropriate number of chips from his/her board making appropriate trades. The first student to remove all of the chips from his/her board is the winner.
14. The game can be expanded to 999 chips, using three dice, or 9,999 chips, using all four dice.

15. Students should record their subtraction problems and the differences obtained.
16. After students learn how to play the various "game options," the materials can be used as an independent learning center activity.

Grouping Strategy

Materials: Base Ten Blocks

Directions: 1. Write the following problems on the chalkboard.

$$\begin{array}{r} 4,862 \\ + 3,898 \\ \hline \end{array}$$

$$\begin{array}{r} 569 \\ 381 \\ 6,295 \\ + 3,736 \\ \hline \end{array}$$

$$\begin{array}{r} 3,957 \\ - 1,732 \\ \hline \end{array}$$

$$\begin{array}{r} 47,810 \\ - 18,396 \\ \hline \end{array}$$

2. Point to each problem, in turn, and ask students to explain how they would solve it. Discuss their strategies. Then ask, "Would one strategy work for all four problems? Why? Why not? Why are strategies used? Is it possible to combine strategies? Could one strategy be easier for one person but harder for another? Why?"
3. Students may need to use Base Ten Blocks to practice some of the strategies. Once they have conceptualized the strategy, they should begin to apply it mentally.



Let's Practice

Check Out the Numbers

Materials: None

Directions: 1. Present students with problems such as the following. Have the students work in small groups to collect, record, and report the data.

- a. How many nonfiction and fiction books are in the school library? What is the difference between the two numbers?
 - b. How many books does the school library own? How many books are currently checked out? How many books should be in the library today?
 - c. How many students are in the fifth grade? How many are buying a hot lunch today? Do you have enough information to determine, without questioning students, how many brought a lunch from home today? Why? Why not? Find out how many did bring a lunch from home to check your answer.
 - d. Repeat "c" but use "fourth grade"
 - e. How many cars pass you (going in both directions) on your way home from school? What is the difference between that number and the number of cars that pass you on your way to school?
 - f. If you live near a busy intersection or highway, count the number of cars that travel on it at a certain time. Return at another time and repeat the activity. What is the difference between the two numbers? (Be sure to play it safe near the busy highway or intersection.)
 - g. How often do you, or the members of your family, change the channels on the TV? Take a count on two different nights and find the difference.
 - h. How often do you, or the members of your family, switch lights on and off in your house? Is one room's light switched on and off more than others? Find the differences.
2. After students have collected and recorded their data, have them share their findings with the rest of the class.
 3. Determine the sum for one set of data and the difference for another set. Example: Six students collected data on cars going through different intersections. How many cars did they see altogether? Look at the fourth and fifth grade lunch counts. How many more fourth graders are buying lunch than fifth graders (or vice versa)?

Figure It Out

Materials: 1"-square ceramic tiles (12 per pair of students)
Black markers (one per student)

- Directions:**
1. Students work in pairs for this activity. Have each pair of students number 10 tiles with the digits 0–9. Have them put a “+” and “–” on the remaining 2 tiles.
 2. Explain that you will write on the board a list of numbers and a list of digits to be used to reach those numbers. Using the tiles with the given digits and the “+” and “–” signs, the partners should come up with a number sentence that will equal each target number. Both addition and subtraction **must** be included in the number sentence. Example: If the target number is 45 and the digits to be used are 1, 3, 4, 5, and 6, the students might make the following number sentence: $56 - 14 + 3 = 45$.
 3. Tell the students that there may be more than one solution for the target numbers.
 4. Be sure students record their findings to share with the class at the end of the activity.
 5. Examples:

Number to Reach	Digits to Use	Possible Solution
83	1, 2, 4, 5, 7	$75 + 12 - 4$
90	0, 1, 2, 4, 6, 9	$49 + 61 - 20$
107	0, 3, 6, 8, 9	$80 - 9 + 36$
610	0, 2, 3, 5, 6, 7, 8	$562 + 78 - 30$

Who Is Faster, You or Your Calculator?

Materials: Calculators (one per pair of students)
"Who Is Faster?" handout (one per student; blackline master follows this activity)

- Directions:**
1. Begin the activity by asking who would like to have a calculation race with you. Choose a volunteer (a student comfortable with basic addition and subtraction facts).
 2. Have another student make up an addition problem with four- or five-digit numbers that does not require regrouping. Have the student write the problem twice, once on one sheet of paper and once on another. Have the student give one sheet to the first student volunteer and one to you. The student volunteer will solve the problem directly on the paper; you will solve it using a calculator.
 3. Select another student to start the race. Whoever finishes first is the winner.

*TEACHER NOTE: The intent of this activity is to show students that it is sometimes faster to solve problems using paper and pencil. Helping students to become discriminating calculator users can be a difficult task, especially if students are still "fascinated" with this tool.

4. Have students brainstorm circumstances when it would be to their advantage to use a calculator and when it would be advantageous to use paper and pencil. Ask students which is more accurate, using the calculator or using paper and pencil. Allow students to justify their responses.
5. Give each pair of students a calculator and two copies of the "Who Is Faster?" handout. Each pair will have their own "contest" to see who is faster. Have the partners decide who will use the calculator first. Then have them look at the worksheet, solve the first problem, and record whether the calculator user or the problem solver finished first. Have them continue on to the other problems, switching roles after problem #5 so that each student will have an opportunity to use the calculator.
6. Have the students discuss their results with the class.



Who Is Faster?

Directions: With your partner, solve the following problems. Each of you should have a copy of this worksheet. One of you should use a calculator to solve the problems; the other should solve the problems using just a pencil and this paper. Determine a fair start. The first one finished may check the appropriate column to show that he/she finished first.

1. $\begin{array}{r} 8,346,451 \\ + 123,316 \\ \hline \end{array}$	2. $\begin{array}{r} 82,975,400 \\ + 7,006,523 \\ \hline \end{array}$	Calculator user finished first	Problem solver finished first
3. $\begin{array}{r} 879,564 \\ - 42,352 \\ \hline \end{array}$	4. $\begin{array}{r} 21,438,953 \\ + 71,251,006 \\ \hline \end{array}$	1.	
		2.	
5. $\begin{array}{r} 73,298,721 \\ + 25,301,178 \\ \hline \end{array}$	6. $\begin{array}{r} 43,643,643 \\ - 21,212,121 \\ \hline \end{array}$	3.	
		4.	
7. $\begin{array}{r} 10,486,259 \\ - 8,265,137 \\ \hline \end{array}$	8. $\begin{array}{r} 90,909,090 \\ + 7,070,707 \\ \hline \end{array}$	5.	
		6.	
9. $\begin{array}{r} 462 \\ 814 \\ 39 \\ 324 \\ 755 \\ 2,738 \\ + 3,140 \\ \hline \end{array}$	10. $\begin{array}{r} 3,820 \\ 6,299 \\ 4,516 \\ 382 \\ 81 \\ + 376 \\ \hline \end{array}$	7.	
		8.	
		9.	

Use with "Who Is Faster, You or Your Calculator?" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Problem Solving

Writing Your Own

Materials: Optional: Calculators (one per student)

- Directions:**
1. Discuss with the students the components that go into writing good word problems and the fact that problems are sometimes written with insufficient or extraneous information.
 2. Ask the students to write five word problems involving addition or subtraction. Have them put the answers on a separate sheet of paper. (The students may want to use calculators to check their work.)

**TEACHER NOTE:* Depending upon the group, problems can be written with insufficient or extraneous information. Students can then determine what information is/is not needed. Discussion should follow.

3. When the students are finished writing their word problems, collect all the problems (have the students keep their own solutions) and redistribute them for other students to solve. If a student has difficulty solving a problem, encourage him/her to discuss the problem with its author.

How Close Can You Get?

Materials: Calculators (one per student)
1" ceramic tiles numbered 1-9 (one set per student)

- Directions:**
1. Have the students arrange their tiles into three 3-digit numbers. The goal is to select numbers that, when added, will have a sum as close to 1,000 as possible.
 2. Explain that the students should try various number combinations, recording each problem they try. However, they should not work the problems; instead, they should use estimation to get an approximate sum.

3. When the students feel they have reached the sum closest to 1,000, have them use a calculator to find the exact answer to their problem.
4. When the students have finished the activity, discuss the estimation strategies they used. Have them refer to their record of problems and ask if they found any patterns in the placement of digits to get them as close to 1,000 as possible. If not, ask, "What did you do to get closer to 1,000? How close was your sum?" Students should compare answers to see who came closest.
5. Have the students think overnight about how they can get their sum even closer to 1,000 than the closest one found in class. Have the students report to the class the next day on their calculations. Discuss whether it helped to be given a second chance, to be given extra time to figure the problem, and to know what number you need to "beat." Ask, "Did these opportunities allow you to find ways to come even closer to the given number? What were the advantages? Disadvantages?"
6. Variation: Have the students use their tiles to figure the sum closest to 2,000.

How Much Money Can You Win?

Materials: Three dice per group
"How Much Money Can You Win?" handout (one per student;
blackline master follows this activity)

- Directions:**
1. Have the students pretend that you are the owner of a new business in town and that, for its grand opening, you are going to award a cash prize to the person who can roll the largest sum of money in 10 dice rolls.
 2. The rules of the contest are simple. A contestant must roll three dice and arrange the dice to make any three-digit number. The contestant will do this 10 times.
 3. After 10 rolls of the dice, the contestant will add the numbers rolled.

4. The contestant with the largest sum will win the amount of money he/she rolled.
5. Have the students in each group take turns rolling the dice and recording the numbers on their handout. Then have them add the numbers and compare sums.
6. Have the students complete the questions at the bottom of the handout. Discuss the strategies the students used to determine the answers.



How Much Money Can You Win?

Directions: Roll the three dice 10 times. After each roll, arrange the numbers to form a three-digit number. Record the numbers below. Add your numbers to find out how much money you might win!

First Throw	\$ _____
Second Throw	\$ _____
Third Throw	\$ _____
Fourth Throw	\$ _____
Fifth Throw	\$ _____
Sixth Throw	\$ _____
Seventh Throw	\$ _____
Eighth Throw	\$ _____
Ninth Throw	\$ _____
Tenth Throw	\$ _____

Total Amount of Winnings \$ _____

- Let's say that, rather than rolling the dice, you could choose the number to record each time.

What is the largest amount of money you could win? _____

What is the smallest you could win? _____

- Now look at the 10 numbers you wrote down. Rearrange each number to determine the following:

What is the largest amount of money you could win? _____

What is the smallest amount you could win? _____

- Compare your total with that of one of your classmates.

Who would win more money? _____

How much more? _____

Use with "How Much Money Can You Win?" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

United States Divisibles

Materials: Almanacs
Map of the United States (one per group of students)
Markers, crayons, or colored pencils
Calculators

- Directions:
1. In small groups, have the students research the population of each state and record it on their map.
 2. Say, "I would like you to divide your maps into six regions with fairly equal population totals. Then color code your maps to denote those regions."
 3. Have the groups share their maps and discuss any differences.
 4. Say, "Now I'd like you to figure out the largest population difference between regions on your map."
 5. Have the students in each group plan a trip throughout one of the regions. Have them find the total distance for the trip.
 6. Have the students write their own questions about the data they collected.

Assessment Activities

SKILL Regrouping to solve addition and subtraction problems through 99 2.3-1

Have the students solve the following problems.

$$\begin{array}{r} 1. \quad 98 \\ - 17 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 45 \\ - 33 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 24 \\ + 53 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 75 \\ - 38 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 57 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 48 \\ + 34 \\ \hline \end{array}$$

$$7. 11 + 65 = \underline{\hspace{2cm}}$$

$$8. 69 - 26 = \underline{\hspace{2cm}}$$

$$9. 29 + 62 = \underline{\hspace{2cm}}$$

$$10. 92 - 58 = \underline{\hspace{2cm}}$$

$$11. 37 + 42 = \underline{\hspace{2cm}}$$

$$12. 74 - 51 = \underline{\hspace{2cm}}$$

SKILL Regrouping to solve addition and subtraction problems through 999 2.3-2

Have the students solve the following problems.

$$\begin{array}{r} 1. \quad 645 \\ + 132 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 438 \\ - 126 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 203 \\ + 199 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 254 \\ - 241 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 389 \\ + 136 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 587 \\ - 149 \\ \hline \end{array}$$

$$7. 242 - 98 = \underline{\hspace{2cm}}$$

$$8. 582 + 17 = \underline{\hspace{2cm}}$$

SKILL Regrouping to solve addition and subtraction problems through 9,999

3.3-3

Have the students solve the following problems.

$$\begin{array}{r} 1. \quad 607 \\ + 224 \\ \hline \end{array}$$

$$2. \quad 812 - 355 = \underline{\hspace{2cm}}$$

$$\begin{array}{r} 3. \quad 6,212 \\ \quad 409 \\ + 2,033 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 6,203 \\ - 1,516 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 5,704 \\ - 2,376 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 782 \\ + 7,643 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 2,753 \\ \quad 1,647 \\ + 1,653 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 2,473 \\ - 2,219 \\ \hline \end{array}$$

SKILL Regrouping to solve addition and subtraction problems through 99,999

4.3-4

Have the students solve the following problems. Remind them to show their work and watch the signs.

$$\begin{array}{r} 1. \quad 412 \\ + 368 \\ \hline \end{array}$$

$$2. \quad 648 - 516 = \underline{\hspace{2cm}}$$

$$\begin{array}{r} 3. \quad 2,385 \\ + 5,076 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 4,361 \\ - 1,250 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 77,989 \\ - 36,709 \\ \hline \end{array}$$

$$6. \quad 4,489 + 5,510 = \underline{\hspace{2cm}}$$

$$\begin{array}{r} 7. \quad 49,803 \\ + 12,928 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 8,000 \\ - 2,105 \\ \hline \end{array}$$

SKILL**Regrouping to solve addition and subtraction problems through 9,999,999****5.3-5**

Have the students solve the problems and circle the correct answer.

1.
$$\begin{array}{r} 59,627 \\ - 6,288 \\ \hline \end{array}$$

- A. 53,341
B. 53,339
C. 65,915
D. 53,441
E. 52,429

2.
$$\begin{array}{r} 4,210,502 \\ - 314,566 \\ \hline \end{array}$$

- A. 3,895,936
B. 4,104,064
C. 3,786,846
D. 4,525,068
E. 3,524,066

3.
$$\begin{array}{r} 48,667 \\ 7,822 \\ 62,144 \\ + 13,681 \\ \hline \end{array}$$

- A. 132,413
B. 131,404
C. 78,413
D. 131,241
E. 132,314

4.
$$\begin{array}{r} 2,688,596 \\ - 604,672 \\ \hline \end{array}$$

- A. 83,924
B. 2,292,168
C. 3,283,278
D. 3,293,268
E. 3,148,268

SKILL Demonstrating a model for multiplication

Skill Analysis

- A. Can interpret the "x" and "=" symbols
- B. Can construct a model for multiplication problems
- C. Can apply the model to find a solution



Concept Manipulative Activities

Array Multiplication

Materials: Overhead projector, blank transparency, Vis-A-Vis pen
 UNIFIX cubes, wooden cubes, chips, or other manipulatives
 suitable for forming arrays
 Construction paper squares
 3/4" graph paper (blackline master may be found in the
 Appendix)

Directions: 1. Using manipulatives, build two lines (groups) with five units in each group. Show your work on the overhead. Then write the addition problem the model represents and the related multiplication problem.

Example:

						5	$2 \times 5 = 10$
						+ 5	
						10	

2. Build several other arrays with manipulatives, each time showing the addition and related multiplication problems. Then have the students practice doing the same.

3. Next, demonstrate array multiplication using construction paper squares. On the overhead, make two rows of construction paper squares with five squares in each row. Write the addition and multiplication sentences next to the array. Example:

<table border="1" style="border-collapse: collapse; width: 100px; height: 40px;"> <tr><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td></tr> </table>											$\begin{array}{r} 5 \\ + 5 \\ \hline 10 \end{array}$	$2 \times 5 = 10$

4. Go over a few examples. Then have the students practice. Instruct them to glue their arrays to another piece of paper and write the addition and multiplication sentences next to them.
5. Another way to demonstrate array multiplication is to use graph paper. On a graph paper transparency, outline two rows with five squares in each row and then color the squares within. Write down the corresponding addition and multiplication problems. Example:

<table border="1" style="border-collapse: collapse; width: 150px; height: 100px;"> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> <tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr> </table>																																									$\begin{array}{r} 5 \\ + 5 \\ \hline 10 \end{array}$	$2 \times 5 = 10$

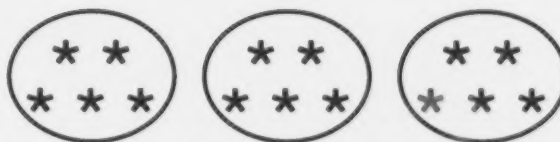
6. Have the students practice making graph paper arrays and writing down the corresponding addition and multiplication problems.

Magnetic Board Sets

Materials: Flannel board or magnetic board
Flannel or magnetic objects

Directions: 1. Place three sets of five objects on the flannel or magnetic board.

Example:



2. Ask, "How many sets do we have? How many objects are in each set? How many objects do we have altogether?"
3. Continue asking questions such as, "How can we write a sentence about these sets?"
 3 sets of 5
 $5 + 5 + 5 = 15$
4. Repeat the activity using other sets and amounts in each set.
5. At this point, if you have not yet introduced the "x" sign, explain that three groups with five in each group is written 3×5 .

Rows and Columns

Materials: UNIFIX cubes, buttons, beans, etc., for modeling
 3/4" graph paper (blackline master may be found in the Appendix)
 Overhead projector

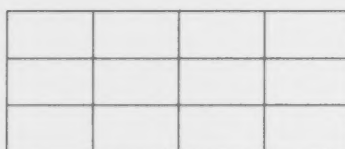
- Directions:**
1. Have the students build three sets of four counters.
 2. Say, "I'd like you to form the sets into a horizontal line."
 Example:



3. Say, "Now I'd like you to place the counters like this."
 (Align the counters to form an array on the overhead.)
 Example:



4. Discuss how, in both of these arrangements, there are still three sets of four counters.
5. Repeat the activity with other arrays (e.g., 2×5 , 1×7 , 3×2).
6. Say, "We could also draw lines around the counters in each row and each column to group them."
7. Draw the lines around the counters on the overhead.
8. Have the students model other arrays with the counters, drawing lines to group the counters in each set.
9. Next, use the $3/4$ " graph paper and the counters to model some problems. Position the counters on the graph paper in an array, draw around the array, and remove the counters.
Example:



10. Ask, "What is left when we remove the counters?" (4 by 3 squares)
11. Ask, "If I give you just the lines of an array, can you tell me how many counters I will need to finish the drawing?"
12. Allow the students to discuss how they would determine their answer.
13. Have the students practice using arrays on the graph paper without the counters.

Equivalent Sets

Materials: None

- Directions:
1. Have the students look around the room to find things that are in equivalent sets. Ask, "What equivalent sets do you see?" Possible responses might be drawers in a file cabinet, desks in groups, legs on tables, legs on chairs, etc.
 2. Ask the students to think about other things that are grouped or come in equivalent sets. Possible answers might be the three wheels on a tricycle, the two wheels on a

250

bicycle, the five sticks of gum in a pack, pop cans in a six-pack, the eggs in a carton, etc. Have students bring in pictures or examples.

3. Discuss with the students the idea of total amount. For example, ask, "If you had three packs of gum, and there were five sticks of gum in each pack, how many sticks of gum would you have in all?" Ask the students how they could find the answer. (Count by ones or fives.) Ask, "How many cans of pop would you have if you had three six-packs?" Continue on in this fashion.
4. Variation: Have each student make a chart listing things that come in sets of two, three, four, etc.
5. Variation: Have the students record a few examples of things that come in multiples and take the sheet home to find five more examples. Example:

Item	Number in group
eggs	12

Straws

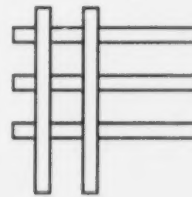
Materials: Cups
Straws or sticks

- Directions:
1. Display three cups.
 2. Ask a student to put two straws in each.
 3. Ask a volunteer to describe the sets.
 4. Ask another student to name the total number of straws.
 5. Repeat the activity with different numbers of straws. Include cups with zero straws and with one straw.

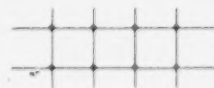
Intersections

Materials: Strips of paper

Directions: 1. Have the students use strips of paper to make the patterns you describe. Begin by saying, "Lay three strips horizontally and then place two strips vertically on top of the horizontal strips to make an intersecting pattern. Example:



2. Ask, "How many points of intersection are there?" (6)
3. Have the students write the multiplication sentence describing "three strips by two strips is six intersections." ($3 \times 2 = 6$) Explain that they can think of the strips as intersecting streets.
4. Give the students other multiplication problems to model as intersecting strips or streets (e.g., 3×4 , 2×5 , 2×7 , 3×6). After they make each model, have them write down the problem and its solution (e.g., $3 \times 4 = 12$), which they would find by counting the intersections.
5. Ask, "If we didn't have paper strips or similar objects, how could we model these problems?" Discuss alternatives with the class. One method would be to model the problems using paper and pencil. For example, for 2×4 , the students could draw:



($2 \times 4 = 8$ intersections)

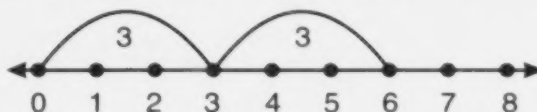
6. Have the students model other problems using paper and pencil.

252

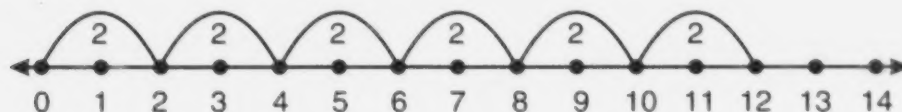
Number Line Multiplication

Materials: Number lines (desk top, walk on, etc.)
 "Number Lines" handout (one per student; blackline master may be found in the Appendix)

- Directions:**
1. Even though the students have had some experience with the number line, a short review would be appropriate. Discuss the following features:
 - a. Equal spaces between numbers
 - b. All numbers are on the line
 - c. The numbers are represented on the line by a dot
 2. Introduce the concept of equal jumps. A frog jumps 3 units each time it hops. If it hops two times, where is it? Example:



3. The students need to count the spaces between each hop in order to make each equivalent jump. The product will be the number at the end of the final jump. 2×3 is 2 jumps of 3 units. (6)
4. The relationship of repeated addition can be shown. Note that 2×3 is the same as $3 + 3$ or 2 jumps of 3.
5. Example: $6 \times 2 = \underline{\quad}$, is six jumps of 2 each. The frog ends at 12.



$$2 + 2 + 2 + 2 + 2 + 2 = 12$$

The repeated addition problem is optional depending upon whether repeated addition is being used as a model.

6. **Variation: Walk on Number Line**
 Give the student a problem (e.g., $3 \times 2 = \underline{\quad}$ and have the student hop two units, three times).

People Multiplication

Materials: None

- Directions:
1. Use the students to demonstrate combining equivalent sets.
 2. Set out six chairs. Have two students sit on each chair. Ask different children to describe what they see. Ask, "How many groups (sets) of two do we have?" (6)
 3. Set out three tables. Have three children get under each table. Ask, "How many groups are there?" (3) "How many in each group?" (3) "What is the total?" (9)
 4. Continue with other examples (e.g., four chairs with zero students; seven children, two feet each; ten hands, five fingers each; etc.).
 5. Besides describing the composition of the sets, students should begin to explore strategies for finding the total.



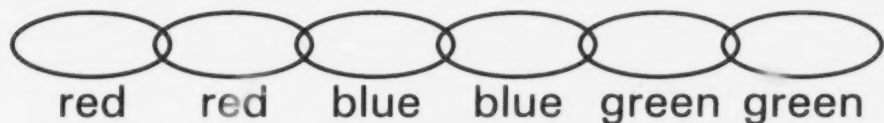
Let's Practice

***TEACHER NOTE:** The intent of this outcome is for the students to become adept at modeling multiplication using the models described in the Concept Manipulative Activities. The students should use models while working the practice activities.

Paper Chains

Materials: Different colored construction paper strips
Task cards
Glue or tape

- Directions:**
1. In this activity, the students will construct paper chains based on the instructions given on task cards. Construct task cards that read, for example, "three colors, two each."
 2. Have the students select a card and make the chain described from strips of colored construction paper. Explain that like colors should be linked together. Example:



3. Have the students exchange their chains with a classmate. The classmate should look at the chain and explain the model (e.g., "You have three groups of two, which is six altogether").

Dot Multiples

Materials: Dot paper (one sheet per student; blackline masters may be found in the Appendix)

- Directions:**
1. Give each student a sheet of dot paper.
 2. Explain to the students that they are going to form sets of dots and find the total number of dots in all the sets.

3. Have the students circle three sets with four dots in each set. Then have them count the total number of dots.
Example:



4. Give the students other problems to model on the dot paper (e.g., two sets of six, three sets of five, five sets of five).

Universal Multiplication

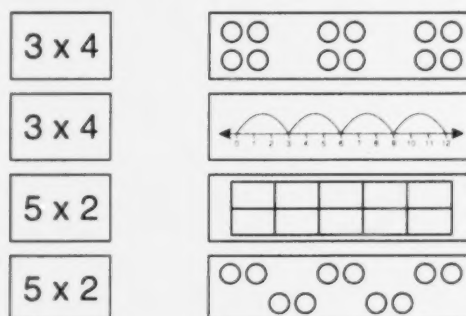
Materials: Universal Game Board (directions for its construction may be found in the Appendix)
Construction paper or tagboard for task cards
Materials for modeling (number lines, counters, rods, graph paper, etc.)
Game markers

- Directions:**
1. This game is played in groups of 2 to 4 students.
 2. Give each group a Universal Game Board and task cards on which you have written easy multiplication facts (8×2 , 7×1 , 3×2 , 4×0 , 5×5 , etc.).
 3. The first student will draw a card and model the problem written on it using counters, rods, arrays, number lines, or graph paper.
 4. Then the student will write the problem and answer on a sheet of paper. If the answer is correct, the student will move his/her game marker two spaces on the game board. If the answer is incorrect, the student will not move the marker.
 5. Play continues until one or all of the students reach the finish line.

Match That Fact

Materials: Blank playing cards or 3" x 5" index cards

- Directions:
1. Give each student 10 blank cards and five multiplication problems.
 2. Have the students write the problems on the cards (one per card) and then make a model for each problem on the remaining cards. Have the students use the model they are most familiar with (e.g., arrays, sets, number lines, rods). The following are some sample cards (students would use only one of the models):



3. Circulate among the students to check the problems and the models.
4. Next, divide the class into groups of four to five students.
5. Have the students in each group mix their cards together and deal out the entire deck. Explain that the students will, in turn, remove all pairs in their hand and lay them face up on the table. For each pair, the students will explain why the model card matches the fact card.
6. Then the first player will draw a card from the hand of the student to his/her right. If the card forms a match with one in the students's hand, the student will lay the match on the table and explain it.
7. Play continues to the right with each student, in turn, drawing a card from the hand of the player to his/her right until all of the cards are laid down in pairs.
8. Variation: An "Old Multiplication" card can be put in the deck and the game played like "Old Maid."

Roll Those Numbers

Materials: Two wooden cubes (yellow and white) numbered 0–5 for each pair of students

Counters, arrays, number lines, or rods for modeling

Two record sheets for each pair of students

Example:

ROLL THOSE NUMBERS				
White		Yellow		
_____	x	_____	=	_____
_____	x	_____	=	_____
_____	x	_____	=	_____
_____	x	_____	=	_____
_____	x	_____	=	_____
_____	x	_____	=	_____
_____	x	_____	=	_____
_____	x	_____	=	_____

- Directions:**
1. Give each pair of students two record sheets and two wooden cubes, one of each color.
 2. Each partner will, in turn, roll the cubes and record the numbers rolled on his/her record sheet.
 3. The student will then model the multiplication problem using counters, arrays, number lines, or rods and record the answer on the record sheet.

***TEACHER NOTE:** The record sheets could be laminated and Vis-A-Vis pens used for recording.

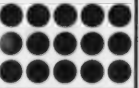
Name That Array

Materials: Blank playing cards or 3" x 5" index cards

- Directions:**
1. Make a set of array cards. Example:



2. Give a card to each student. The students should write down the multiplication problem represented by the array and find the answer for the problem. Example:

B.  $3 \times 5 = 15$

**TEACHER NOTE:* Allow the students to write either $3 \times 5 = 15$ or $5 \times 3 = 15$. Learning to use the model is more important than the order of the factors.

3. Have the students exchange cards and repeat the activity. The students should keep a record sheet like the following so their work can be checked.

Array	Multiplication Problem
A	$2 \times 3 = 6$
B	$3 \times 5 = 15$

4. Variation: The cards can be used with a Universal Game Board. Each student would, in turn, draw a card and write down the problem and answer. If the student is correct, he/she would move forward on the game board.

Word Problems

Materials: UNIFIX cubes

- Directions:
1. Give the students story problems to model with their cubes.
 2. Problems are more fun when they are directly related to students in the class. In the following examples students' names can be used to fill in the blanks.
 - a. _____ baked six cakes. Each cake had two layers. How many layers are there in all?
 - b. _____ built five buildings. Each building had three stories. What is the total number of stories?
 - c. _____ ate two plates of spaghetti. Each plate had six meatballs on it. How many meatballs did _____ eat?
 - d. _____ baked five M&M cookies. Each had six M&M's. How many M&M's did _____ use in all?

- e. _____ builds model trains. Each has six cars. He built four trains today. How many cars were there?
- f. _____ is having a birthday party tomorrow. Everyone will get a cup of nuts. There will be 10 nuts in a cup. Seven people will be at the party. How many nuts does _____ need?
3. Have the students make up their own stories to be modeled and solved by the rest of the class.

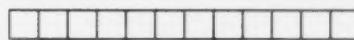


Problem Solving

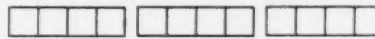
Number Family Graphs

Materials: 3/4" graph paper (blackline master may be found in the Appendix)
UNIFIX cubes, wooden blocks, etc., for modeling
Glue
Colored pencils or crayons

- Directions: 1. Begin this activity by having students use different colored cubes or blocks to model different sets representing the same number.
2. Ask, "How many different ways can you model 12 using equivalent sets?"



1 group of 12



3 groups of 4



2 groups of 6

Also, 4 groups of 3, 6 groups of 2, and
12 groups of 1 could be modeled.

3. Have the students color graph paper strips to match their models and glue them onto a sheet of paper as a way of recording their work.
4. Discuss the models with the students. Tie in the concept of different names for the same number.
5. Repeat the activity using other numbers (e.g., 8, 9, 15, 18).

260

Multiples of Money

Materials: Real or play money (blackline masters may be found in the Appendix)

- Directions:**
1. In this activity the students are going to form equivalent sets of money and determine the total value of the money.
 2. Ask, "If ____ pennies are equal to \$1.00, how many pennies are equal to \$3.00?" (By modeling, the students can determine that 100 pennies are equal to \$1.00 and that 300 pennies are equal to \$3.00.)
 3. Ask other questions such as:
 - a. If ____ nickels are equal to \$1.00, how many nickels are equal to \$2.00?
 - b. If ____ dimes are equal to \$1.00, how many dimes are equal to \$4.00?
 - c. If ____ quarters are equal to \$1.00, how many quarters are equal to \$4.00?
 - d. If ____ half-dollars are equal to \$1.00, how many half-dollars are equal to \$3.00?
 - e. If ____ dimes are equal to 50¢, how many dimes are equal to \$1.50?
 - f. If ____ quarters are equal to 50¢, how many quarters are equal to \$2.50?

**S
T
O
R
Y**

**P
R
O
B
L
E
M
S**

Directions: A circus parade is going through your town. Draw pictures to help you find answers to the following questions about the parade.

1. There were five clowns in the parade. Each clown carried three balloons. How many balloons were there?
2. Four cages went by. There were two lions in each cage. How many lions were there altogether?
3. Next came the elephants. There were six altogether. Two riders were on each elephant. How many people were riding the elephants?
4. A funny-looking bicycle came by. It had four clowns on it. We saw how many clowns this time?
5. Following the clowns were two jugglers. Each of them was juggling three balls. How many balls were being juggled?
6. Write two of your own number stories about the circus. Draw pictures for your stories.



*TEACHER NOTE: The students may need to use manipulatives to help them solve these problems. Encourage them to draw pictures, too. Drawing pictures is a problem solving strategy.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Pattern Block Multiplication

Materials: Pattern Blocks

Directions: 1. Have the students use Pattern Blocks to find the answers to the following, and similar, problems.

- a. What is the total number of sides on four green triangles?
- b. What is the total number of sides on five orange squares?
- c. What is the total number of sides on one red trapezoid?
- d. What is the total number of sides on two yellow hexagons?
- e. What is the total number of sides on three blue parallelograms?
- f. What is the total number of angles on five red trapezoids?
- g. What is the total number of angles on two tan parallelograms?
- h. What is the total number of angles on three green triangles?
- i. What is the total number of angles on four orange squares?
- j. What is the total number of angles on one yellow hexagon?

2. Have the students keep a record sheet like the following.

	Number of pieces	Number of sides or angles	Total	Multiplication Sentence
a.	4	3	12	$4 \times 3 = 12$
b.				
c.				
d.				

B
R
E
A
K

T
H
E

C
O
D
E

Directions: Solve these problems using materials for modeling. Then break the code in the saying at the bottom of the page by writing the letter that corresponds with each problem on the line above the answer for that problem.

$\begin{array}{r} 2 \\ \times 8 \\ \hline \end{array}$ M	$7 \times 3 = \underline{\quad}$ U	$0 \times 9 = \underline{\quad}$ H
$5 \times 3 = \underline{\quad}$ S	$1 \times 6 = \underline{\quad}$ A	$\begin{array}{r} 4 \\ \times 5 \\ \hline \end{array}$ F
$4 \times 1 = \underline{\quad}$ I	$\begin{array}{r} 6 \\ \times 2 \\ \hline \end{array}$ N	$\begin{array}{r} 8 \\ \times 1 \\ \hline \end{array}$ T

Write the appropriate letter in each blank.



<u>16</u>	<u>6</u>	<u>8</u>	<u>0</u>	<u>4</u>	<u>15</u>
<u>20</u>	<u>21</u>	<u>12</u>			



SKILL Demonstrating a model for division

Skill Analysis

- A. Can interpret the " $\div$ " and " $\overline{)}$ " symbols
- B. Can construct a model for division problems
- C. Can apply the model to find a solution
- D. Understands the concept of division



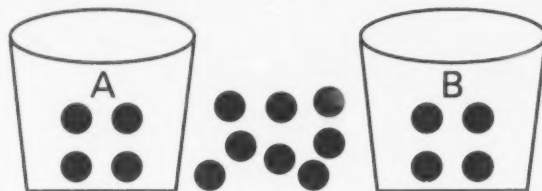
Concept Manipulative Activities

Set Division

Materials: Counters (UNIFIX cubes, buttons, keys, etc.)

- Directions:
1. Give each student 20 counters.
 2. Present the problem $12 \div 3$ to the students. Discuss the symbols used in this problem.
 3. Have the students put 12 objects in three equal sets or build sets with 3 objects in each. Do not have the students do both with the same problem; doing so may be confusing at this time.
 4. Provide practice with other problems, such as $8 \div 2$, $15 \div 3$, $15 \div 5$, $16 \div 8$, and $20 \div 4$. Have the students model the problems and record the results.
 5. Divide the class into groups of two or three students. Have the students in each group work together on larger problems (e.g., $27 \div 3$, $35 \div 7$, $42 \div 6$, $21 \div 7$, $30 \div 5$, $48 \div 8$, $32 \div 4$). Again, have the students model the problems and record the results.
 6. Variation: Use cups for the sets. For the problem $16 \div 2$, give the students two cups and have them distribute the

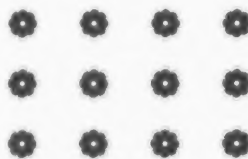
counters equally by putting one in cup A and one in cup B, a second in cup A, a second in cup B, etc., until all the counters are in the cups. Then have the students count the objects and complete the problem: $16 \div 2 = 8$. Repeat the activity with other problems. For $18 \div 3$, for example, use three cups; for $15 \div 5$, use five cups. Finally, present problems and let the students (1) determine the number of cups needed, and (2) solve the problem.



Array Division

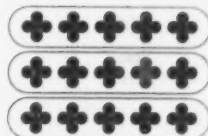
Materials: Counters (UNIFIX cubes, buttons, blocks, etc.)

Directions: 1. Review multiplication with arrays. For example, give the students the problem 3×4 and have them make the model (3 rows of 4 objects).



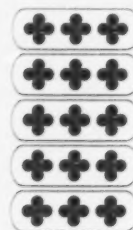
2. To model division with arrays, write the problem ($15 \div 5$) on the board. Count out 15 items and then model the problem by making 3 rows of 5 objects and then 5 rows of 3 objects. Make sure the students understand both models and can name 3 as the quotient.

Example:



$$15 \div 5 = 3$$

Repeated Subtraction Model
 "How many groups (rows) of 5 are there in 15? There are 3 rows."



$$15 \div 5 = 3$$

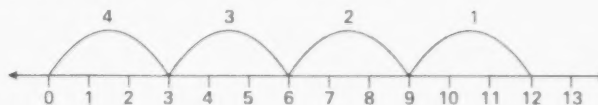
Partition Model
 Partition the set into 5 groups (rows). "How many are in each row? There are 3 in each row."

3. Give the students other problems to model (e.g., $16 \div 4$, $18 \div 3$, $18 \div 6$, $21 \div 7$, $8 \div 1$, $42 \div 7$, $35 \div 5$). Have them model the problems and record their results.

Division on the Number Line

Materials: Number lines (walk-on, individual, overhead projector, large)
 "Number Lines" handout (blackline masters may be found in the Appendix) and transparency
 Overhead projector
 Vis-A-Vis pen

- Directions:**
1. If possible, have students use a walk-on number line.
 2. Have a student stand on the number 12. Say, "Take jumps of 3 until you get to 0. How many jumps did you make?" Have another student keep a record of the number of jumps made.
 3. Model the jumps on the overhead.



$$12 \div 3 = 4$$

4. Model how to write the division problem $12 \div 3 = 4$. Explain that 12 is where the student started, 3 is how big each jump was, and 4 is how many jumps the student made in all.

5. Select another student to model the problem $6 \div 3$. Say, "Start at 6 and take jumps that are 3 spaces long. How many jumps did you make in all?"
6. Repeat the activity with other division problems until the students are comfortable with number line division.
7. Give each student a "Number Lines" worksheet. Have the students practice modeling problems and writing the corresponding number sentences. For some problems, give the students the number sentence to model on the number line; for other problems, give the number line model on the overhead and have the students copy it onto their worksheets and then write the number sentence.

Rows

Materials: Counters (beans, UNIFIX cubes, blocks, etc.)

- Directions:
1. Give the students 12 counters.
 2. Say, "Make rows of 3 counters. How many rows can you make?" (4)
 3. Continue the activity using other practice problems (e.g., given 14, make rows of 2; given 26, make rows of 4 [2 will be left over]; given 30, make rows of 10). Give the students some problems that will not have a remainder and some that will. Discuss both situations with the students.

Draw a Picture

Materials: Counters (blocks, UNIFIX cubes, etc.)

- Directions:
1. Give the students division word problems to solve by either building models or drawing pictures on a piece of paper. Use students' names and familiar situations in the word problems.
 2. Examples:
 - a. Matt had six cans. He gave them to his two friends Scott and Devin. Each friend has the same number of cans. How many cans does each have?

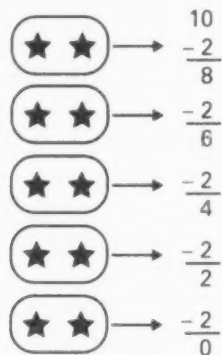
268

- b. Jason had six books. He put them in three piles. Each pile had the same number of books. How many books were in each pile?
- c. Amy had 11 cookies. She put them on three plates. Each plate had the same number of cookies. How many cookies were on each plate? (Discuss what happens when there is a remainder.)

Take Aways

Materials: Counters (beans, UNIFIX cubes, chips, etc.)

- Directions:
1. Have the students make a set of 10 counters. Ask, "How many groups of two can be taken away from 10? How could we find out?"
 2. Have the students remove sets of two, keeping a written record of their work. Example:



3. Ask, "How many groups of two did you take away? Were there any left over?"
4. Repeat the activity using some problems that will have a remainder and some that will not. Discuss remainders with the students emphasizing that there is a remainder when there are not enough objects remaining to make another equivalent set the size of the divisor. Ask questions like the following:
 - a. If you have 12 objects, how many groups of three can you make?
 - b. If you have 10 objects, and make two equivalent sets, how many objects will be in each set?

- c. If you have 13 objects, and make three equivalent sets, how many objects will be in each?
- d. If you have 14 objects; how many groups of five can you make?

Block Problems

Materials: Counters (blocks, UNIFIX cubes, etc.)

- Directions:
1. Create division word problems for students to solve using their counters. Use your students' names and familiar situations in the problems.
 2. Examples:
 - a. Dave had six chairs. He put them at three tables. He put the same number of chairs at each table. How many did he put at each table?
 - b. Karen had eight presents. She put them on the table in two stacks. She put the same number of presents in each stack. How many did each stack contain?
 - c. Kara had 10 cans. She put them into five cupboards. She put the same number in each cupboard. How many cans did she put in each cupboard?



Let's Practice

People Problems

Materials: Classroom props

- Directions:
1. Create division story problems for the students to act out. Use your students' names, neighborhood places, and other familiar names in the problems to make them more meaningful.
 2. Examples:
 - a. Marcia had six books. Using the books she had, she gave Marilyn and Barb the same number of books. How many did each girl receive?

270

- b. Don had eight cookies. Using the cookies he had, he gave the same number of cookies to Mike, Joe, Chuck, and Tim. How many cookies did he give each child?
 - c. Matt had five pencils. He put them in equal groups in five boxes. Each box had the same number of pencils in it. How many pencils were in each box?
3. Have the students act out the problems to solve them.
 4. Have the students practice with a variety of problems. You may want to extend the activity by having the students make up their own problems for the rest of the class to solve.

Picture of Division

Materials: Counters (beans, bottle caps, popcorn, etc.)

- Directions:**
1. Give each student a different number (e.g., 4, 6, 8, 9, 10, 12, 14, 16, 18).
 2. Ask the students to use their counters to sort out different ways their number can be divided equally. Have them share the different configurations with the class.
 3. Then ask the students to show, by illustration, the different ways their number can be separated into equal groups.
 4. Have them write the division number sentence beneath each illustration.
 5. The completed pictures can be used for a bulletin board display. Examples:



$$8 \div 1 = 8 \text{ or } 1 \overline{)8}$$



$$8 \div 8 = 1 \text{ or } 8 \overline{)8}$$



$$8 \div 2 = 4 \text{ or } 2 \overline{)8}$$



$$8 \div 4 = 2 \text{ or } 4 \overline{)8}$$

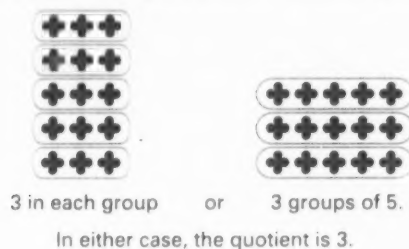
Blimpo

Materials: Universal Game Board (directions for its construction may be found in the Appendix)
 Division cards
 Game markers
 Counters

- Directions: 1. The game is played by four to six students.
 2. Place a pile of division cards on the game board. Each card should contain a different division problem. Example:

$$15 \div 5$$

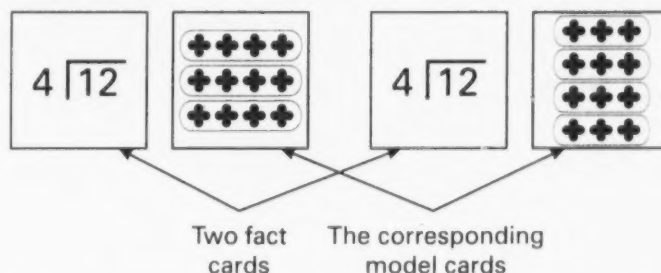
3. The first student draws a card, constructs the model for the division problem using manipulatives, and finds the answer.
 4. If the model and answer are correct, the child moves his/her game marker forward two spaces on the game board. If the model or answer is incorrect, the child does not move his/her marker.
 5. Play continues with students taking turns drawing a card until one or more students reach the finish line.
 6. Variations:
 a. At the start of each turn, the student rolls a die with sides numbered 1, 1, 2, 2, 3, 3. If the student's model and answer for the division problem are correct, he/she moves forward that many spaces.
 b. The models can be drawn on the back of the cards so the students can self-check their work. Both models should be included for each problem. Example: $15 \div 5 = 3$



Match Up

Materials: Blank playing cards

- Directions:
1. This game is played in groups of three to four students.
 2. Give each group a deck of cards that contains division problems and models for them. For each problem, there should be two identical fact cards and two different model cards. Example:

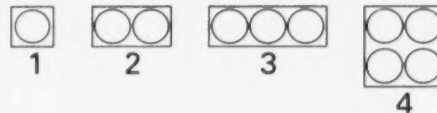


3. Have the students mix the cards and deal out the entire deck. Explain that they will, in turn, remove all of the pairs in their hand, laying them face up on the table. For each pair laid down, the student needs to explain why the model card matches the fact card.
4. When all pairs are matched and explained, the first player will draw a card from the hand of the student to his/her right. If the card makes a match with one in the player's hand, he/she should lay the cards down and explain the match.
5. Play continues to the right with each student drawing a card from the hand of the player to the right until all of the cards are laid down in pairs.

Egg Cup Division

Materials: Egg cartons
 Counters (beans, UNIFIX cubes, colored macaroni, etc.)
 Division cards

Directions: 1. Cut egg cartons into sections containing different numbers of cups (from one to nine cups per section). Examples:



2. Make a set of division cards. Example:

$$12 \div 3$$

3. Have a student draw a card, select the appropriate egg carton section, and model the division problem.

Example: $12 \div 3$

a. The student selects the three-cup egg carton section.



b. He/she counts out 12 counters.

c. The student distributes the 12 counters into the three cups.

d. The student draws a picture of the results.

e. Then, he/she writes the answer ($12 \div 3 = 4$).

OR

a. The student counts out 12 counters.

b. He/she separates them into sets of 3.

271

- c. The student chooses the four-cup egg carton section.



- d. He/she distributes a set of 3 counters into each cup.



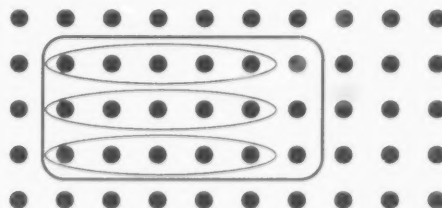
- e. Then, the student writes the answer ($12 \div 3 = 4$).

4. Continue the activity with different students drawing cards and modeling the problems.

Hurray for Arrays

Materials: A large laminated array of dots
Vis-A-Vis pens, at least two colors

- Directions:** 1. Circle a random number of dots on the array with a Vis-A-Vis pen. Call on a student volunteer to divide the dots into groups of 1, 2, 3, etc., circling the groups with another color pen. The student decides the number of dots per group. Example: 18 dots were circled. The student groups them into sets of five.



2. Have the student volunteer count the groups to determine the quotient. Ask the student if there is a remainder.
3. Have the students keep a record of their work. Example:

Number	Number in each group	Number of groups	Left over
18	5	3	3



Problem Solving

Word Problems

Materials: Materials for modeling division

Directions: 1. Using the overhead projector or a handout, present word problems that are associated with the model for division you have been using. Have the students construct the model for each problem and solve it.

2. Sample questions for typical models are listed below.

A. Sets

1. John had 18 pieces of candy. If he divided them among six people, how many pieces would each person receive?
2. If there are 14 human feet around the table, how many people are around the table?
3. A group of seven Cub Scouts handed out newspapers. If they were paid \$28 for handing out the papers and the money was divided equally among the Scouts, how much money did each Scout receive?

B. Arrays

1. If the 24 students in a class were put into six rows, how many students would be in each row?
2. A box of chocolate-covered cherries has 18 pieces in it. If the candy is in three rows, how many pieces are in each row?
3. There were 42 people at a meeting. After the meeting their chairs were stacked six high. How many stacks were there?

C. Number Lines

1. A robin hopped 24 spaces in 3 hops. How many spaces did the robin cover in 1 hop?
2. Billy the frog jumped 7 hops. Each hop was 4 units long. When he was finished hopping, how far was Billy from where he started?
3. A kangaroo jumped 4 times to go 32 meters. If the kangaroo jumped the same distance each time, how long was each jump?

D. Cuisenaire Rods

1. A bridge is 16 units long. If the bridge was built in four equal sections, how long is each section?
2. If four of the same type of car take up 28 feet when parked end to end, how long is one car?
3. In 5 days, a paving crew refinished 30 miles of highway. How many miles did the crew refinish on a typical day?

Calculator Fun

Materials: Calculators (one per pair of students)

- Directions:**
1. Have the students work in pairs.
 2. Say, "I'd like everyone to predict how many 9s there are in 72 and write your prediction on a piece of paper."
 3. Say, "Now I'd like one person in each pair to use the calculator to subtract 9 from 72. Continue subtracting 9 until the calculator gets to 0. As you are doing that, your partner should tally every 9 that is subtracted."
 4. When they are finished, have the students check the actual number of 9s with their predicted number.
 5. Repeat the activity with other numbers. Have the partners switch roles for each new problem.

*TEACHER NOTE: For enrichment, larger numbers involving hundreds can be used.

Many Ways

Materials: Materials for modeling division

- Directions:
1. Have the students use manipulatives to find as many ways as possible to divide the number 18 (e.g., $18 \div 1 = 18$, $18 \div 2 = 9$, $18 \div 3 = 6$, $18 \div 6 = 3$, $18 \div 9 = 2$, $18 \div 18 = 1$). You may want to have the students draw pictures of the models they have made for each number.
 2. The students should record all of the possibilities they have found. Example:

18	$\div$	1	=	18
18	$\div$	2	=	9
18	$\div$	3	=	6

3. Repeat the activity using other numbers (e.g., 12, 21, 24, 30, 36, 39, 42, 48).

Picture Graphs

Materials: Materials for modeling division

- Directions:
1. Tell the students that two second grade classes took a poll to see how the students get to school. Here's what they found:

12 children ride the bus
 6 children ride bikes
 15 children walk to school
 9 children ride in cars

Write the data on the board or overhead for the students to refer to.

278

2. Graph the data, as follows. Explain that each head represents 3 children.

Car	  
Bike	 
Walk	    
Bus	   

3. Have the students select their own topic for a class survey and collect the data.
4. Then have them graph the data as you did earlier, with each head representing 3 children. The students should use manipulatives to solve the problems. They will need to determine some way to represent $\frac{1}{3}$ or $\frac{2}{3}$ of a person for those data for which there is a remainder.
5. Variation: Have the students decide how they want to represent their findings on the graph. They can use objects other than heads, and each object can represent more or less than 3 people.

Mix and Match

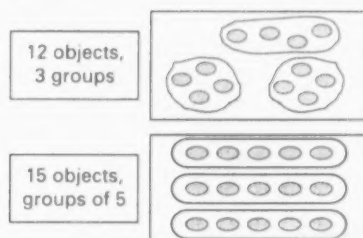
Materials: 4" x 6" index cards

Beans, buttons, or similar objects or magazines containing pictures to cut out

Glue

- Directions: 1. Give each student two 4" x 6" index cards on which you have written division statements (one problem per card). Also give each student two blank index cards.
2. Instruct the students to illustrate the division statements on their blank cards by gluing the appropriate number of sets to their cards. They can use objects such as beans or buttons or pictures they cut out of magazines.

- Have the students circle the sets to help identify the statement. Examples:

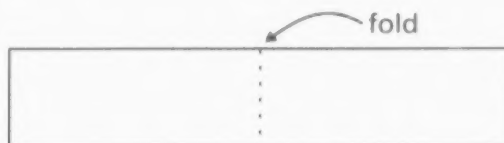


- When the students have finished illustrating their division statements, have them mix up their cards and exchange them with a classmate. Each student should match the statements and illustrations for the cards they've been given.
- Variation: Have the students exchange illustration cards and write the division statements for the cards they receive.

Division Parts

Materials: 2" x 12" strips of paper
Gummed dots or stamps and stamp pads

- Directions:**
- Give each pair of students several strips of paper and either a set of dots or a stamp and ink pad.
 - Say, "Fold one strip into two parts, like this."
Example:



- Say, "Using 12 dots (or stamps), put the same number of dots (or stamps) on each half." It may be helpful to have students use counters to work the problem before sticking down the dots or stamping the paper with ink.
- Ask, "How many groups do you have?" (2) "How many dots (or stamps) are in each group?" (6)

250

5. Say, "Fold another strip into three parts. Using 9 dots, put the same number of dots in each section."
6. Ask, "How many sections do you have?" (3) "How many dots did you put in each section?" (3)
7. Repeat the activity using different numbers. Use some combinations that will have a remainder.
8. Discuss the relation between fractions and division (e.g., explain that dividing by 2 is like halves; dividing by 3 is like thirds, etc.).

Dividing Coins

Materials: Real or play coins (blackline master may be found in the Appendix)

- Directions:**
1. Give each student a pile of real or play coins (nickels, dimes, quarters, and half-dollars).
 2. Say, "Find a nickel in your pile of coins. Show me how you could divide this amount of money into equal groups." (5 groups, one penny each; 1 group, one nickel)
 3. Say, "Find a dime. Show me how you could divide this amount of money equally." (5 sets of two pennies; 10 sets of one penny each; 2 sets of one nickel each; 2 sets of five pennies each)
 4. Proceed with higher value coins. Have the students find as many ways as possible to divide, or partition, the specified amount of money.

SKILL Describing and building a model for multiplication

Skill Analysis

- A. Understands the concept of multiplication
- B. Understands the "x" symbol
- C. Can interpret and model multiplication problems
- D. Can state the product of multiplication problems



Concept Manipulative Activities

UNIFIX Cube Multiplication

Materials: UNIFIX cubes

- Directions:
1. Have the students count 10 UNIFIX cubes of the same color and form a stack or train with them. The stack (or train) will be used as a reference for checking the length of other stacks they build. Students could also trace around the stack of 10 on a piece of paper and use the outline as a reference.
Example:



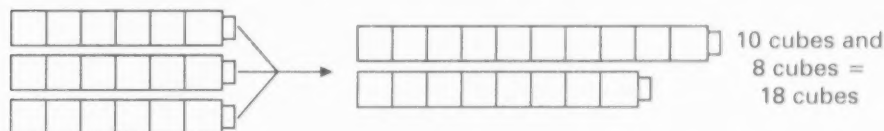
2. Using cubes, show the students that 3×6 is three groups or stacks of 6. Example:



3. Discuss with the students that counting all of the cubes to determine the product can be time consuming as well as

tedious. Building groups of 10 will expedite this process, especially for larger products.

4. Using the stacks you made for the problem 3×6 , take one stack of 6 and add to it 4 of the cubes from another stack to make 10. Take the 2 cubes that are left over and join them with the remaining stack of 6. Use the reference stack of 10 cubes to validate that there are 10 cubes in the first stack. Explain that by adding the cubes in the two new stacks (easy addition because one of the numbers is 10), the students will find the product of 6×3 .



5. Give the students a variety of practice problems. Be sure to allow sufficient time for them to model the original problem and then make their new stacks.

Multiplication With Cuisenaire Rods

Materials: Cuisenaire rods (one set per student or pair of students)

- Directions:**
1. Give each student or pair of students a set of rods. If the students have not previously used the Cuisenaire rods, provide some free exploration time.
 2. Say, "Using the white rod as a reference, determine the length of each of the other rods. For example, the red rod is two white rods long and the light green rod is three white rods long."
 3. Have the students make a chart listing the lengths of each rod (in reference to the white rod) or place a piece of tape on each rod and number it. (With both methods, white would be 1, red would be 2, light green would be 3, and so on.)
 4. Use the rods to model 3×4 . Read the problem as three rods of length 4 (purple). Place the rods end to end. Have the students model the problem at their desks.

Example:

Purple (4)	Purple (4)	Purple (4)
------------	------------	------------

5. Show the students how to use the rods to determine a product of 3×4 . Demonstrate using 1 orange rod and 1 red rod to match the length of the model. Then demonstrate using 12 white rods. Example:

Orange (10)										Red (2)	
Purple (4)				Purple (4)				Purple (4)			
W	W	W	W	W	W	W	W	W	W	W	W

6. Show how you can then add the lengths of the new rods (easy addition) to get the product of 3×4 .
7. Give the students other multiplication problems to solve with Cuisenaire rods.
8. Variation: Have the students place the original rods beside a centimeter ruler or number line to determine the product. Example:

Purple (4)				Purple (4)				Purple (4)							
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	

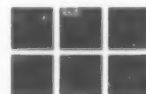
Array Multiplication

Materials: Counters of the same size and shape to make arrays (UNIFIX cubes, chips, beans, paper clips, etc.)

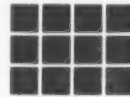
Overhead projector, blank transparency, Vis-A-Vis pen

Graph paper (blackline masters may be found in the Appendix)

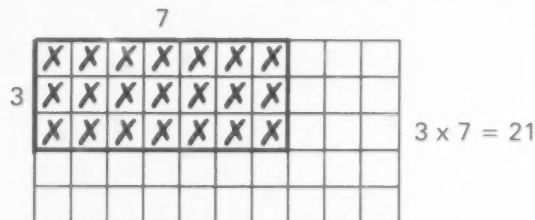
Directions: 1. Make the following array on the overhead projector.



2. Ask, "How many cubes are in the top row?" (3) "How many cubes are in the next row?" (3) "How many cubes do we have altogether?" (6) Say, "In 2 rows of 3 we have 6 cubes."
3. Put the following array on the overhead.



4. Ask the students to describe this array and find the product. (There are 3 rows of 4. The product is 12.)
5. Write the sentence "Three rows of 4 cubes is 12 cubes" on the overhead. Say, "This sentence can be written mathematically as $3 \times 4 = 12$."
6. Repeat the activity with several other models. Select student volunteers to build the models on the overhead.
7. Give the students a problem, such as 4×5 , and have them use their counters to build a model for the problem.
8. Have the students state in words what the number sentence means (e.g., " 4×5 means 4 rows of 5 cubes") and how many cubes they used ($4 \times 5 = 20$).
9. Repeat steps 7 and 8 with different problems.
10. After the students have practiced modeling arrays, introduce a pictorial (semi-concrete) model using graph paper. Give the students the problem 3×7 . Have them make 3 rows of 7 by shading the graph paper boxes, making x's in them, or circling the array. Have them write the number sentence next to the graph paper model. Example:

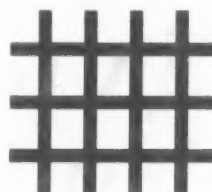


11. Give the students other practice problems.

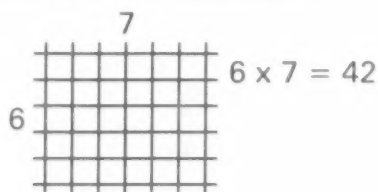
Intersecting Lines

Materials: Construction paper cut into $1/2$ " x 12" strips.

- Directions:
1. Give each student 20 strips of construction paper.
 2. Model the problem 3×4 by placing 3 strips in rows and 4 strips in columns that intersect the rows. Example:



3. Have the students count the number of intersections to determine the product. (12)
4. Give the students a number of problems to model with construction paper strips. Have them write the corresponding number sentence next to each model.
5. Variation: This technique can also be used by simply drawing intersecting lines. Example:



6. Variations: Display a model and have the students write the number sentence. You can also have students construct and exchange models and write the number sentence for the model they've been given.

Equivalent Sets

Materials: Counters or other objects for constructing equivalent sets

- Directions:**
1. Give the students a set of counters. Show them how they can model the problem $3 \times 4 = \underline{\quad}$.
 2. Demonstrate that this problem can be modeled as 3 sets of 4 objects. Example:



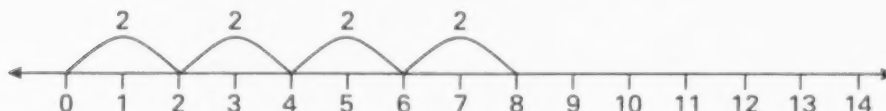
3. Have the students construct the same model at their desks. Ask, "How many objects are there altogether?" (12)
4. Give the students the problem $2 \times 5 = \underline{\quad}$ and ask them to construct a model for it. It is customary to read and model the problem as 2 sets of 5. However, it is not incorrect for a student model the problem as 5 sets of 2, since multiplication is commutative.
5. Repeat the activity with a variety of problems. For each problem, have the students read and record the problem, construct the model, and record the product. Call on student volunteers to explain their models.

Multiplication With Number Lines

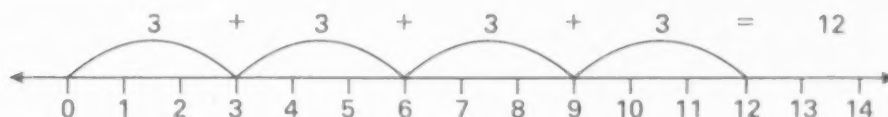
Materials: Number lines (desktop, walk-on, meter stick, etc.)
 "Number Lines" handout (one per student; blackline master may be found in the Appendix)

- Directions:**
1. Even if students have had some experience with the number line, a short review of the importance of equal spaces between the numbers may be appropriate.
 2. Say, "Today I'm going to show you how the concept of multiplication can be modeled as a certain number of equivalent jumps on the number line."

3. Model the problem $4 \times 2 = \underline{\hspace{2cm}}$. Starting at 0, make 4 equivalent jumps of 2. Explain that the product is the number associated with the last jump (in this case, 8).



4. Give the students the problem 5×3 . Have them explain what this problem means (5 jumps of 3). Have them model the problem on their number lines.
5. Allow the students to generate and solve their own problems.
6. Give the students some number line models and have them write the corresponding number sentence and product.
7. Using the number line, show the students the relationship between multiplication and repeated addition. For example, show that 4×3 is the same as $3 + 3 + 3 + 3$, or 4 jumps of 3.



8. Further work with the repeated addition problems is optional unless repeated addition has been used as a model.
9. Variations:
- Walk-On Number Lines—Have students hop on the number line to determine the product.
 - Overhead Transparency—Make a frog transparency and color it green. Call on student volunteers to demonstrate multiplication problems on the overhead using the frog and number line transparency.
 - Desktop Number Lines—Purchase or make laminated number lines and have the students tape them to their desks. Have them use Vis-A-Vis pens to model multiplication on their number lines. The number lines can be easily cleaned after each use.



Let's Practice

Multiplication Modeling

Materials: 3" x 5" cards with multiplication problems written on them or multiplication flash cards

Materials for modeling multiplication (counters, cubes, rods, graph paper, number lines, etc.)

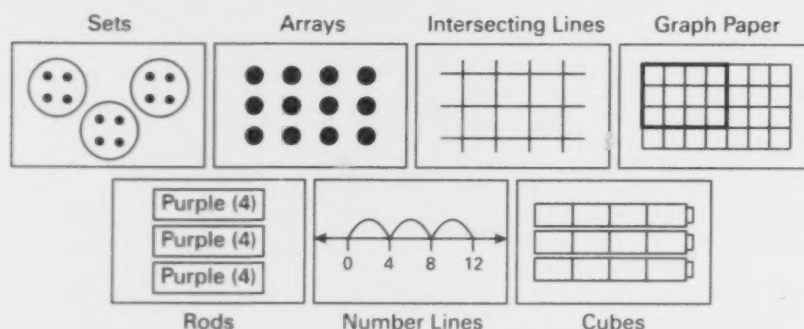
- Directions:
1. Give each student or small group of students materials for modeling the multiplication problems written on the cards.
 2. Have each student draw one card from the deck and construct the model for the problem from the materials provided.
 3. The students should record their model and the number sentence on a piece of paper.
 4. Have the students draw additional cards and repeat the activity.
 5. As the students are constructing their models, circulate among them and question them about their work. Ask the students to explain their models to you.
 6. Variation: Have the students work in pairs. One student draws a card and models the problem. The partner, who should not see the card, describes the model by writing down the number sentence. The partners switch roles and repeat the activity.

Match Model

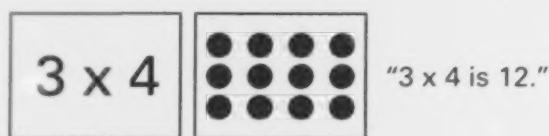
Materials: 30-50 blank playing cards

- Directions:
1. The game is played by small groups of students.
 2. Give each group a deck of cards that contains multiplication problems and models for them. When making the deck of cards, use the model(s) the students

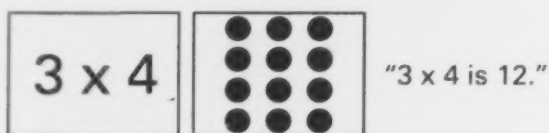
have been using (e.g., sets, arrays, rods)—one model per deck. For example, for 3×4 you would use one of the following models:



- Someone in each group should deal out all of the cards. The student to the left of the dealer begins the game. If the player has a match, he/she lays it down and states the problem and the answer. Example:



- The other players check to see if the match is correct. Commutative pairs are allowed. Example:



- A player can lay down only one matching pair per turn.
- If a player cannot make a match, he/she draws a card from the person on his/her left.
- Play continues to the left. The game is over when all of the cards have been matched.
- Optional: Each player gets a point for every match he/she makes during each game. The winner is the player with the most points at the end of the playing time (one or several games).

Game Board

Materials: Universal Game Board (directions for its construction may be found in the Appendix)
Construction paper or tagboard for game cards
Game markers
Die

- Directions:**
1. Two to four students may play this game.
 2. Give the students a Universal Game Board and a stack of game cards each of which has a multiplication number sentence on one side and its model on the reverse side. Use a model that has been discussed in class, and use only one model per deck of game cards. Examples of models may be found in the "Match Model" activity in this objective. The cards should be placed on the game board model side up.
 3. The first player draws the top card and writes the number sentence for the model.
 4. The player then turns the card over to check his/her work. If it is correct, the student rolls the die and moves his/her marker that number of spaces forward on the game board. If the student's answer is incorrect, the student does not advance.
 5. Play continues until one or more players reach the finish line.
 6. Variation: Use a deck of multiplication flash cards. Have the students, in turn, draw a card and construct the corresponding model using available material. If the model is correct, the student moves his/her marker two spaces on the game board.



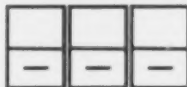
Problem Solving

Classroom Models

Materials: None

- Directions:
1. Show the students how different things in the classroom model multiplication. For example, point to two 4-drawer file cabinets and show how they model the problems $4 \times 2 = 8$ and $2 \times 4 = 8$.
 2. Divide the class into pairs or teams. Have the groups of students find other examples that model multiplication in the classroom.
 3. For each example, students should name the model, draw it, and write the multiplication sentence. Example:

"windows"



" $2 \times 3 = 6$ " (or $3 \times 2 = 6$)

4. Other examples in the classroom could include floor tiles, ceiling tiles, tote trays, coat hooks, desk drawers, books on shelves, shoes worn by a group of students, boxes, and so forth.
5. After a specified time, have the teams share the models they have found and their number sentences.

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

Directions: Draw a picture and write the number sentence for each problem.

1. Sarah bought three packages of hot dog buns for the picnic. Each package contained eight buns. How many buns did she buy?
2. Veronica bought two sets of plastic building blocks and used them to make a wall that is seven blocks long and four blocks high. How many blocks did she use in the wall?
3. A pack of gum has five sticks in it and costs 30¢. How many sticks of gum are in nine packs?
4. There are seven windows in a sunroom. If each window has four panes of glass, how many window panes are there altogether?
5. Mary has five friends coming to her party. She will give each friend three yellow balloons and two red balloons. How many red balloons will she need?

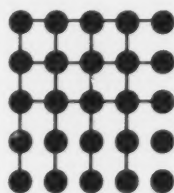
How many yellow balloons will she need?



Lines and Points

Materials: Geoboards (one per student)
Red and green rubber bands

Directions: 1. Demonstrate a geoboard model for 3×4 by placing three green rubber bands in a horizontal pattern on the geoboard and then placing four red rubber bands in a vertical pattern on top of the green bands to form intersecting points.
Example:



2. Ask, "How many green rubber bands are there?" (3) "How many red rubber bands are there?" (4)
3. Ask, "How many places on the geoboard do the red and green rubber bands intersect?" (12) (Or ask, "How many nails are holding both red and green rubber bands?")
4. Say, "Record the number sentence for this model."
($3 \times 4 = 12$)
5. Give the students problems to model on their geoboards. For each problem have them record the corresponding number sentence.
6. Have each student make up a problem, construct the model, and exchange geoboards with a partner. Instruct the partners to write the corresponding number sentences.

SKILL Describing and building a model for division

Skill Analysis

- A. Can construct a given number of equivalent sets
- B. Can construct sets with a specified number of members
- C. Understands the symbols " $\div$," " $\overline{\hspace{1cm}}$," and " $=$ " signs
- D. Can interpret and model division problems
- E. Can state the quotient of division problems



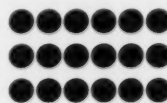
Concept Manipulative Activities

Partitioning Sets

Materials: 50 counters per student (beans, buttons, cubes, etc.)
 Workspace (9" x 12" construction paper, place mat, carpet square, etc.; one per student)
Optional: Overhead projector

- Directions:**
1. In this activity, students will model division problems on their workspaces. (Using a workspace will help students focus on the counters that are being used.)
 2. Say, "Put 18 counters on your workspace. Using these counters, make three equivalent sets." Some students may immediately put 6 counters into each of the three sets, while others may count out 1 counter at a time to make the three sets.
 3. Ask, "How many counters are in each of your three sets?"
 (6)

4. Write $18 \div 3 = 6$ on the board. Discuss the parts of the number sentence and what each part represents. For example, say, "Eighteen counters divided into three sets equals six counters in each set. This problem can also be written
$$\begin{array}{r} 6 \\ 3 \overline{)18} \end{array}$$
"
5. Repeat the activity with other problems, having the students model the problems and write the division sentence in either format. The students should clear their workspace after each problem. Problems can be shown on the overhead.
6. Say, "Put 21 counters on your workspace. Using these counters, make four equivalent sets." Have the students discuss what they think should be done with the 1 counter that is left over. This provides an excellent opportunity for student sharing and the expression of creative solutions.
7. Discuss how the students should write their answers when there is a remainder involved.
8. Repeat the activity with other problems, some of which should have remainders.
9. Variation: Have the students place their counters in rows and columns so that their finished model resembles an array. Example: For $18 \div 3 = 6$, they would put their 18 counters in three rows and explain that there are 6 counters in each row.



Give them some problems that have remainders.

Partitioning the Class

Materials: 35 counters per student (beans, buttons, cubes, etc.)
Workspace (9" x 12" construction paper, place mat, carpet square, etc.; one per student)

- Directions:
1. Have eight students stand at the front of the room. Tell them, "I want you to form two equivalent groups. How many of you will be in each group?" Have the rest of the students model the problem on their workspaces using counters.
 2. After the students have modeled the problem at their desks, have the group of students at the front of the class form the two groups.
 3. Ask, "How many students are in each group?" (4)
 4. Write the number sentence $8 \div 2 = 4$ on the board. Discuss each part of the sentence and what it represents.
 5. Have a different group of eight students stand at the front of the room. Tell them, "I want you to form three equivalent groups. How many of you will be in each group?" Have the rest of the students model the problem on their workspaces using counters.
 6. Discuss what should be done with the two "leftovers."
 7. Have the students at the front of the room model the problem.
 8. Record the number sentence on the board.
 9. Repeat the activity with other problems including several involving remainders.
 10. Variation: Instead of having students model the problems at the front of the room, select student volunteers to model the problems with counters on the overhead.

Make Sets

Materials: 50 counters per student (beans, buttons, cubes, etc.)
 Workspace (9" x 12" construction paper, place mat, carpet square, etc.; one per student)
 Optional: Overhead projector

- Directions:
1. Say, "Put 12 counters on your workspace. Using these counters, make sets with 4 members in each set."
 2. Ask, "How many sets did you make?" (3)

3. Write $12 \div 4 = 3$ on the board. Discuss the parts of the number sentence and what each part represents. For example, say, "Twelve counters divided into sets with 4 members in each equals 3 sets. This problem could also be written $4 \overline{)12}^3$."
4. Repeat the activity with other problems, having the students model the problems and write the division sentence in either format. The students should clear their workspace after each problem. Problems can be shown on the overhead.
5. Say, "Put 16 counters on your workspace. Using these counters, make sets with 3 members in each set." Have the students discuss what they think should be done with the 1 counter that is left over.
6. Repeat the activity with other problems, some of which should have remainders.
7. Variation: Have the students place their sets in rows and columns so that their finished model resembles an array. Example: For $12 \div 4 = 3$, they would put their 12 counters in four rows and explain that there are 4 counters in each row.



Give them some problems that have remainders.

Graph Paper Division

Materials: Graph paper cut into strips (blackline masters may be found in the Appendix)

Scissors

Brightly colored construction paper

Glue

Crayons or markers

Directions: 1. Have each student fold a piece of construction paper into fourths. Example:



2. Say, "Make a graph paper strip of 15 squares. Cut off sets of 3 squares. Glue all the sets to one section of your construction paper."
3. Ask, "How many sets of 3 squares do you have?" (5)
4. Write $15 \div 3 = 5$ on the board. Discuss each part of the number sentence and what each part represents (e.g., "Fifteen squares divided into sets with 3 squares in each set equals five sets. This problem can also be written
$$\begin{array}{r} 5 \\ 3 \overline{)15} \end{array}$$
".
5. Have the students record the problem in the section of their construction paper where they glued the graph paper sets.
6. Repeat the activity with other problems, having the students glue graph paper models to their construction paper and write the division sentence in either format. Include problems that involve remainders.
7. Students can use the reverse side of their construction paper to record additional problems, or they can use other sheets of construction paper to form a booklet.
8. Variation: Give each student a piece of graph paper. Have the students trace around a strip of 15 squares with a black crayon. Then, using different colors of crayons, have them color sets of 3 squares to determine the quotient of $15 \div 3$. Repeat the activity with other problems.

Number Line Division

Materials: "Number Lines" handout (one per student; blackline master may be found in the Appendix)
Overhead projector
Vis-A-Vis pen

- Directions:**
1. Say, "We're going to practice hopping backward on the number line to get to 0. I will say where to begin hopping and how big the hops are to be."
 2. Say, "Let me show you. I'll begin at 8 and make hops of 2 spaces until I reach 0." Demonstrate the problem on the transparency.
 3. Ask, "How many hops did I make?" (4)
 4. Write $8 \div 2 = 4$ by the number line. Discuss the parts of the number sentence and what each part represents. The problem can also be written
$$\begin{array}{r} 4 \\ 2 \overline{)8} \end{array}$$
 5. Have the students work several problems on the "Number Lines" handout. Tell them where to begin on each number line and how large their hops are to be.
 6. Discuss the number sentence for each problem and have the students record the sentence on their paper.
 7. Say, "I want you to begin at 9 and make hops of 4 spaces back to 0. What happens?" Allow the students to discuss why it is impossible to land on 0.
 8. Ask, "How many hops did you make?" (2, the quotient)
"How many spaces are left?" (1, the remainder)
 9. Continue with a variety of problems including some with remainders.

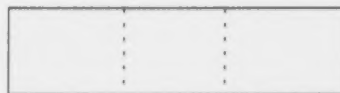


Let's Practice

Division Models

Materials: Materials for modeling division (stickers, graph paper, number lines)
 3" x 5" cards with division problems written on them using both " $\div$ " and " $\overline{\hspace{1cm}}$ " (include problems with remainders)
 9" x 12" construction paper
 Glue
 Crayons or markers

Directions: 1. Have the students fold their construction paper into thirds.
 Example:



2. Have them label the first section "Problem," the second section "Model," and the third section "Number Sentence."
3. Each student will draw a card and write the problem in the first section. They will then construct a model for the problem in the second section using sets, graph paper, number lines, etc.—whichever model they are most familiar with.
4. After the students have completed the model, they should write the complete number sentence in the third section.
 Example:

Problem	Model	Number Sentence
$15 \div 3$		$15 \div 3 = 5$

5. When the students finish the first problem, they should draw another card and repeat the activity. They can write several problems, models, and number sentences on the same sheet of construction paper.

Division Games

Materials: 50 game cards (25 cards on which you have written division problems using both " $\div$ " and " $\overline{\hspace{1cm}}$ " formats, and 25 cards with the corresponding model; include problems with remainders)

Universal Game Board (directions for its construction may be found in the Appendix)

Game markers

Die

Manipulatives for modeling division problems (counters, UNIFIX cubes, graph paper, Cuisenaire rods, number lines, etc.)

- Directions:**
1. This game is played by groups of two students.
 2. Each player, in turn, draws a division problem card, constructs a model of the problem using the manipulatives of his/her choice, and explains the model. The players need to explain their model so that there is no confusion about whether it is correct. Example: For $12 \div 4$ both



are correct.

3. If the problem is correctly modeled and explained, the player rolls the die and moves his/her game marker forward that number of spaces. If the problem is incorrectly modeled, the player to the left attempts to model the problem correctly.
4. Have the students use all 50 game cards for games such as Old Maid and Go Fish. In these games, the students make matches of models and division problems.

Number Line Record

Materials: "Number Lines" handout (one per student; blackline master may be found in the Appendix)

- Directions: 1. In this activity, you will give the students division problems to solve by recording their hops backward on a number line.
2. Say, "For the problem $21 \div 7$, start at 21 on the number line and make hops of 7 spaces backward." Example:

Start	21
Hop	-7
Land	14
Hop	-7
Land	7
Hop	-7
Land	0

3. Ask, "How many hops did you make?" (3) "Did you land at 0?" (yes)
4. Have the students write the complete number sentence. ($21 \div 7 = 3$)
5. Repeat the activity with other problems, including some with remainders. Example: For $8 \overline{)19}$

Start	19
Hop	-8
Land	11
Hop	-8
Land	3

6. Ask, "How many hops did you make?" (2) "Did you land at 0?" (no, 3)
7. Have the students write the complete number sentence.

$$\left(8 \overline{)19} \begin{matrix} 2 \\ \text{R} 3 \end{matrix} \right)$$

*TEACHER NOTE: By practicing repeated subtraction with the use of the number line, the students can see a relationship between division and subtraction, not just an algorithm.



Problem Solving

Division Strips

Materials: Graph paper (blackline masters may be found in the Appendix)

Scissors

- Directions:
1. In this activity, the students will cut graph paper into one-row strips and use the strips to solve division problems. A different strip will be used for each problem.
 2. Give the students the problem $12 \div 2$. Have them cut a strip to a length of 12 squares. Then have them fold the strip into two equal parts. Ask, "How many squares are in each part?" (6)
 3. Begin with easy problems, such as $12 \div 2$, so that the students can determine the exact quotient from the number of squares in each section. Divisors of 2 and 4 will be the easiest.
 4. Discuss with the students how to fold the strips into three parts and then six parts. (Since the strips are short, 2, 3, 4, and 6 may be the only divisors appropriate for this activity.)
 5. Repeat the activity using many problems with divisors of 2, 3, 4, and 6.
 6. Give the students a problem with a remainder, such as $17 \div 2$. Discuss the number of squares in each section. The students may "see" $8\frac{1}{2}$. Other problems with fractional parts may be inappropriate for students at this level.

Create a Story

Materials: Manipulatives for modeling division problems (counters, UNIFIX cubes, graph paper, Cuisenaire rods, number lines, etc.)

- Directions:**
1. Say, "I am going to give you a story problem, and I want you to model it using your (manipulative)."
 2. Say, "There are 18 students in Mrs. Brown's reading class. Mrs. Brown wants to form reading groups. She puts 6 students in each group. How many groups does she form?"
 3. Allow the students to model the problem. Have a volunteer show and explain his/her model. Ask, "Can you write a number sentence to show us what you did?" ($18 \div 6 = 3$ or
$$\begin{array}{r} 3 \\ 6 \overline{)18} \end{array}$$
)
 4. Next, give a story problem that involves partitioning. An example would be: "Sam has 24 stickers. If he puts them into four groups, how many stickers are in each group?"
 5. After reviewing several problems, have one of the students create a story problem and share it with the class. Have the other students model the story problem and write down the corresponding number sentence.
 6. Have several other students make up problems for the class to model and solve.
 7. You may want to have the students write additional story problems to be used in other activities or at an independent learning center.

How Many Rectangles?

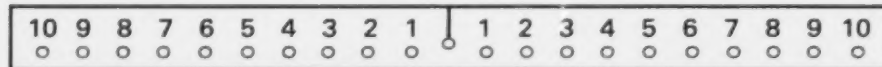
Materials: Counters (beans, buttons, cubes, etc.)
Graph paper (blackline masters may be found in the Appendix)
Construction paper, each sheet folded in half
Scissors
Glue
Markers or crayons

- Directions:**
1. Give each student 30 counters.
 2. Say, "We will be trying to place our counters in the shape of a rectangle. Can anyone arrange 8 of the counters into rows and columns (like an array) so that the shape is a rectangle?"
 3. If one array is discovered, ask, "Can anyone find a different arrangement?" (There are four possibilities: 1×8 , 2×4 , 4×2 , and 8×1 .)
 4. Have the students cut their arrays out of a piece of graph paper and glue them onto one section of their construction paper. Have them label this section "8." Have the students write the corresponding number sentence by each array (e.g., $8 \div 1 = 8$, $8 \div 2 = 4$, $8 \div 4 = 2$, and $8 \div 8 = 1$).
 5. Repeat the activity with other numbers. Have the students glue the arrays to different sections of their construction paper. See if the students can find patterns when they are looking for all the possible arrays. Ask, "Which number has the fewest possible arrays?" "Which has the most?"

Invicta Balance

Materials: Invicta balance (A homemade version can be made by taking a strip of pegboard, balancing it from the center hole, and numbering the holes 1–10 from the center hole.)

Example:



Set of equal weights to hang on the balance

- Directions:**
1. Have the students model and solve division problems with the balance. For example, $8 \div 2$ would be modeled by placing one weight on the left side of the balance at “8” and then placing weights at “2” on the right side until the balance “balances.” Four weights would be placed at “2,” so $8 \div 2 = 4$.
 2. To model a number larger than 10, weights are placed on the balance in terms of the place value of the number (e.g., 21 would be modeled by placing two weights at 10 and one weight at 1). Students can therefore use the balance to model problems such as $21 \div 7$.
 3. Challenge the students to discover how to deal with remainders when modeling on the balance. Example: For $7 \div 3$ the scale would balance with a weight at 7 on the left, and two weights at 3 **and** one weight at 1 on the right.
 4. You may want to also challenge the students to discover, using the balance, as many ways as possible to divide a given number.

Assessment Activities

SKILL Demonstrating a model for multiplication

2.4-1

Materials: Materials for modeling multiplication problems (counters, arrays, rods, number lines, etc.) that the students are used to working with
Blank cards

- Directions:**
1. Write a multiplication problem on each card. Have at least one card for each child.
 2. Give each student a card and modeling materials. Ask the students to model the multiplication problems and give you the answer.
 3. Examples of problems: $5 \times 3 = \underline{\quad}$, $2 \times 6 = \underline{\quad}$, $\begin{array}{r} 7 \\ \times 3 \\ \hline \end{array}$, etc.
 4. Have the students exchange cards. Repeat the activity a sufficient number of times to allow you to observe each student modeling some problems.

SKILL Demonstrating a model for division 2.4-2

Materials: Materials for modeling division problems (counters, arrays, rods, number lines, etc.) that the students are used to working with
Blank cards

- Directions:**
1. Write a division problem on each card. Have at least one card for each child.
 2. Give each student a card and modeling materials. Ask the students to model the division problems and give you the answer.
 3. Examples of problems: $12 \div 3 = \underline{\quad}$, $16 \div 4 = \underline{\quad}$,
 $\begin{array}{r} 6 \overline{)30} \\ 3 \overline{)18} \end{array}$

4. Have the students exchange cards. Repeat the activity a sufficient number of times to allow you to observe each student modeling some problems.

SKILL Describing and building a model for multiplication

3.4-3

Materials: Materials for modeling multiplication problems (counters, number lines, graph paper, UNIFIX cubes, etc.) that the students are used to working with
Blank cards

Directions: 1. Write several different multiplication problems on each card, such as: $3 \times 8 = \underline{\quad}$, $5 \times 7 = \underline{\quad}$, $\begin{array}{r} 7 \\ \times 4 \\ \hline \end{array}$, $\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$

Be sure to write problems both horizontally and vertically.

2. Give a card and modeling materials to each student.
3. Ask the students to make a model for each problem and write the answers on a piece of paper.
4. As they are working, ask each child, in turn, to model a problem for you. Have the students exchange cards if they need more problems.
5. Have the students model a sufficient number of problems to allow you to assess their mastery of the process.

SKILL Describing and building a model for division

3.4-4

Materials: Materials for modeling division problems (counters, number lines, graph paper, place value blocks, etc.)
Blank cards

Directions: 1. Write several different division problems on each card, such as:

$$56 \div 8 = \underline{\quad}, \quad \begin{array}{r} 7 \overline{)40} \\ 399 \end{array}, \quad 36 \div 5 = \underline{\quad}, \quad \begin{array}{r} 7 \overline{)49} \end{array}$$

2. Give a card and modeling materials to each student.
3. Ask the students to make a model for each problem and write the answers on a piece of paper.
4. As they are working, ask each child, in turn, to model a problem for you. Have the students exchange activity cards if they need more problems.
5. Have the students model a sufficient number of problems to allow you to assess their mastery of the process.

SKILL Describing products of numbers through
 10×10 ***Skill Analysis***

- A. Understands the concept of multiplication
- B. Can model the concept of multiplication
- C. Understands the "x" and "=" symbols and can use them when recording work

**Concept Manipulative Activities*****Equivalent Sets***

Materials: Counters that can be linked together to form sets (e.g., UNIFIX cubes)

- Directions:
1. Give each student 30 counters.
 2. Ask the students to link together sets of three until they have used a total of nine objects. Ask, "How many sets of three do you have?" (3)
 3. Ask, "Does anyone know how to write a multiplication sentence for this model?" Let a student write the sentence $3 \times 3 = 9$ on the board. If no one in the group can write the sentence, write it for them.
 4. Discuss the symbols and their interpretations. Relate the symbols to the model. Say, "Three sets of three equals nine," or "The product of 3 times 3 is 9."
 5. Have the students clear away the three sets. Say, "Lay out sets of three until you have used a total of 15 objects." Ask, "How many sets of three do you have?" (5)

6. Ask a student to write the multiplication sentence that goes with the model (i.e., $5 \times 3 = 15$).
7. Discuss the interpretation of the number sentence. (Five sets of three equals 15, or the product of 5 and 3 is 15.)
8. Repeat the activity having the students form different quantities with sets of three.
9. Variation: Have the students link the counters into sets of four, five, six, etc., to form different quantities.

Different Models

Materials: None

Directions: 1. Some suggested strategies that may help students learn/memorize their multiplication facts are discussed in this activity. It is neither required nor expected that every student learn these strategies. However, they can be an effective and efficient aid for students who are having difficulty learning their facts or who are just starting to learn their facts.

a. Skip Counting

Skip counting is an efficient strategy for memorizing the ones, twos, fives, and tens facts.

1, 2, 3, 4, . . .

2, 4, 6, 8, . . .

5, 10, 15, 20, . . .

10, 20, 30, 40, . . .

It is more difficult to use skip counting for other numbers because there are few, if any, "natural" situations for skip counting by numbers other than those listed above.

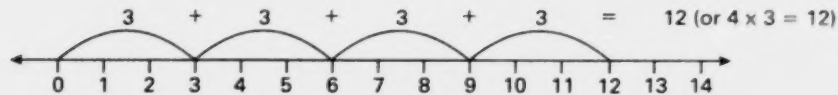
b. Repeated Addition

Repeated addition helps some students to understand the concept of multiplication. It can be modeled with sets,

$$\begin{array}{c} \triangle \triangle \\ \triangle \\ 3 \end{array} + \begin{array}{c} \triangle \triangle \\ \triangle \\ 3 \end{array} + \begin{array}{c} \triangle \triangle \\ \triangle \\ 3 \end{array} + \begin{array}{c} \triangle \triangle \\ \triangle \\ 3 \end{array} = 12 \text{ (or } 4 \times 3 = 12)$$

31

with a number line,



or with other models. This strategy is not very efficient when there are a large number of sets. In such cases, memorization of multiplication facts makes finding the solution faster and easier.

c. Multiplying by 2—Doubling

Most children have an easy time remembering doubles. However, some may not make the connection on their own that multiplying by 2 is the same as **doubling** a number.

d. One More Set

When students first learn their facts, a good strategy is to show them how to build on to what they already know (e.g., “You know $4 \times 5 = 20$. You can use this to help you with 5×5 , which is one more set of five. So, $5 \times 5 = 20 + 5 = 25$.”).

e. Commutative Property

Understanding the commutative property for multiplication reduces the number of facts that must be learned to 55.

1 x 1	1 x 2	1 x 3	1 x 4	1 x 5	1 x 6	1 x 7	1 x 8	1 x 9	1 x 10
2 x 1	2 x 2	2 x 3	2 x 4	2 x 5	2 x 6	2 x 7	2 x 8	2 x 9	2 x 10
3 x 1	3 x 2	3 x 3	3 x 4	3 x 5	3 x 6	3 x 7	3 x 8	3 x 9	3 x 10
4 x 1	4 x 2	4 x 3	4 x 4	4 x 5	4 x 6	4 x 7	4 x 8	4 x 9	4 x 10
5 x 1	5 x 2	5 x 3	5 x 4	5 x 5	5 x 6	5 x 7	5 x 8	5 x 9	5 x 10
6 x 1	6 x 2	6 x 3	6 x 4	6 x 5	6 x 6	6 x 7	6 x 8	6 x 9	6 x 10
7 x 1	7 x 2	7 x 3	7 x 4	7 x 5	7 x 6	7 x 7	7 x 8	7 x 9	7 x 10
8 x 1	8 x 2	8 x 3	8 x 4	8 x 5	8 x 6	8 x 7	8 x 8	8 x 9	8 x 10
9 x 1	9 x 2	9 x 3	9 x 4	9 x 5	9 x 6	9 x 7	9 x 8	9 x 9	9 x 10
10 x 1	10 x 2	10 x 3	10 x 4	10 x 5	10 x 6	10 x 7	10 x 8	10 x 9	10 x 10

The diagonal line through the table (from 1×1 to 10×10) represents the line of symmetry. The multiplication facts on either side of the line are “commutative pairs” (e.g., 3×2 and 2×3 , 4×2 and 2×4). Students should be encouraged to look at a problem from left to right first, but if they do not know the product, to then look at the problem from right to left to see if they know that product.

f. Distributive Property

As with "One More Set," with this strategy students use what they know to solve what they don't know.

Example:

6×4

I do not know.



I do know that $3 \times 4 = 12$.

I can think of 6×4 as

3×4 plus 3×4 :



$3 \times 4 = 12$



$3 \times 4 = 12$

6×4 is $12 + 12$ or 24.

Students will not be able to use the distributive property effectively until they have had many opportunities to explore physical models for multiplication. This exploration may consist of activities where the students are given a model and asked to investigate the number of ways the sets in the model can be divided. Example:

6×4



1×4

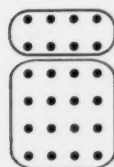
4

+

5×4

+ 20

24



2×4

8

+

4×4

+ 16

24

etc.

g. Patterns

Students should be encouraged to find and explain as many patterns as possible for the 1-10 multiplication tables. Once a student has found a pattern for a specific table, he/she can use the pattern to remember that particular multiplication table.

Picture Graphs for Facts

Materials: Pictures of objects that come in equivalent sets
Glue

Directions: 1. Have the students collect pictures of objects that come in equivalent sets. Then have the class glue the pictures to a graph (make sure the squares are large enough to accommodate different sized pictures) according to whatever characteristics the students choose. In the following example, the students chose shoes as the object and categorized them by type of shoe.

Type of Shoe					Number Sentence
Tennis					$3 \times 2 = 6$
High Heel					$2 \times 2 = 4$
Boot					$4 \times 2 = 8$

- As shown in the example above, the students should write the number sentence to accompany each category. In this example, they could count by twos to check their answers.
- The relationship of multiplication to addition can be shown by having the students first write the repeated addition sentence and then the multiplication sentence. Example:

Type of Shoe					Addition Sentence	Multiplication Sentence
Tennis					$2 + 2 + 2 = 6$	$3 \times 2 = 6$
High Heel					$2 + 2 = 4$	$2 \times 2 = 4$
Boot					$2 + 2 + 2 + 2 = 8$	$4 \times 2 = 8$

- Have the students repeat the activity using pictures of items that are grouped into different-sized sets, such as threes, fours, or fives.

Geometric Shapes and Facts

Materials: Toothpicks, straws, Popsicle sticks, etc.

- Directions:
1. In this activity, students discover number patterns by building geometric shapes.
 2. Tell the students to build a triangle using the objects you have given them (e.g., toothpicks). Ask, "How many triangles do you have?" (1) "How many toothpicks did you use?" (3)
 3. Tell the students to build two triangles. Ask, "Now that you have two triangles, how many toothpicks have you used?" (6)
 4. Tell the students that you want them to make a table to help them keep track of their findings and to help them find patterns. The following are some examples of tables they could use. Show them one of these or a version of your own.

a.

1	2	3	4	
Δ	$\Delta\Delta$	$\Delta\Delta\Delta$	$\Delta\Delta\Delta\Delta$	
3	6	9	12	

b.

Triangles	1	2	3	4		
Toothpicks	3	6	9	12		

5. Encourage the students to look for patterns. By building the geometric shape (triangles), the students can **visualize** the physical number pattern in a concrete way, while the tables allow them to see more abstract representations or numbers.
6. Show the students how the information in the tables can be rewritten as number sentences (e.g., $1 \times 3 = 3$, $2 \times 3 = 6$, $3 \times 3 = 9$, etc.).
7. Repeat the activity using other shapes for different multiplication facts (e.g., squares for 4s, pentagons for 5s, etc.).

316

Hundred Squares

Materials: "Hundred Square" handout (several per student; blackline master may be found in the Appendix)

Crayons

- Directions: 1. Give each student a "Hundred Square" handout.
2. Say, "Choose a crayon that you would like to use. Use this crayon to color every second number on your chart."

Example:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3. After the students have colored their charts, discuss any patterns that they see when looking at the colored areas. Discussion points include:
- "The colored squares are in straight lines."
 - "The colored squares are all under the 2, 4, 6, 8, and 10."
 - "The colored squares are all even numbers."
4. Have the students use a new handout to color in multiples of another number. Encourage them to share their observations about the colored squares. Post their charts and findings on a bulletin board.
5. The following are a sample chart and discussion points for multiples of three:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- a. "The colored squares form diagonals."
- b. "If you follow the diagonal from the 3, the number of colored squares is three. If you follow the diagonal from the 6, the number of colored squares is six."
- c. "Does the pattern seen for '6' and '3' hold for all the diagonals? Can you make it work for all the diagonals?"
- d. "What other interesting phenomena or patterns can you find?"



Let's Practice

Product

Materials: Deck of cards (50 cards on which you have written multiplication facts)

- Directions:**
1. This game, played by two to four students, follows the same basic rules as the card game War.
 2. All the cards are dealt out, face down. Each player turns over one of his/her and names the product. The person with the largest product takes all the cards. (Students should be allowed to use manipulatives to model the problems, if necessary.)
 3. If there is a tie—that is, if two or more students have equivalent products—those players turn their next card over and name the product. The player with the largest product takes all the cards for that round.
 4. Play continues for a set amount of time or until one player has all the cards.

Multo Roll

Materials: "Multo Roll Record Sheet" (one per student; blackline master follows this activity)

Two cubes, one with sides numbered 4, 5, 6, 7, 8, and 9, and one with sides numbered 3, 4, 5, 6, 7, and 8

Directions: 1. Each student, in turn, tosses the cubes, multiplies the numbers rolled, and records the answer in an appropriate space on the "Multo Roll Record Sheet." Only one box may be filled in at a time. Example:

For a roll of
5 and 6:

	4	5	6	7	8	9
3						
4						
5			30			
6		30				
7						
8						

or

2. The activity continues until a player fills his/her card or time is called.

**Multo Roll Record Sheet**

	4	5	6	7	8	9
3			18			
4						
5						
6						
7						
8						

Use with "Multo Roll" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

3.5-1

Multiplication Toss

Materials: Tagboard
Beanbags

Directions: 1. Using tagboard, make a target with sections numbered from 1 to 10, as shown below.



The students will be tossing beanbags onto the target, so be sure its sections are larger than the beanbags that will be used.

2. Have the students, in turn, toss two beanbags onto the target and multiply the numbers in the sections on which the beanbags have landed. Explain that the goal is to first get 1 as the product, and then 2, and then 3, and so on.
3. The only way to get the product 1 is for both beanbags to land in the "1" section, because only $1 \times 1 = 1$.
4. If a student gets the product 1, he/she tosses the beanbags again and tries to get the next product (2). One bag would need to land on 2 and one would need to land on 1. If the student is successful again, he/she then tries to get the product 3 (3×1) by tossing one bag on 3 and one bag on 1.
5. If the student is unsuccessful, that is, if he/she does not get the beanbags to land on the appropriate sections to yield the desired product (1, 2, 3, 4, 5, etc.), then play passes to another student.
6. Note that some products offer more flexibility than others. For the product 4, for example, the bags can land on 4 and 1 or twice on 2.

Product	Problem
1	1×1
2	2×1
3	3×1
4	4×1 or 2×2
5	5×1
:	:
12	6×2 or 4×3
:	:

7. Have the students try to get all products, in order, from 1 to 20 with the exception of 11, 13, 17, and 19 (prime numbers greater than 10 that would be impossible to get on this game board).
8. Variations:
 - a. Have the students try to get the even-numbered products, in order, from 2 to ____.
 - b. Have the students work backwards from 25 to 1 (again skipping the products 11, 13, 17, and 19).
 - c. Have the students aim for the product 2, then for products that are multiples of 2. Do the same for 3, 4, etc.
 - d. Have the students make a list of all of the products from 1 to 100 that are impossible to get on this gameboard.

Number Scramble

Materials: Set cards numbered 0 to 9 made from large pieces of construction paper (one card per student)

- Directions:**
1. Choose four students to be the recorders. The recorders will be responsible for checking and recording number sentences. These students should position themselves around the classroom.
 2. The purpose of the game is for the students to form multiplication sentences (e.g., $6 \times 3 = 18$) using the numeral cards.

322

3. Give each student (other than the recorders) a numeral card. Then have the students move about the classroom to create multiplication sentences. For example, a student with a 3 card could find a student with a 6 card, and then they could find students holding a 1 and an 8. (The multiplication and equals signs are assumed.) All four students would go to the nearest recorder.
4. The recorder would check the multiplication sentence to see if it is correct. If it is, the recorder would write the number sentence on the back of each of the students' numeral cards.
5. The students would continue to form multiplication facts.
6. At the end of a given time, have the students count the number of multiplication sentences they were able to help form. The students with the greatest number are the winners.

Multi-Cards

Materials: Three decks of regular playing cards (no face cards)

- Directions:**
1. Have the students work in pairs.
 2. Mix the cards from the three decks and place all the cards, face down, in a pile in the center of the playing area.
 3. Draw two cards from the deck. Have the students multiply the values of the two cards and state the product.
 4. Then have each pair of students draw three cards. The students should multiply either two or all three of their cards to come as close as possible to the product of the two "teacher cards."
 5. The pair that comes closest to, or equals, the given product "banks" the cards they used. All of the other cards are placed in a discard pile.
 6. The partners with the greatest number of cards in their bank at the end of the playing time are the winners.



Problem Solving

Calculo

Materials: "Calculo" handout (one per student; blackline master follows this activity)

- Directions:
1. Challenge the students to find as many multiplication problems as they can for each product listed on the handout. The students should write the problems in the handout squares. Let the students know that more than two factors can be used in the problems (e.g., $2 \times 2 \times 3$).
 2. After a set amount of time, meet with the students to share and discuss their solutions.
 3. This activity provides a good opportunity to look at multiplication by 1 and 0.



Calculo

6		28		9	
10		8		12	
18		14		36	
20		25		5	
0		30		24	
45		16		40	

Use with "Calculo" activity. Students can use calculators to check their work. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Side Match

Materials: "Side Match" handout (one per student; blackline master follows this activity)

Directions: 1. Give each student a "Side Match" handout. Have the students cut apart the squares and fit them back together so that the edges that touch name the same number.

2. Solution:

14	12	8	15
2010	2 × 516	4 × 421	3 × 730
16	18	25	7
2 × 8	3 × 6	5 × 5	7 × 1
18	22	2 × 112	3 × 42 × 918
4	0	2 × 7	3 × 5
2 × 2	6 × 0	14	15
6	93	3 × 35 × 420	7 × 535
2 × 10	6	2 × 8	8
20	2 × 3	16	2 × 4
1	12	2 × 60	0 × 730
5 × 6	9	15	12

3. Variation: Glue the "Side Match" solution to a piece of tagboard and then glue a picture to the other side. Cut the pieces apart and have the students rearrange them. If they find the correct solution, the picture on the backside will be intact.



Side Match

20	2×2	14	12
1	12 6	9 20	10 2×5 16
15	2×10	16	18
8	15	3×6	2×8
4×4	21 3×7	30 2×1	12 18 2
25	7	0	4
5×5	6×0	14	15
3×4	2×9	3×5	5×4 20 7×5 35 8
2×7	6	2×8	8
2×4	7×1	2×3	16
5×6	9 18	6 2×6	0 0×7 30
12	3×5	16	10

Use with "Side Match" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Describing quotients of numbers through $100 \div 10$

Skill Analysis

- A. Understands the concept of multiplication
- B. Understands the concept of division
- C. Understands the relationship between multiplication and division
- D. Understands and can use the symbols "x," " $\div$," " $\overline{\hspace{1cm}}$," and "=" when recording work
- E. Can recall the easy multiplication facts (0, 1, 2, 5, and 10)



Concept Manipulative Activities

Division Sets

Materials: Counters

- Directions:
1. Tell the students to count out 10 counters. Say, "Separate the counters into two equivalent sets. How many counters are in each set?" (5) "Are any counters left over?" (no)
 2. Ask if anyone in the class can write a division sentence for the model he/she just built. If no one knows how to write the sentence, write $10 \div 2 = 5$ on the board.
 3. Discuss the symbols and their relationship to the model. For example, say, "Ten can be divided into two equivalent sets of five. The **quotient** of 10 divided by 2 is 5."

4. Tell the students to look again at the model they built.
Example:



Say, "This could also be a model for multiplication. Can anyone tell me what multiplication sentence describes this model?" ($2 \times 5 = 10$) Write the sentence on the board if no student volunteers the answer.

5. Ask the students if they see a relationship between the two problems. Guide the discussion to help them see that division separates a total into equivalent groups and that multiplication combines equivalent groups to get the total.
6. Ask, "How can you use what you know about the multiplication facts for 5 to help you solve the following division problems?"

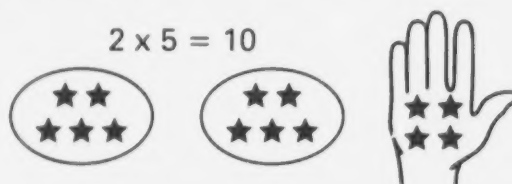
$$20 \div 5 = ? \quad (\text{THINK } 5 \text{ times } ? \text{ is equal to } 20.)$$

$$35 \div 5 = ? \quad (\text{THINK } 5 \text{ times } ? \text{ is equal to } 35.)$$

Give the students other examples until they are comfortable with this concept. The students could build models for each problem to check their answers.

7. Say, "Make a set of 14 counters. Separate your set of 14 into sets of 5. How many sets of 5 do you have?" (2) "How is this problem different from the previous problems?" (There are 4 left over.)
8. Ask, "Can anyone in the group can write a division sentence for this model?" ($14 \div 5 = 2 \text{ R}4$) Write the sentence on the board if no student knows what to write.
9. Discuss the symbols and their relationship to the model. Say, for example, "Fourteen can be divided into 2 sets of 5 with 4 remaining. The **quotient** of 14 divided by 5 is 2, with a remainder of 4."
10. Tell the students to look again at the model they just built. Say, "This model is similar to the models we have built for multiplication. Cover up the remainder (leftovers) with

your hand. What multiplication problem fits this model?"
 ($2 \times 5 = 10$) Example:



11. Ask, "What number did we start with?" (14) Say, "But, 2×5 equals 10. What can we do to get back to the 14 we started with?" (Add on the 4 remainders.)
12. Write the equation $(2 \times 5) + 4 = 14$ on the board. Explain to the students that you put 2×5 in parentheses to show that they multiply those numbers **first** and then add the remainder.
13. Show them that they could also have worked the problem vertically. Example:

$$\begin{array}{r} 5 \\ \times 2 \\ \hline 10 \\ + 4 \\ \hline 14 \end{array}$$

Either method is correct. Discuss the fact that in division there will oftentimes be a remainder.

14. Ask, "How can you use what you know about the multiplication facts for 5 to help you solve the following division problem?"

$16 \div 5 = ?$ (**THINK** $2 \times 5 = 10$. That might work.)

(**THINK** $3 \times 5 = 15$. That's very close.)

(**THINK** $4 \times 5 = 20$. That's too much.)

Say, "Count out 16 counters. Now use them to build a model for 2×5 . How many counters did you use?" (10)

"How many counters do you have left?" (6) "Do you have enough counters left over to make another set of 5?" (yes)

"Build another set of 5."

15. Have the students write a multiplication sentence for this new model. (They could write $3 \times 5 = 15$.) Ask, "Do you have enough counters left from the 16 you started with to make

the model for $4 \times 5 = 20$?" (no) Say, "The closest multiplication fact, without going over 16, to help you solve the problem $16 \div 5 = ?$, is $3 \times 5 = 15$. There will be a remainder of 1."

16. Show the students how to record this problem. Example:

$$16 \div 5 = 3 \text{ R}1 \quad \text{OR} \quad \begin{array}{r} 3 \text{ R}1 \\ 5 \overline{)16} \end{array}$$

17. Have the students write out their multiplication facts for 5 from 0×5 to 10×5 and use this table to help them solve the following problems:

$0 \times 5 = 0$
$1 \times 5 = 5$
$2 \times 5 = 10$
$3 \times 5 = 15$
$4 \times 5 = 20$
$\vdots$
$\vdots$
$10 \times 5 = 50$

$$27 \div 5$$

(25 is close and 30 is too much, so I'll use $5 \times 5 = 25$)

$$5 \overline{)33}$$

$$6 \div 5$$

$$5 \overline{)18}$$

$$49 \div 5$$

$$5 \overline{)13}$$

18. Have the students build models for the problems if they are having trouble seeing the relationship between multiplication and division. For example, the following model shows 3 sets of 5 or $3 \times 5 = 15$, but it also shows 15 divided into sets of 5 or $15 \div 5 = 3$.



19. Discuss each problem with the students. Check their understanding of the relationship between multiplication and division.

*TEACHER NOTE: Students who have memorized the easy multiplication facts will have an easier time with these problems. For other students, and for all students working on harder multiplication facts, writing out the related multiplication table will be a very useful aid.

Equivalent Sets

Materials: Counters that can be linked together to form sets

- Directions:**
1. Students may work in pairs. Give each student (or pair of students) 50 counters.
 2. Have the students write multiplication tables or use tables they have already constructed for multiplying by 1s, 2s, 5s, and 10s.
 3. Tell the students they will use these tables to help them solve some division problems.
 4. Have the students use their counters to model the following problems. They should refer to the multiplication tables to help them with the construction of the sets. After solving the problems with the aid of the models, the students should record the problems and quotients on a piece of paper.

$$15 \div 2 = \underline{\quad}$$

$$5 \overline{)38}$$

$$23 \div 10 = \underline{\quad}$$

$$2 \overline{)10}$$

$$17 \div 5 = \underline{\quad}$$

$$2 \overline{)13}$$

$$27 \div 5 = \underline{\quad}$$

$$10 \overline{)36}$$

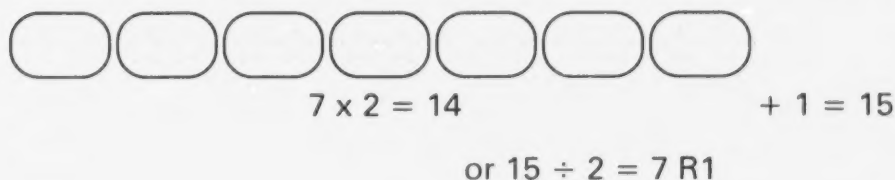
$$6 \div 1 = \underline{\quad}$$

5. While the students are working on the problems, ask questions similar to the following:
 - a. "What does this problem (point to $15 \div 2$) mean?" (15 divided by 2)
 - b. "For $15 \div 2$, what multiplication table should you refer to for help?" (the twos)
 - c. "Which multiplication fact would you use for this problem?" ($7 \times 2 = 14$)
 - d. "Why would you use 7×2 instead of 6×2 ?" (7×2 is closer) "Why would you use 7×2 instead of 8×2 ?" (8×2 is too much)
 - e. "Do you have any remainders?" (yes)

- f. "How can there be remainders?" (I don't have enough for another set.)
6. The students may need guided direction to learn how to use the multiplication tables to help them build the models for the division problems. Example:

$0 \times 2 = 0$ $1 \times 2 = 2$ $2 \times 2 = 4$ $3 \times 2 = 6$ $4 \times 2 = 8$ $5 \times 2 = 10$ $6 \times 2 = 12$ $7 \times 2 = 14$ $8 \times 2 = 16$ $9 \times 2 = 18$ $10 \times 2 = 20$	$15 \div 2 \rightarrow$	a. Have the students start with 15 counters. b. Ask, "How many groups of 2 can you make using 15 counters?" c. Tell the students to find the answer in the twos multiplication table. d. Say, "The table shows that 7 groups of 2 equals 14 and that 8 groups of 2 equals 16. 7 $\times$ 2 is the closest multiplication fact you can use without going over 15."
---	-------------------------	--

7. For the above example, building the model for $7 \times 2 = 14$ will demonstrate the meaning of having a remainder because there will be one counter left over from the 15. The same model works for $15 \div 2$.



Remainders

Materials: Counters
 Multiplication tables for reference

- Directions:
1. Ask the students to make a set of 16 counters. Write $16 \div 3$ on the chalkboard.
 2. Say, "I would like you to model this problem with your counters. You may use the multiplication table for threes to help you determine the model you should build."
 3. The students may build either 3 sets or sets of 3. Both models are correct. If they build 3 sets, ask, "How many are in each set?" (5) If they build sets of 3, ask, "How many sets of 3 did you make?" (5)
 4. Ask, "Do you have any counters left?" (yes, 1)

5. Tell the students that you are going to show them how to record their division work.

a. Write $3 \overline{)16}$ on the board. Ask, "How many 3s are there in 16?" (5)

b. Write $3 \overline{)16}^5$. Ask, "What is the total number you have if you have five 3s?" (15)

c. Write $5 \times 3 = 15$ to the side of the division problem. Say, "I am going to show you how to record this information."

$$\begin{array}{r} 5. \\ 3 \overline{)16} \end{array}$$

d. Write $\underline{15}$. Point to the 5 and the 3. Tell the students that they should multiply these numbers. (Point to the multiplication sentence, $5 \times 3 = 15$, written to the side.) Show them where you have recorded 15 in the division problem. Explain that the product is written there because it aligns the total used in forming equivalent sets (the product) under the total number given.

$$\begin{array}{r} 5. \\ 3 \overline{)16} \\ - 15 \\ \hline \end{array}$$

e. Write $\underline{1}$. Say, "Look at the models you built. How many counters did you use in the sets?" (15) Say, "If you used 15 of the 16 counters, what can we do with these two numbers to see how many counters are left over?" (subtract)

f. Point to the subtraction in the division problem. Explain to the students that using the related multiplication problem and then subtracting is how to keep track of how many counters were used and the number left over.

$$\begin{array}{r} 5R1. \\ 3 \overline{)16} \\ - 15 \\ \hline 1 \end{array}$$

g. Write $\underline{1}$. Show the students how to write the 1 as a remainder in the quotient. The quotient is 5, remainder 1.

6. Repeat steps a–g for the following problem: $5 \overline{)35}$
 - a. Have the students refer to the multiplication table for fives to find the quotient.
 - b–d. Record $7 \times 5 = 35$ next to the problem.
 - e. Subtract $35 - 35 = 0$. Discuss with students that this means there is no remainder.
 - f–g. Explain to the students that since there is no remainder you don't have to write anything else in the quotient. The quotient is 7.
7. Repeat the activity using other problems with and without remainders. After working through a few examples with the students, have them do the recording themselves.

Counter Division

Materials: Counters

- Directions:
1. Give the students 20 counters each and have them divide their counters into equal groups (e.g., 2 piles of 10, 4 piles of 5, etc.).
 2. Once they have decided how they wish to divide their counters, ask them what multiplication sentence they could write about their sets (e.g., 2×10 , 4×5 , etc.).
 3. Discuss how they arrived at their answers. Have the students write down their multiplication sentences.
 4. Say, "Look at your model again. This time think of a division problem to go with the model. Think of how many counters you started with and how many groups you divided them into. Then write the division sentence under the multiplication sentence." (Example: $20 \div 2 = 10$, $20 \div 4 = 5$, etc.)
 5. Repeat the activity using the sets of 24, 18, 15, etc. Discuss how each model can be used for both multiplication and division.

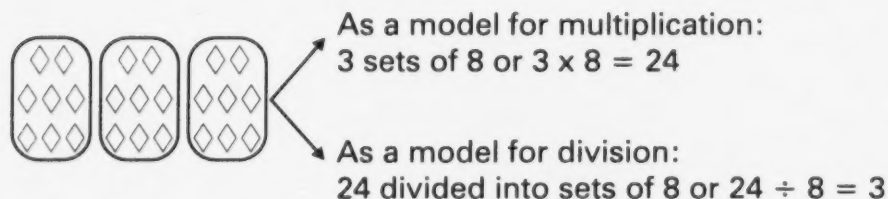


Let's Practice

Stack 'Em Up

Materials: UNIFIX cubes

- Directions:
1. Give each student 24 UNIFIX cubes. Write $3 \times 8 = 24$ on the chalkboard and ask the students to use their UNIFIX cubes to model 3 sets of 8.
 2. Say, "You divided your 24 cubes into groups of 8. Another way of saying that, mathematically, is 24 divided by 8." Write the division sentence $24 \div 8 = \underline{\quad}$ on the board.
 3. Ask, "How many 8s are there in 24?" (3) Write a 3 on the blank in the problem. Read the sentence for the students: "24 divided by 8 equals 3."
 4. Discuss with the students how the model they made could be used for **either** multiplication or division. Example:



5. Encourage the students to think of multiplication as joining the equivalent "parts" to get the total. Encourage them to think of division as starting with the total and dividing it into equal parts.
6. Repeat the activity with other problems that don't have remainders. Examples:

TOTAL	PARTS	MULTIPLICATION	DIVISION
30	3 sets of 10	$3 \times 10 = 30$	$30 \div 10 = 3$
15	5 sets of 3	$5 \times 3 = 15$	$15 \div 3 = 5$
32	8 sets of 4	$8 \times 4 = 32$	$32 \div 4 = 8$

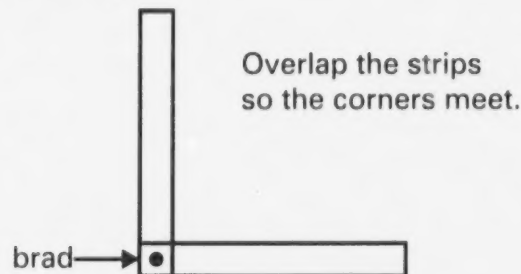
7. Give the students some problems with remainders.
8. Have the students record **all** of their work.
9. Variation: Have the students start with a total and do the division first (e.g., 24 divided into 4 equivalent sets). Have them look at the model and write both the division and related multiplication sentences.

L Strips

Materials: Tagboard and brads for making "L" cards (see directions below)

"Multiplication and Division Hundreds Chart" (one per student; blackline master follows this activity)

- Directions: 1. Have each student construct an "L" card by cutting two strips of $1/2$ " x 12" tagboard and hooking them together with a brad. Example:



2. Have the students complete the "Multiplication and Division Hundreds Chart," filling in the empty circles with the relevant products.
3. Say, "I'm going to show you how to use your 'L' cards to find the intersection of two numbers that have been multiplied." Explain that the students need to turn their "L" cards over so that they make a backwards L. (┘)

4. Have the students place their backwards "L" cards on their "Multiplication and Division Hundreds Chart" so that the 7 at the top of the page and the 4 on the left side are just inside the card. Example:

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100

5. Tell the students to follow the two strips to the place where they meet (at the corner with the brad).
6. Ask, "What number is in this corner?" (28) "Why is 28 here?" (Because 4×7 or 7×4 equals 28.)
7. Ask, "How can you use this chart to help you prove that 4×7 or 7×4 equals 28?" Possible responses:
- "If you count all the circles inside the 'L' card, you get 28."
 - "You have 4 rows with 7 in each row. If you look at the circles in the last column you can count by 7s: 7, 14, 21, 28."
 - "You have 7 columns with 4 in each. If you go along the bottom row you can count by 4s: 4, 8, 12, 16, 20, 24, 28."
 - "You made a rectangle that is 4 by 7, so you get 28."
8. Say, "Think about how you could use this chart for division. How could you use the chart to find the answer to $28 \div 4$?" (Make a backward "L" so that the 28 is in the corner and one side is under the 4. Follow the other side up to the top row. You are at 7, so $28 \div 4 = 7$.)
9. Ask, "Why do you think this works?" Possible responses:
- "If you go to the top it shows how many are in each row. You divided 28 into 4 rows and there are 7 in each one."
 - "Because the 'L' card showed you what you multiplied."

- c. "I don't know."
10. Allow the students time to experiment with the "L" card in groups of two or three.
 11. Ask each group to share something they learned or were able to figure out using their charts and "L" cards.
 12. Have the students find solutions to various problems using their charts and "L" cards. Give them both multiplication and division problems. For multiplication problems, have them find the product and then write a related division problem. For division problems, have them find the quotient and then write a related multiplication problem. Make sure they record all of the problems and their solutions.
 13. Discuss the problems and how the students found the solutions.



Multiplication And Division Hundreds Chart

1	2	3	4	5	6	7	8	9	10
2									
3									
4									
5									
6									
7									
8									
9									
10									

Use with "L Strips" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Write to Learn

Materials: Counters

Directions: 1. Give the students a division problem (e.g., $9 \div 3$). Have them use counters to build a model of the division problem. Then have them draw a picture of their model and record the division problem beneath it. Next, have them write the related multiplication sentence. Examples:



$$9 \div 3 = 3$$

$$3 \times 3 = 9$$



$$7 \div 2 = 3 \text{ R}1$$

$$2 \times 3 = 6, 6 + 1 = 7$$

2. When they are finished, have the students write an explanation for what they have done. Explain that they should be as organized and concise as possible in their written explanations. Examples:
 - a. "My division problem is $9 \div 3 = 3$. I took nine cubes and made three sets. Each set had three cubes in it. My multiplication problem is $3 \times 3 = 9$ because three sets of three equals nine."
 - b. "I had seven buttons. I put them in groups of two. I made three groups and had one button left over. My division problem is $7 \div 2 = 3 \text{ R}1$, which means I had one left. My multiplication problem is $2 \times 3 = 6$, but I have to add on my one left over. Then I get 7."
3. Encourage the students to think about what they are writing. They should use their own wording. Read their explanations to check concept understanding.
4. Repeat the activity with several different problems. Include some with remainders.

Contig

Materials: "Contig Game Sheet" (one per group of students; blackline master follows this activity)

Dice (three per group)

Game markers

- Directions:
1. This game is played in groups of two to five students. Give each group a game sheet, game markers, and three dice.
 2. Have each student roll a die to see who goes first. The student who rolls the largest number goes first.
 3. That student rolls the three dice and adds, subtracts, multiplies, or divides the three numbers in any order. Some possible problems and answers for a roll of 2, 5, and 3 are listed below.

$$2 \times 5 + 3 = 13$$

$$5 - 2 + 3 = 6$$

$$5 \times 3 \times 2 = 30$$

$$5 - 3 + 2 = 4$$

$$5 \times (3 - 2) = 5$$

$$2 + 3 + 5 = 10$$

4. The object is to create a problem whose answer is the number on an unoccupied square on the game board. Only one square may be filled in per turn.
5. The student whose turn it is needs to write the problem and answer on a piece of paper. The other students then check the problem to make sure it is correct. If it is correct, the student places a marker on the game square. If it is wrong, the student loses his/her turn. You may want to collect the problems at the end of the game.
6. The game continues with each student, in turn, rolling the dice and attempting to fill an unoccupied square.
7. Students receive one point for placing a marker on an unoccupied square plus one additional point for each contiguous (touching) square—above, below, to either side, or diagonal—that is covered. The most points that can be scored in any one turn is nine. Most students will score three or four points after a few turns.

8. The student with the most points after a set amount of time or when the sheet is almost full is the winner.
9. Variation: Have the students play the game in cooperative teams. Each team takes 40 turns in a row and sees how many points they can get as a team.

313

**Contig Game Sheet**

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	44	45	48	50	54	55
60	64	66	72	75	80	90	96
100	108	120	125	144	150	180	216

314

Use with "Contig" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Universal Division

Materials: Universal Game Board (directions for its construction may be found in the Appendix)
 Game markers
 Blank cards

Directions: 1. Write (or have the students write) the following problems on blank cards:

$3 \overline{)13}$	$4 \overline{)25}$	$5 \overline{)24}$	$58 \div 7$	$36 \div 4$	$46 \div 5$
$4 \overline{)17}$	$3 \overline{)10}$	$3 \overline{)22}$	$26 \div 6$	$8 \div 8$	$31 \div 4$
$5 \overline{)26}$	$5 \overline{)17}$	$2 \overline{)13}$	$59 \div 8$	$21 \div 3$	$40 \div 6$
$6 \overline{)41}$	$7 \overline{)44}$	$9 \overline{)83}$	$40 \div 9$	$24 \div 6$	$61 \div 8$
$7 \overline{)51}$	$6 \overline{)39}$	$4 \overline{)16}$	$35 \div 8$	$18 \div 4$	$29 \div 5$
$3 \overline{)20}$	$8 \overline{)25}$	$7 \overline{)42}$	$77 \div 9$	$35 \div 8$	$10 \div 8$
$2 \overline{)7}$	$4 \overline{)33}$	$4 \overline{)8}$	$52 \div 7$	$7 \div 3$	$38 \div 4$
$3 \overline{)20}$	$3 \overline{)19}$	$2 \overline{)12}$	$27 \div 3$	$15 \div 7$	$16 \div 2$
$6 \overline{)44}$	$9 \overline{)57}$	$8 \overline{)16}$	$15 \div 5$	$5 \div 2$	$22 \div 5$

- This game should be played in groups of two to four students. Have each student draw a card. The student whose card has the largest (or smallest) divisor starts the game.
- All of the cards are returned to the deck. The cards are shuffled and placed face down in a pile on the game board, and the markers are placed at Start.
- The first player draws the top card and solves the problem. (The players may use multiplication tables to help them find the solutions to the division problems.) He/she states the answer. If it is correct, the student moves his/her marker the number of spaces named by the remainder. For example, if the student drew the problem $7 \div 3$, he/she would move one space because $7 \div 3 = 2 \text{ R}1$. If the student's answer is incorrect, the student loses his/her turn.
- Some of the problems have a remainder of zero. Have the students decide ahead of time how to treat these problems. They could either not move at all when they draw a "zero

remainder" card or move ahead a predetermined number of spaces.

6. Play progresses to the right, and the used cards are returned to the bottom of the deck.
7. Play continues until a student reaches the finish or time is called.



Problem Solving

How Many Remainders?

Materials: Counters

- Directions:
1. Give each student 30 counters. Say, "You will be using these counters to learn some things about the remainders you can get when dividing."
 2. Say, "Make a set of 1. Can you divide this set into two equal groups?" (Expect answers of "No" or "You can cut the counter in half.") Tell the students that, for now, they cannot cut or break the counters. They are to make sets of **whole** counters only.
 3. Write $2 \overline{)1}$ on the board. Ask, "What is the answer to this problem?" Students may need to discuss the fact that zero is a number and that they can have an answer of zero.
 4. Ask one of the students to go to the board and complete the problem.

$$\begin{array}{r} 0 \text{ R}1 \\ 2 \overline{)1} \end{array}$$

5. Next, have the students make a set of 2. Write $2 \overline{)2}$ on the board. Ask, "What is the quotient for this problem?" (1) "Is there a remainder?" (no)
6. Continue with sets of 3, 4, 5, etc., all divided by 2. After several examples, ask the students to share what they have observed about the remainders you get when dividing by 2. If the students don't come up with any ideas on their own, lead the

discussion by saying, "When dividing by 2, there is either no remainder or a remainder of 1."

7. Ask, "Why do you think we never get a remainder of 2 or 3?" (You would be able to subtract another set of two, or if you were partitioning, you would be able to put one more in each set.)
8. Repeat the activity for divisors of 3 (remainders of 1 and 2), 4 (remainders of 1, 2, and 3), etc.
9. Have the students keep a written record of the work they do as they model the problems to help them connect the physical model to the mathematical sentence. Examples:

$$\begin{array}{r} 0 \text{ R} 1 \\ 3 \overline{)1} \end{array} \quad \begin{array}{r} 0 \text{ R} 2 \\ 3 \overline{)2} \end{array} \quad \begin{array}{r} 1 \\ 3 \overline{)3} \end{array} \quad \begin{array}{r} 1 \text{ R} 1 \\ 3 \overline{)4} \end{array} \quad \begin{array}{r} 1 \text{ R} 2 \\ 3 \overline{)5} \end{array} \quad \begin{array}{r} 2 \\ 3 \overline{)6} \end{array}$$
10. Have the students also make a remainder chart. Example:

DIVISOR	POSSIBLE REMAINDERS
1	
2	1
3	1, 2
4	1, 2, 3
:	:

11. Tell the students to look at their remainder chart. Ask, "Do you see a pattern? What can you say about remainders?" (They can be any number that is less than the divisor.)

Invicta Multiplication and Division

Materials: Invicta balance (one per group of two or three students)

- Directions:**
1. On one side of the balance, have the students hang one weight at 10 and another weight at 5. Ask, "What number does this represent?" (15)
 2. Tell the students that you want them to balance the scale by hanging weights on the other side. However, this time, all of the weights must be placed at the same number. Possible solutions: three weights at 5 or five weights at 3.

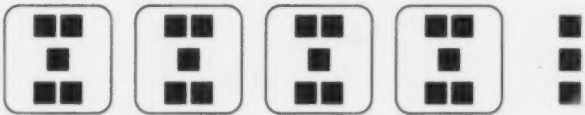
3 17

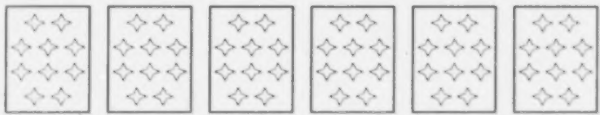
3. When the students have discovered one of the solutions, ask them to write a division number sentence describing the model they made using the weights (e.g., $15 \div 3 = 5$ or $15 \div 5 = 3$, since 15 can be divided into three sets of 5 or five sets of 3).
4. Ask the students to look at the model again and write a multiplication number sentence that describes the model (e.g., $3 \times 5 = 15$ [for three weights at 5], or $5 \times 3 = 15$ [for five weights at 3]).
5. Repeat the activity having students hang two weights at 10 and one weight at 1. Ask, "What is the value represented on the scale?" (21)
6. Have the students balance the scale by once again, hanging all of the weights at the same number on the other side. Possible solutions: three weights at 7 or, if they can, seven weights at 3.
7. Have the students write both a division and a multiplication number sentence for the model (e.g., $21 \div 7 = 3$ or $21 \div 3 = 7$, and $3 \times 7 = 21$ or $7 \times 3 = 21$).
8. Next, have the students hang one weight at 10 and one weight at 8. Ask, "What number does this represent?" (18)
9. Ask, "Can you balance the scale by hanging weights only at 5?" (no) Lead a discussion that will help the students see that three weights at 5 would not be enough but four weights at 5 would be too much.
10. Have the student hang three weights at 5. Then say, "I want you to add just one more weight to the scale (on the side with three 5s) so that it balances." The students should find that they need to hang the weight at 3. Explain that the weight of 3 represents a remainder of three.
11. Have the students write a division problem and a related multiplication problem for this model (e.g., $18 \div 5 = 3 \text{ R}3$; and $5 \times 3 = 15$, $15 + 3 = 18$, or $3 \times 5 = 15$, $15 + 3 = 18$).
12. Repeat the activity using other totals for the students to balance.

MULTIPLICATION & DIVISION

Directions: The following drawings can be used as models for both multiplication and division. Write one multiplication number sentence and two division number sentences for each model.

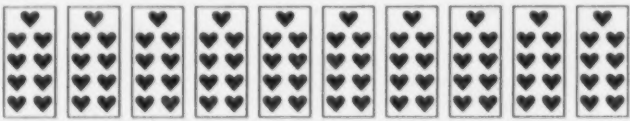
1.  _____

2.  _____

3.  _____

4.  _____

5.  _____

6.  _____



*TEACHER NOTE: Some students may need help initially in recording remainders with multiplication problems. They can use either of two formats: $T = a \times b + R$ or $a \times b = c$ and $c + R = \text{Total}$.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Describing basic facts for multiplication and division

Skill Analysis

- A. Understands the concept of multiplication
- B. Understands the concept of division
- C. Understands the symbols for multiplication and division ($\times$, $\div$, $\overline{\hspace{1cm}}$) and can use them to solve problems in vertical and horizontal form
- D. Understands the terms "product," "quotient," "factor," "divisor"
- E. Understands the inverse relations of multiplication and division



Concept Manipulative Activities

Looking at Sets

Materials: UNIFIX cubes, blocks, Learning Links
Index cards or tagboard cut into 3" x 5" pieces

Directions: 1. Review the use of sets for modeling multiplication and division. For example, say, "Using UNIFIX cubes, show me what 4×3 means." Students might build something like this:



and then say that 4×3 means 4 sets of 3.

2. Ask, "How do the same blocks show division? What problem do they represent?" Students might respond in either of two ways:

- a. There are 12 UNIFIX cubes that have been divided into sets of 3. There are 4 sets, so the problem is $12 \div 3 = 4$.
 - b. There are 12 UNIFIX cubes that have been divided into 4 sets. There are 3 UNIFIX cubes in each set, so the problem is $12 \div 4 = 3$.
3. Continue the review with several other examples.
 4. Divide the class into 10 groups. Give each group a number from 1 to 10. Have each group use counters to build the multiplication table for its assigned number. For example, the group given the number 3 would build the "3 times table" with UNIFIX cubes and then label each set of cubes by writing the appropriate fact on an index card.



5. Have each group share its "times table" model. After all of the groups have shared their models, spend time discussing what the students noticed and what patterns they saw.

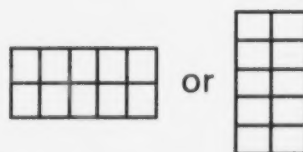
**TEACHER NOTE:* To construct all of the "times table" models, the students would need 3,025 UNIFIX cubes. If that number is not available, spread the activity over several days by having students model the lower numbers first and then focus on the larger numbers.

Arrays

Materials: UNIFIX cubes, blocks, or graph paper (blackline masters for various sizes of graph paper may be found in the Appendix)
Index cards or tagboard cut into 3" x 5" pieces

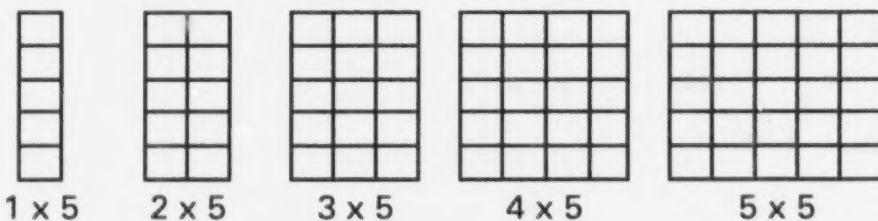
Directions: 1. Review the use of arrays for modeling multiplication and division. For example, say, "Using UNIFIX cubes or graph paper, build or draw an array that will show me what 2×5

means." The students might build or draw an array like this:



and then say 2×5 means 2 rows of 5 or 2 columns of 5.

2. Ask, "How could your array show division? What problem does it represent?" Students might respond in either of two ways:
 - a. The array shows 10 boxes (or blocks or cubes). There are 2 rows with 5 boxes in each row. The problem is $10 \div 2 = 5$.
 - b. The array shows 10 boxes (or blocks or cubes). There are 5 rows with 2 boxes in each row. The problem is $10 \div 5 = 2$.
3. Continue the review with several other examples.
4. Divide the class into 10 groups. Give each group a number from 1 to 10. Have each group use cubes, blocks, or graph paper to build the multiplication table for its assigned number. For example, the group given the number 5 would build the "5 times table" with arrays and would label each array with the appropriate fact using index cards or tagboard cut in 3" x 5" pieces.

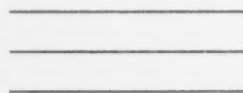


5. Have each group share its "times table" model. As a class, spend time discussing what the students noticed, what patterns they saw, and which facts formed perfect squares.

Crosslines

Materials: Long strips of paper, yarn, or string, yardsticks, etc. (Students could also use any straightedge and draw the lines)

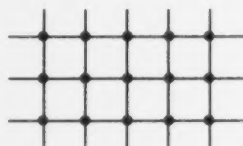
- Directions:**
1. Write the problem 3×5 on the board.
 2. Tell the students to form horizontal lines to represent the first factor (3). Example:



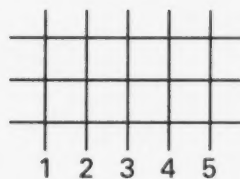
3. Then tell the students to form vertical lines on top of the horizontal lines already in place to represent the second factor (5).



4. Tell the students to count the places that the lines cross or intersect. (There are 15.)



5. Go over several other problems with the students.
6. Have the students work in pairs, with each student giving his/her partner other problems to illustrate in this manner.
7. Discuss how this model can be used for division. Give the students the problem $15 \div 3 = \underline{\quad}$. Say, "We know factors can be represented by lines. In the problem $15 \div 3$, the 3 indicates one factor and we need to find the second factor. We know that there should be 15 intersections altogether. Draw three horizontal lines to represent the number 3. Then begin drawing vertical lines over the horizontal lines. Stop when you have made 15 intersections."



8. Explain that because five lines were made to show the 15 intersections, $15 \div 3 = 5$.
9. Have the students continue using crosslines to illustrate other division problems, again working in pairs.

Balance

Materials: Invicta balance (one per group of students)

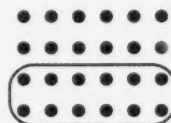
- Directions:**
1. Have the students work with a partner or in small groups. Give each group an Invicta balance.
 2. Have the students hang one weight on one side of the balance at 9.
 3. Ask the students to find where they could hang one weight or a combination of weights on the other side of the balance to make both sides balance. They must hang all of the weights on the same number. Possible solutions: They could hang one weight on 9 (which means one set of 9, or 1×9), three weights on 3, or nine weights on 1, if possible.
 4. Ask, "What facts did you model?" Students may state multiplication or division facts.
 5. Give the students another problem to work in this manner.
 6. Write several numbers on the board that can be modeled on the balance. For each, ask, "How many different ways can you model this number?"
 7. Have the students share the models they found with the rest of the class.

***TEACHER NOTE:** Giving students numbers greater than 10 will reinforce the concept of place value since they need to be represented with more than one weight. For example, 24 would be represented with two weights on 10 and one weight on 4. Allow the students to determine how to represent these numbers. Also, check to see how many weights a number on the balance will hold. Some problems may be impossible to model.

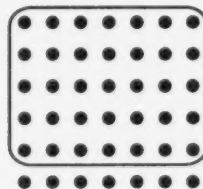
Ways to Remember

Materials: Graph paper or centimeter dot paper (blackline masters may be found in the Appendix)

- Directions:**
1. Conduct this activity with a group of students or when a student says, "I can't remember $\underline{\quad} \times \underline{\quad} = \underline{\quad}$."
 2. Give the students the problem 4×6 .
 3. Tell them to pretend they can't remember the product of that problem.
 4. Ask the students if there might be a smaller "6 fact" that they might remember. Then use the facts the students generate to help them solve the unknown problem.
 5. Example: "I know $2 \times 6 = 12$. If I make an array of the unknown fact 4×6 and I circle the fact I do know, how might that help me find the answer?" (Another array of the exact same size is evident, so the answer must be $(2 \times 6) + (2 \times 6)$, or $12 + 12 = 24$.)



6. Continue with other facts if more work is needed.





Let's Practice

Contig

This is a game that students find fun and reviews all the basic facts. It is described in the third grade materials.

Square Scores Game

Materials: "Square Scores Game Sheet" (blackline master follows this activity)

Vis-A-Vis pens or erasable markers

8-1/2" x 11" tagboard

- Directions:
1. Divide the class into groups of two to three students. Give each group a "Square Scores Game Sheet" that you have mounted on tagboard and laminated or covered with clear Con-Tact paper.
 2. Have the students in each group take turns drawing a line connecting two dots horizontally or vertically.
 3. When a student's line encloses a square, he/she should answer the fact written in that square. If the answer is correct, that student should write his/her initials on that square. If the answer is incorrect, the next player may give the correct answer and initial the square.
 4. If a student encloses two squares, he/she may answer both facts and initial both squares.
 5. Play continues until all of the squares are enclosed.
 6. Students score a point for each square they initial. The student with the most points wins.



Square Scores Game Sheet

•	•	•	•	•	•	•	•	•
3×3	7×3	$45 \div 9$	8×3	3×9	$56 \div 7$	$54 \div 9$	$24 \div 8$	
•	•	•	•	•	•	•	•	•
$72 \div 8$	$54 \div 6$	6×3	$48 \div 6$	7×4	$36 \div 4$	3×8	4×9	
•	•	•	•	•	•	•	•	•
8×4	3×4	$32 \div 4$	$63 \div 7$	$30 \div 6$	$81 \div 9$	6×7	$27 \div 3$	
•	•	•	•	•	•	•	•	•
$63 \div 9$	$24 \div 6$	9×3	$32 \div 8$	$24 \div 3$	$42 \div 6$	$27 \div 9$	$36 \div 6$	
•	•	•	•	•	•	•	•	•
6×4	$21 \div 3$	$35 \div 7$	$18 \div 3$	$28 \div 7$	7×6	$64 \div 8$	$49 \div 7$	
•	•	•	•	•	•	•	•	•
$15 \div 3$	3×6	$12 \div 3$	$72 \div 9$	6×6	4×7	8×8	4×8	
•	•	•	•	•	•	•	•	•
4×4	$3 \div 3$	8×9	$21 \div 7$	5×5	3×7	9×6	6×9	
•	•	•	•	•	•	•	•	•
7×7	$35 \div 5$	$36 \div 9$	$28 \div 4$	9×8	$18 \div 6$	$24 \div 4$	$4 \div 4$	
•	•	•	•	•	•	•	•	•
$18 \div 2$	8×5	9×4	$12 \div 4$	9×7	$42 \div 7$	$30 \div 5$	$48 \div 8$	
•	•	•	•	•	•	•	•	•
7×8	$40 \div 5$	4×6	5×7	7×9	$20 \div 4$	8×7	$16 \div 4$	
•	•	•	•	•	•	•	•	•
9×5	6×8	9×9	8×6	5×9	$56 \div 8$	$9 \div 9$	$45 \div 5$	
•	•	•	•	•	•	•	•	•

Use with "Square Scores Game" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Multiplication Dice

Materials: Three dice, one numbered 0–5, one numbered 3–8, and one numbered 4–9.

“Multiplication Dice” handout (one per pair of students; blackline master follows this activity)

Calculators (one per group of students)

- Directions:**
1. This game is played by pairs of students.
 2. The first player tosses the three dice, chooses two of the numbers rolled to multiply, and gives the product.
Example:

$$\boxed{4} \boxed{7} \boxed{8} \quad 4 \times 7 = 28$$

3. The student's partner may challenge the answer and check it with a calculator.
4. If the answer is correct, the player scores a point and initials the box that contains the answer on the handout.
5. If the box the player initials is touching an already initialed box or a “free” box—above, below, to either side, or diagonally—the player scores an additional point. Up to eight points may be scored on a turn.
6. The player with the most points when time is called wins.



Multiplication Dice

3	0	27	9	28	45	4
12	free	20	24	36	42	16
64	10	8	54	35	5	40
7	72	72	12	free	18	25
36	25	14	56	20	0	21
32	30	63	24	49	8	15
4	free	16	32	15	48	6

Use with "Multiplication Dice" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Multiplication War

Materials: Two decks of cards per pair of students (each deck should contain four cards of each number 0–9)
Calculators (one per pair of students)

- Directions:**
1. This game is played by pairs of students.
 2. Each student selects and shuffles one of the decks of cards.
 3. Then, at the same time, the two players turn over the top card from their deck.
 4. The first player to correctly identify the product of the two numbers “banks” the two cards used. The other player may challenge the answer and then check it with a calculator.
 5. The winner is the player who has collected the most cards at the end of the game or the time allowed.

Zero Game

Materials: Pair of dice, one numbered 0–5 and one numbered 3, 4, 6, 7, 8, 9
Number strip, like the following, constructed from paper or tagboard

0	1	2	3	4	5	6	7	8	9
---	---	---	---	---	---	---	---	---	---

Game markers (one per student)

- Directions:**
1. Any number of students can play this game.
 2. Have all of the students place their markers at zero.
 3. Each player, in turn, rolls the two dice, multiplies the numbers, and moves his/her marker to the square representing the last digit of the product.
 4. On subsequent turns, the players roll the dice, find the product, and add the product to the number of the square on which their marker is currently sitting. The players then move their markers to the **new** last digit.

Example:

Numbers Rolled	Product	Square
3, 6	18	8
7, 8	56	$\begin{array}{r} 56 \\ + 8 \\ \hline 64 \end{array} \rightarrow 4$
3, 4	12	$\begin{array}{r} 12 \\ + 4 \\ \hline 16 \end{array} \rightarrow 6$
4, 5	20	$\begin{array}{r} 20 \\ + 6 \\ \hline 26 \end{array} \rightarrow 6$
2, 2	4	$\begin{array}{r} 26 \\ + 4 \\ \hline 30 \end{array} \rightarrow 0$

5. The first player to get back to zero is the winner.



Problem Solving

Division Tangrams

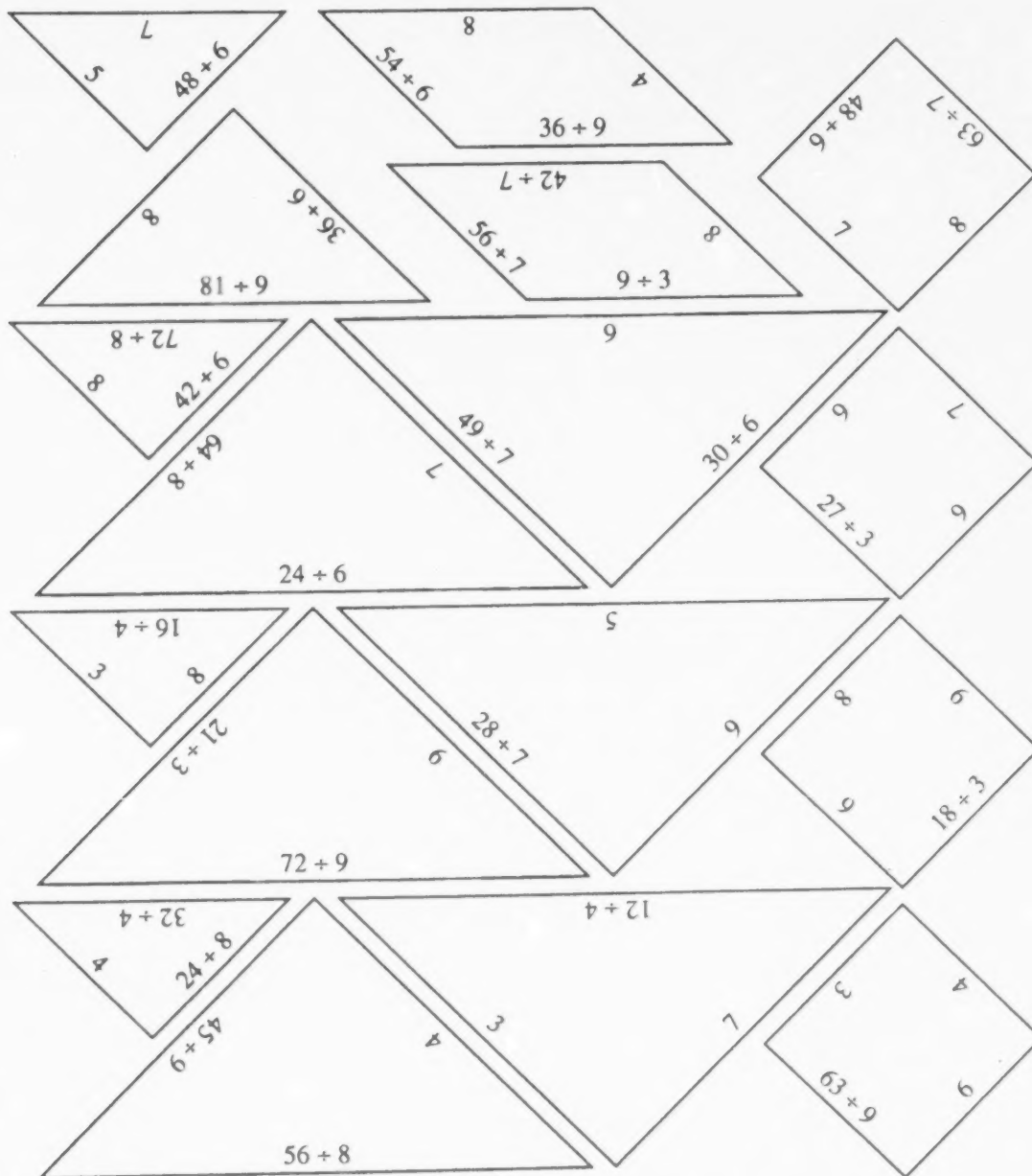
Materials: Tangram division pieces (blackline master follows this activity)

- Directions:
1. Give each student at least one set of tangram pieces.
 2. Ask the students to form a figure by connecting the sides of the pieces that have matching problems and answers.
 3. The students may paste down their figure or draw around its outline and add background.



Tangram Division Pieces

Cut the pieces out and use them with the Division Tangrams game.



Use with "Division Tangrams" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Classy Facts

Materials: Optional: Calculators

- Directions:
1. Divide the class into small groups of students.
 2. Have each group count the total number of children in a different class in the school.
 3. Then have the students create number sentences using multiplication and/or addition that equal the total they obtained. For example, if a group counted 30 children in a class, some possible problems would be:

$$3 \times 10$$

$$5 \times 6$$

$$6 \times 5$$

$$10 \times 3$$

$$(7 \times 4) + (2 \times 1)$$

$$(3 \times 8) + (2 \times 3)$$

$$(2 \times 6) + (2 \times 6) + (2 \times 3)$$

$$(4 \times 6) + (2 \times 3)$$

etc.

4. Have the groups exchange the totals they obtained and develop problems for the new number.
5. Groups that have worked with the same totals should compare the problems they developed, discussing how their problems are alike and different.
6. Variation: Extend students' investigation to the total number of students in a grade level.

SKILL Describing basic facts for multiplication and division

Skill Analysis

- A. Understands the concept of multiplication
- B. Understands the concept of division
- C. Can identify the "x," "+," and "—" symbols and use them to solve equations
- D. Understands and uses the terms "product," "quotient," "remainder," "factor," "divisor," and "multiple"



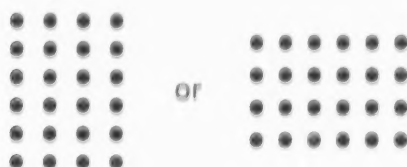
Concept Manipulative Activities

Multiplication Into Division

Materials: UNIFIX cubes
Overhead projector
Loops of string or yarn

- Directions:**
1. Have the students work in groups of four. Give each group 100 UNIFIX cubes.
 2. Have the students count out 24 cubes. On the board write $4 \times 6 = 24$. Ask the students to model this problem with their cubes. Examples:

Arrays:



Sets:

6 UNIFIX cubes in 4 sets



3. Have one student share his/her model on the overhead. Ask if anyone else made a different model. Have other students share their models on the overhead.
4. Ask, "How could we represent a division problem with these same cubes?" Instruct the students to use string or yarn to show groups. Discuss the results.
5. Ask the students to represent 24 using other multiplication problems. Have them discuss and show all the possibilities.
6. Have each group use their UNIFIX cubes to create all of the possible multiplication models for a specified number. Have the students record their models and number sentences on a piece of paper. Example: 6

$$\bullet \bullet \bullet \bullet \bullet \bullet \quad 1 \times 6$$

$$\begin{array}{cc} \bullet & \bullet \\ \bullet & \bullet \end{array} \quad 2 \times 3$$

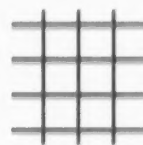
$$\begin{array}{c} \bullet \\ \bullet \\ \bullet \end{array} \quad 3 \times 2$$

$$\begin{array}{c} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{array} \quad 6 \times 1$$

Criss-Cross Multipliers

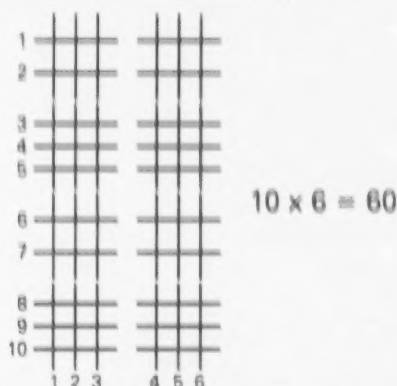
Materials: Spaghetti or sticks
Overhead projector

Directions: 1. Create the following array on the overhead using spaghetti or sticks.

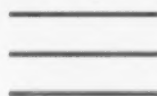


$$3 \times 5$$

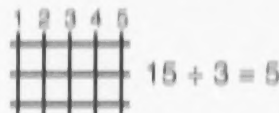
2. Ask the students what multiplication problem this array shows. (4 sticks x 3 sticks) Ask, "How can we determine the answer to 4×3 ?" (Count the places where the sticks cross.)
3. Distribute sticks or spaghetti to the students. Have them use the sticks or spaghetti to model a variety of multiplication problems.
4. Have the students represent more difficult problems by making a number of arrays. Example:



5. Ask, "How can I use the spaghetti (or sticks) to model division? If I have a problem like $15 \div 3$, how many pieces of spaghetti (or how many sticks) should I start with?" (3)
6. Place three pieces horizontally on the overhead.



7. Begin placing vertical pieces over the three horizontal pieces. Have the students count the number of intersections by 3s until they reach 15. Be sure they understand that the number of vertical pieces when they reach 15 intersections is the answer to the problem.



8. Have the students model other division problems using spaghetti or sticks.
9. Variation: In a very large area have students use yardsticks to construct arrays.

Multiplication Table

Materials: "Multiplication Table" handout (one per student; blackline master follows this activity)

Materials for modeling multiplication problems (counters, graph paper, number lines, etc.)

Markers, crayons, or colored pencils

- Directions:
1. Distribute copies of the multiplication table. Review with students how to complete the table.
 2. Have the students fill in the squares for the problems that they think are the **easiest**, those facts they know very well.
 3. When the students have completed these problems, have them lightly color the squares in red.
 4. Next, have the students fill in the squares for the problems they think are a **little harder**, those facts they know fairly well. Have them lightly color those squares in blue.
 5. Discuss which problems the students colored red and why they did so. Ask, "Why do you think these facts are easy for you?" Then do the same for the problems colored blue.
 6. Ask the students to look at their table to determine if they have shaded in the commutative pairs of these "known" facts. If they haven't, have them do so now.
 7. Discuss the remaining problems on the students' table. Have them circle each of the remaining squares they have not shaded and list the unsolved facts on a piece of paper. Review the commutative property and have them eliminate one of the commutative pairs for each fact.
 8. Review strategies the students can use to assist them in learning these harder facts. Have the students define a strategy or strategies they could apply to solve various problems.



Multiplication Table

x	0	1	2	3	4	5	6	7	8	9	10
0											
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											

Use with "Multiplication Table" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Let's Practice

Multiplication Check

Materials: Two cubes, one numbered 3–8 and one numbered 4–9 (one set for each pair of students)
Stopwatch or clock with second hand

Directions: 1. Have the students work in pairs. Have each pair make a record sheet like the following.

	Student #1	Student #2
Round 1		
Round 2		
Round 3		
Round 4		
:		
Round 10		

2. Explain that the students should, in turn, roll the cubes and multiply the numbers rolled.
3. If the product is given within 5 seconds and is correct, the student should place a checkmark on the record sheet for that round. If the product is incorrect or is not given within the 5-second time limit, the student should record the problem on the record sheet. Example:

	Student #1	Student #2
Round 1	✓	✓
Round 2	✓	9 x 6
Round 3	7 x 8	✓
Round 4	✓	✓
:		

369

4. The winner is the student with the most check marks after 10 rounds.
5. Students can repeat the game with the same or a different partner.

Multiplication-Division Match

Materials: Multiplication-Division Match Cards (blackline master follows this activity)

Vis-A-Vis pens

Multiplication table(s) with answers (student made or one from Appendix)

- Directions:**
1. Duplicate the blackline master "Multiplication-Division Match Cards" and laminate the copies. Cut the cards apart.
 2. Individually or at a center, have the students use Vis-A-Vis pens to match each problem on a card with its answer.
 3. Have the students use multiplication table(s) to check their answers.



Multiplication-Division Match Cards

Card #1	Card #2	Card #3
1. $3 \times 6 =$ A. 9	1. $54 \div 9 =$ A. 36	1. $4 \times 8 =$ A. 2
2. $81 \div 9 =$ B. 6	2. $9 \times 9 =$ B. 5	2. $7 \overline{)63}$ B. 14
3. $7 \overline{)42}$ C. 20	3. $7 \times 7 =$ C. 0	3. $8 \div 4 =$ C. 6
4. $8 \times 8 =$ D. 8	4. $28 \div 4 =$ D. 49	4. $8 \times 9 =$ D. 4
5. $4 \times 5 =$ E. 7	5. $3 \overline{)24}$ E. 8	5. $2 \times 7 =$ E. 8
6. $2 \times 8 =$ F. 16	6. $40 \div 8 =$ F. 9	6. $5 \times 8 =$ F. 7
7. $6 \overline{)48}$ G. 18	7. $7 \times 8 =$ G. 81	7. $3 \overline{)21}$ G. 72
8. $63 \div 9 =$ H. 0	8. $6 \times 6 =$ H. 7	8. $30 \div 5 =$ H. 9
9. $0 \times 7 =$ I. 45	9. $2 \overline{)18}$ I. 6	9. $1 \times 8 =$ I. 40
10. $9 \times 5 =$ J. 64	10. $0 \times 0 =$ J. 56	10. $36 \div 9 =$ J. 32
Card #4	Card #5	Card #6
1. $6 \overline{)54}$ A. 8	1. $1 \times 6 =$ A. 8	1. $35 \div 7 =$ A. 5
2. $0 \times 4 =$ B. 7	2. $9 \times 6 =$ B. 16	2. $4 \times 6 =$ B. 6
3. $4 \times 7 =$ C. 30	3. $36 \div 4 =$ C. 4	3. $3 \overline{)18}$ C. 16
4. $48 \div 8 =$ D. 72	4. $9 \overline{)36}$ D. 54	4. $5 \times 6 =$ D. 64
5. $3 \times 8 =$ E. 9	5. $3 \times 4 =$ E. 5	5. $9 \times 9 =$ E. 81
6. $6 \overline{)42}$ F. 24	6. $4 \times 4 =$ F. 7	6. $48 \div 6 =$ F. 9
7. $6 \times 5 =$ G. 3	7. $20 \div 4 =$ G. 28	7. $2 \times 8 =$ G. 30
8. $27 \div 9 =$ H. 6	8. $7 \times 4 =$ H. 6	8. $5 \overline{)45}$ H. 8
9. $8 \times 9 =$ I. 28	9. $32 \div 4 =$ I. 9	9. $12 \div 3 =$ I. 24
10. $56 \div 7 =$ J. 0	10. $7 \overline{)49}$ J. 12	10. $8 \times 8 =$ J. 4

Use with "Multiplication-Division Match" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

371

Tic-Tac-Toe

Materials: "Tic-Tac-Toe Game Board" (one per student; blackline master follows this activity)
Colored markers

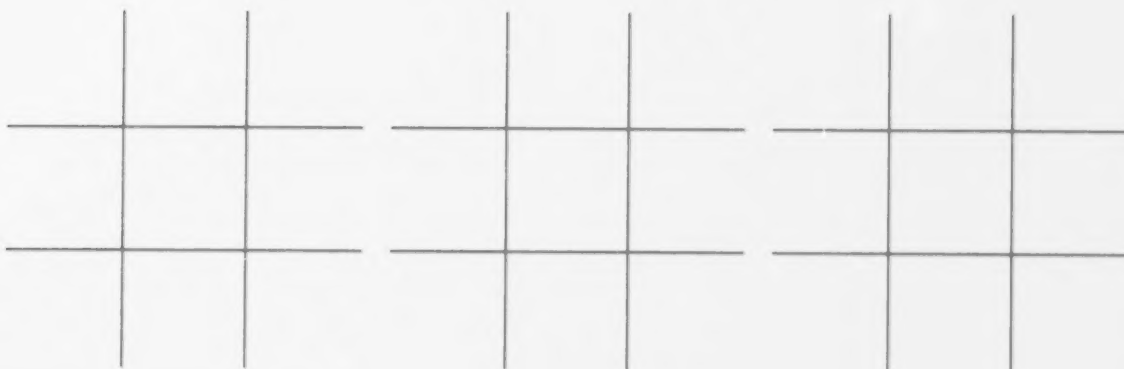
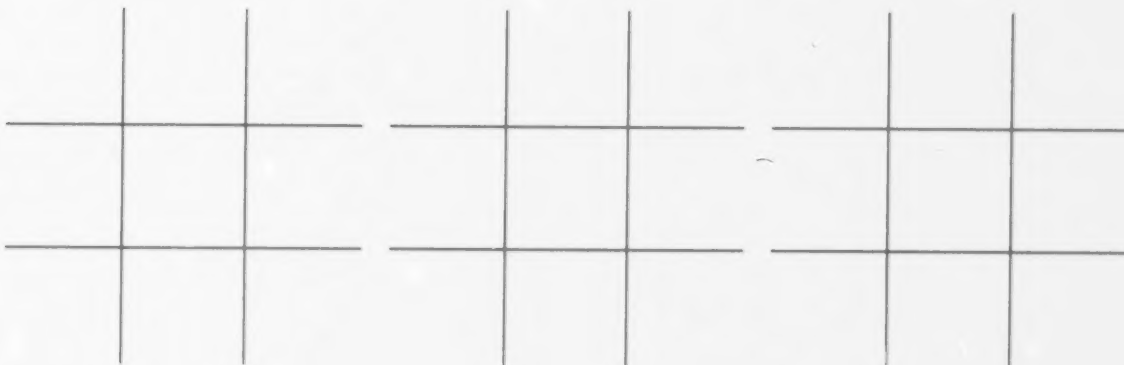
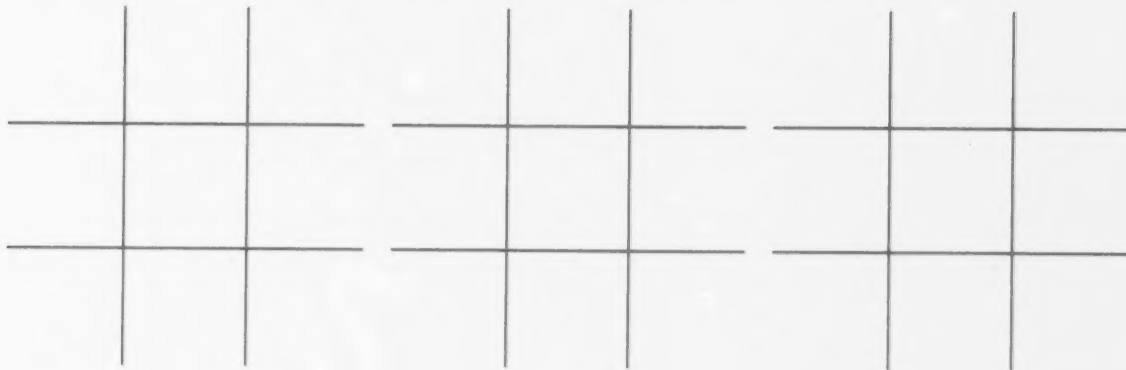
Directions: 1. Have the students write a division or multiplication problem in each square of each tic-tac-toe game on their game board. (The students may refer to a basic facts list or multiplication chart for problems.) Example:

$2 \times 3 =$	$6 \times 7 =$	$81 \div 9 =$
$30 \div 5 =$	$2 \times 8 =$	$24 \div 8 =$
$4 \times 7 =$	$21 \div 3 =$	$5 \times 3 =$

- In pairs, have the students play one game at a time, taking turns filling in the answers and attempting to get three in a row vertically, horizontally, or diagonally.
- The students should use different colored markers to distinguish between their answers.
- The student who gets tic-tac-toe begins the next game.
- The students can either complete all of the games on their two sheets or complete a predetermined number of games.
- Variation: Have the students write the answers to multiplication and division problems in the squares of the game boards, then have them take turns filling in problems to match the answers.



Tic-Tac-Toe Game Board



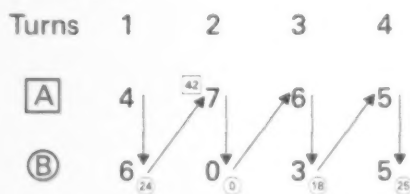
Use with "Tic-Tac-Toe" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Tic-Tac Multiplication

Materials: "Tic-Tac Multiplication" handout (one per pair of students;
blackline master follows this activity)

Colored markers

- Directions:**
1. This game is played by two students.
 2. Player **A** is the student whose first name is closer in the alphabet to A.
 3. Player **A** chooses a number (0–9) from his/her set on the handout and crosses it out.
 4. Player **B** chooses a number from his/her set, crosses it out, and multiplies by player **A**'s number. If the product is on the game board, player **B** colors in the square with his/her marker. If the product is not on the board, player **B** loses his/her turn.
 5. Play continues with player **A** choosing a number from his/her set that he/she has not used before and multiplying it by the last number player **B** chose.
 6. Players take turns until a player gets tic-tac-toe or until a player has used all the numbers in his/her set.
 7. Sample play:



GAME 1		
36	16	25
42	24	27
0	63	18

- A** chooses 4.
- B** chooses 6 ($4 \times 6 = 24$) and marks 24 on the game board.
- A** chooses 7 ($6 \times 7 = 42$) and marks 42 on game board.
- B** chooses 0 ($7 \times 0 = 0$) and marks 0 on game board.
- A** chooses 6 ($0 \times 6 = 0$) but cannot mark the game board and loses his/her turn.
- B** chooses 3 ($6 \times 3 = 18$) and marks 18 on game board.

- ☐ A chooses 5 ($3 \times 5 = 15$) but cannot mark the game board and loses his/her turn.
- ☐ B chooses 5 ($5 \times 5 = 25$), marks 25 on game board, and wins (tic-tac-toe).
8. The students may create their own game boards and play at an independent learning center.



Tic-Tac Multiplication

GAME 1

36	16	25
42	24	27
0	63	18

A plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

B plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

GAME 2

21	2	0
45	56	18
8	30	54

A plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

B plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

GAME 3

40	28	35
9	0	6
4	1	48

A plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

B plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

GAME 4

A plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

B plays {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Use with "Tic-Tac Multiplication" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Multiplication Patterns

Materials: None

- Directions: 1. By examining multiplication tables, students may discover patterns that can assist them in remembering harder facts. The following pattern, for example, can be used to find the answer when multiplying any number by any other number and is based on the distributive property.
2. Begin with any number (e.g., 6). Then follow the "doubling" and distributive property process. Example:

6	1×6
12	2×6 (double 1×6)
18	$3 \times 6 = (1 \times 6) + (2 \times 6)$; add $6 + 12 = 18$
24	4×6 (double 2×6)
30	$5 \times 6 = (2 \times 6) + (3 \times 6)$; add $12 + 18 = 30$
36	6×6 (double 3×6)
42	$7 \times 6 = (3 \times 6) + (4 \times 6)$; add $18 + 24 = 42$
48	8×6 (double 4×6)
54	$9 \times 6 = (4 \times 6) + (5 \times 6)$; add $24 + 30 = 54$
60	10×6 (double 5×6)

3. Discuss with students the options available within each distributive "step." Ask, "Why were the addends indicated chosen for each number? Why was 7 expressed as $3 + 4$ and not $2 + 5$?"
4. Go over the pattern using other multiplication tables.
5. Extension: Have the students use this pattern to create the multiplication tables for larger numbers (e.g., 15, 52, 123, and 752).



Problem Solving

Can You Make 18?

Materials: 1" ceramic tiles (13 per student)
 Black markers
 4" x 4" tagboard cards divided into fourths (students will write numbers you give them in each section)

Card #1

1	2
4	4

- Directions:**
1. Give each student 13 tiles. Have them number 9 tiles with the numerals 1-9. On the remaining 4 tiles, have the students mark "+," "-", "x," and "÷."
 2. Next, give the students 4" x 4" cards and four numbers to mark on each card. (The four numbers should total 18 when a variety of processes are used.) Examples:

Card	Possible Solution
#1 (1, 2, 4, 4)	$(4 \times 4) + (1 \times 2) = 18$
#2 (1, 2, 3, 5)	$(3 \times 5) + (1 + 2) = 18$
#3 (1, 2, 4, 7)	$(7 \times 2) + (1 \times 4) = 18$
#4 (3, 6, 9, 9)	$(9 - 9) + (3 \times 6) = 18$
#5 (3, 4, 5, 6)	$(5 \times 6) - (3 \times 4) = 18$
#6 (1, 3, 6, 8)	$(8 \times 3) - (6 \times 1) = 18$

3. Explain that the object of the game is to arrange all four numbers on a card into a number sentence that equals 18. Each number may be used only once. The numbers may be added, subtracted, multiplied, and/or divided.
4. Have the students work in pairs. Each pair picks a card and the corresponding number tiles. Using the number and sign tiles, the students form a number sentence equal to 18.
5. Have the students record their number sentences for five cards (or any other designated amount) on a record sheet.

6. At the end of the activity, have the students share and compare their number sentences.

Party Planning Time

Materials: None

- Directions:
1. Have the students pretend that they are professional caterers. Explain that an important part of their job is to plan parties, which includes figuring amounts of, and purchasing, all the needed items.
 2. Explain that you want them to work in small groups to plan a party and that they may choose a theme for their party if they wish.
 3. They should decide what refreshments and supplies they will need for a designated number of guests.
 4. While planning their parties, the groups should record in story problem form the amount of items needed for meals, games, prizes, etc. Multiplication and division fact problems should be included in the story problems.
Examples:
 - a. Fifteen people will be attending a Fourth of July party. Of the 15, there will be 9 boys. The boys will participate in an apple dunking contest. Each boy will be able to dunk for apples twice. How many apples will we need to buy?
 - b. At the same party, the girls will play a "hit-the-target" game with water balloons. Each girl will toss four water balloons to determine her score. How many balloons will we need to purchase?
 5. Have the students share their party ideas and story problems. Have each group solve another group's problems.
 6. If feasible, schedule one of the groups' parties.
 7. Variation: Have the students use newspaper advertisements to determine the cost of their parties. Ask how they would alter their party if they were working within a given budget.

Assessment Activities

SKILL Describing products of numbers through 10×10

3.5-1

Have the students solve the following problems.

1.
$$\begin{array}{r} 6 \\ \times 2 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 10 \\ \times 1 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 5 \\ \times 5 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 2 \\ \times 8 \\ \hline \end{array}$$

5. $1 \times 4 = \underline{\quad}$

6. $10 \times 10 = \underline{\quad}$

7. $0 \times 7 = \underline{\quad}$

8. $5 \times 10 = \underline{\quad}$

9.
$$\begin{array}{r} 0 \\ \times 6 \\ \hline \end{array}$$

10.
$$\begin{array}{r} 7 \\ \times 2 \\ \hline \end{array}$$

11.
$$\begin{array}{r} 4 \\ \times 5 \\ \hline \end{array}$$

12.
$$\begin{array}{r} 1 \\ \times 9 \\ \hline \end{array}$$

SKILL Describing quotients of numbers through $100 \div 10$

3.5-2

Have the students find the quotient for the following problems.

1. $6 \overline{)30}$

2. $25 \div 5 = \underline{\quad}$

3. $9 \div 3 = \underline{\quad}$

4. $2 \overline{)14}$

5. $14 \div 4 = \underline{\quad}$

6. $2 \overline{)17}$

7. $6 \overline{)30}$

8. $27 \div 5 = \underline{\quad}$

SKILL Describing basic facts for
multiplication and division

4.5-3

Have the students fill in the quotients or products.

1. $3 \times 8 = \underline{\quad}$

2. $6 \overline{)24}$

3. $\begin{array}{r} 9 \\ \times 5 \\ \hline \end{array}$

4. $81 \div 9 = \underline{\quad}$

5. $8 \overline{)56}$

6. $\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$

7. $27 \div 3 = \underline{\quad}$

8. $9 \times 4 = \underline{\quad}$

9. $\begin{array}{r} 10 \\ \times 10 \\ \hline \end{array}$

10. $48 \div 8 = \underline{\quad}$

11. $4 \times 7 = \underline{\quad}$

12. $2 \overline{)18}$

SKILL Describing basic facts for
multiplication and division

5.5-4

Have the students fill in the quotients or products.

1. $3 \times 8 = \underline{\quad}$

2. $36 \div 4 = \underline{\quad}$

3. $\begin{array}{r} 9 \\ \times 6 \\ \hline \end{array}$

4. $6 \overline{)42}$

5. $15 \div 5 = \underline{\quad}$

6. $\begin{array}{r} 5 \\ \times 7 \\ \hline \end{array}$

7. $6 \overline{)36}$

8. $8 \times 7 = \underline{\quad}$

SKILL Multiplying a one-digit number by multiples of 10 and 100***Skill Analysis***

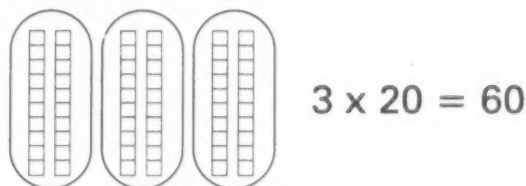
- A. Understands the process of multiplying one-digit numbers
- B. Can apply the commutative property of multiplication
- C. Understands the place value for a three-digit number
- D. Can regroup tens to hundreds and hundreds to thousands
- E. Can count by 10s and 100s

**Concept Manipulative Activities*****Multiples Using Base 10 Patterns***

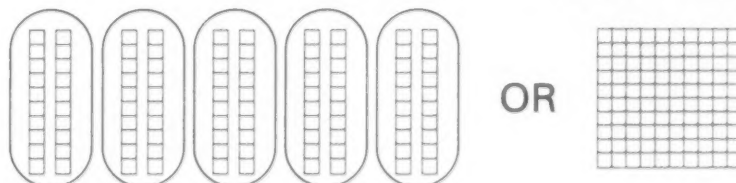
Materials: Base 10 patterns (blackline masters may be found in the Appendix)

- Directions:**
1. Have the students work in pairs. Give each pair 15 ten strips, 12 hundred squares, and 1 thousand cube.
 2. Give the students multiplication problems that use 10 as a factor (e.g., 3×10 and 5×10).
 3. Have the students model the problems by selecting the appropriate number of 10 strips and then give the result (e.g., $3 \times 10 = 30$). The students record the problem and answer.
 4. After the students have practiced modeling problems using 10 as a factor, ask them about the relationship of the problems and their answers. (The students should understand that 3×10 means 3 sets of 10 and that 30 means 3 sets of 10.) Be sure the students understand how to use the commutative property for problems like 40×3 (i.e., $40 \times 3 = 3 \times 40$).

5. Repeat steps 2-4 for problems that use 100 as a factor (e.g., 7×100). The students should understand that 7×100 means 7 sets of hundred squares and that 700 means 7 groups of 100.
6. Next, have the students model problems that use multiples of 10 and 100 as a factor (e.g., 3×20 and 3×200). For 3×20 , the students should make one set of 20 and then two more of these sets. Example:

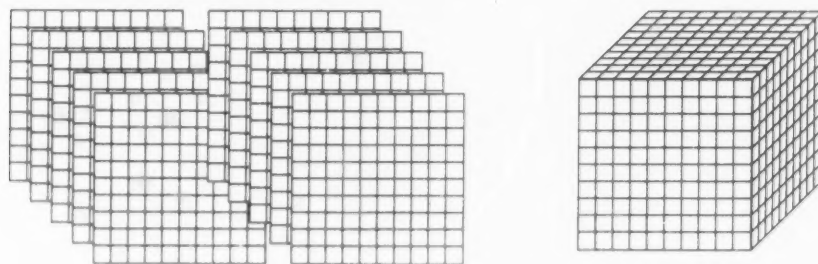


7. Ask, "Why do you get 60 for your answer?" The students need to see that there are 3 strips of 2 tens (20), which is the same as 6 strips of 10, or 60. Since these are all groups of 10, there will be zero ones.
8. Repeat the activity for several other problems with answers less than 100.
9. Have the students model 5×20 . Ask, "What is the result?" (100) Have the students stack the 10 ten strips side by side to show that there are 100 units in a hundred square. Have them trade the 10 ten strips for a hundred grid. Example:



10. Have the students work other problems that use multiples of 10 as a factor. Whenever the answer is larger than 100, they need to trade in their 10 ten strips for a hundred square. Sample problems include: 2×30 , 3×30 , 4×20 , 6×20 , 3×40 , 5×30 , 2×60 , 2×70 , 3×50 , and 5×20 . The students record the problems and answers.
11. The next step is for students to model problems that use 100 or multiples of 100 as a factor. Explain that when you

have 10 hundred squares, you need to trade the hundreds for a thousand cube. Example:



10 hundreds

make 1 thousand

12. Have the students construct models for problems such as 2×300 , 4×200 , 5×100 , 6×200 , 3×400 , 3×300 , 2×500 , 2×600 , 1×400 , and 4×300 .
13. Combine pairs of students so that each group has a large set of modeling materials. Have the students take turns making models and writing the solutions to problems like 4×40 , 3×70 , 2×60 , 7×20 , 2×90 , 6×30 , 5×40 , 8×20 , 3×60 , 7×30 , 6×50 , 7×60 , and 8×60 . Then give them problems that use the hundred squares, such as 4×400 , 2×600 , 3×600 , 7×300 , 4×700 , 8×200 , 6×500 , 7×600 , 8×600 , and 2×900 . The students keep a record of the problems and answers that they model.

Commutative Property of Multiplication

Materials: Base Ten Blocks, bean sticks and rafts, graph paper, etc.

- Directions:
1. Write problems on the board that demonstrate the commutative property of multiplication (e.g., $\overset{10}{\underset{\times}{3}}$ and $\overset{3}{\underset{\times}{10}}$ or 3×10 and 10×3).
 2. Discuss the problems with the group. Examples:

$$3 \times 10 \text{ is 3 sets of 10.} \quad \begin{array}{r} 3 \\ \times 10 \end{array}$$

$$10 \times 3 \text{ is 10 sets of 3.} \quad \begin{array}{r} 10 \\ \times 3 \\ \hline \end{array}$$

Ask, "How do we work these problems? Are the results the same?"

- Be sure the students understand that because of the commutative property, the order of the numbers makes no difference. Three sets of 10 is 30, and 10 sets of 3 is also 30. Have the students build a model for each problem if they are not sure that the results will be the same. Example:



$$3 \times 10 = 30$$



$$10 \times 3 = 30$$

- Give the students several other problems. Have them rewrite those that are difficult for them in a form that is easier for them to work with. Examples:

$$\begin{array}{r} 3 \\ \times 40 \\ \hline \end{array} \longrightarrow \begin{array}{r} 40 \\ \times 3 \\ \hline \end{array} \quad 40 \times 3 \longrightarrow 3 \times 40$$

The students need to know that they can change the order of the numbers in multiplication problems without changing the results.

Bean Sticks and Rafts

Materials: Bean sticks (10 beans per stick) and rafts (10 bean sticks per raft) or stacks of 10 UNIFIX cubes

Directions: 1. Give the students multiplication problems that have 10 as a factor (e.g., 3×10 , 5×10).

355

2. Have the students construct the model for each problem using bean sticks, find the result, and record the problem and answer on a piece of paper.
3. Help them understand the relationship between the problem and answer (e.g., 3×10 is 3 sets of 10. The answer is 30, which means 3 groups of 10).
4. For problems like 10×4 , tell the students that the commutative property may be used. They can rewrite the problem as 4×10 so that they can model 4 sets of 10 instead of 10 sets of 4.
5. After the students have modeled problems with 10 as a factor, give them problems to model that have multiples of 10 as a factor (e.g., 3×20 , 2×40 , and 2×30). For 3×20 , they would build 3 sets of 20 beans (3 sets of two bean sticks), which equals 60.
6. Model a problem like 6×20 , which requires regrouping. Be sure the students understand how to trade 10 bean sticks for 1 raft (100 beans). Give the students many problems to model that require trading 10 bean sticks for 1 raft (e.g., 3×40 , 2×70 , 2×90 , 3×60 , 3×50 , 4×40 , 4×50 , 3×70 , 3×80 , etc.).
7. Have the students write down the problems and record the answers after building each model. Make sure they practice using both horizontal and vertical notations.
8. Have the students discuss the relationship between each problem and its answer. Possible comments include: " 3×40 will give me 120 because there will be 12 tens," or "The answers will be groups of 10. All we need to do is find out how many tens there will be." Detailed discussion about trading 10 tens for 1 hundred should also take place.
9. After the students have practiced regrouping tens into hundreds, show them how to regroup hundreds into thousands. Have the students construct models for problems like 5×300 , 6×200 , 3×400 , and 2×700 . They will need to exchange 10 rafts (100 beans) for a 10-raft bundle (1,000 beans). (You can form the bundles by stacking 10 rafts, one on top of the other, and holding the pile together

with two rubber bands. Or, you can simply place 10 rafts in a box. You may need to have the students work in groups so there will be enough rafts.) Some people call the bundle of rafts a boat or ship.

10. Have the students discuss the relationship between each problem and its answer. For a problem like $600 \times 3 = 1,800$, ask, "How do we get 1,800?" Expect responses similar to those discussed step 8.

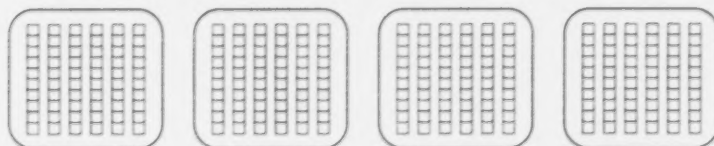


Let's Practice

Stamp It

Materials: 10s and 100s stamps or base 10 patterns (blackline master may be found in the Appendix)

- Directions:**
1. Give the students stamps that represent 10s and 100s or have them cut out 30 ten strips and 25 hundred squares from the base 10 patterns.
 2. Give the students the problem 4×60 . Have them make the model and solve the problem. It may be helpful for the students to circle the sets of tens (or hundreds). Example:



3. Have the students trade in any groups of 10 tens for groups of 100 and then record the problem with the answer (e.g., $4 \times 60 = 240$).
4. Give the students several other problems that use 10 or multiples of 10 as a factor to model and solve in this way (e.g., 3×10 , 2×30 , 1×40 , 2×60 , 0×30 , 5×20 , 4×50 , 3×70 , 3×80 , 90×3 , 60×2 , 40×4 , 7×20 , 5×60 , 8×20 , 1×90 , 3×90 , 9×10 , 30×7 , etc.).
5. Ask the students to describe how to multiply by 10 or multiples of 10.

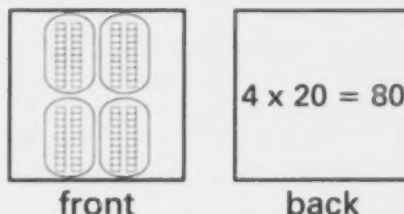
387

6. Expand the activity to include practice in multiplying by multiples of 100. Give the students the problem 4×300 . Have them make the model and write down the problem with the solution.
7. Have the students practice with other problems that use multiples of 100 as a factor (e.g., 2×200 , 4×200 , 300×2 , 6×300 , 500×2 , 3×400 , 5×300 , 2×700 , 900×2 , 3×800 , 5×500 , etc.).
8. Ask the students to describe how to multiply by 100 or multiples of 100.

Name the Model

Materials: 3" x 5" index cards
10s and 100s stamps or base 10 patterns (blackline master may be found in the Appendix)

- Directions:**
1. On one side of the index cards make models for problems that use multiples of 10 as a factor.
 2. On the other side, write the corresponding problem.
- Example:



3. Place the cards in a pile, model side up.
4. Have the students take turns drawing a card, looking at the model, writing down the problem on a piece of paper, and giving the answer. By turning the card over, the students can self-check their work.
5. Repeat the activity using cards with problems and models involving multiples of 100. (The hundred squares may need to be overlapped.)

Calculator Zero

Materials: Calculators (one per student)

Directions: 1. Give each student a calculator. Then write the following four problems on the board.

$$\begin{array}{r} 10 \\ \times 4 \\ \hline \end{array} \quad 8 \times 20 = \underline{\quad\quad} \quad \begin{array}{r} 200 \\ \times 2 \\ \hline \end{array} \quad 100 \times 7 = \underline{\quad\quad}$$

2. Have the students work the problems on their calculators. Check their answers to be sure that they are entering the problems correctly.
3. Give the students another set of problems to work on their calculators. These problems, like the first set, should have 10, 100, or multiples of them as a factor (e.g., 300×5 , 700×2 , 7×200 , 30×6 , 40×7 , 80×6 , 30×4 , 20×9 , 60×7 , 100×3 , 4×300 , 7×700 , 8×900 , 9×900 , 8×600 , etc.). Have the students write down each problem and answer. Then have them discuss how they might find the answers quickly without using a calculator.
4. Have the students solve the following problems without using calculator. (After they have completed the problems, they may check their answers using the calculator.)

$$\begin{array}{r} 20 \\ \times 6 \\ \hline \end{array} \quad \begin{array}{r} 10 \\ \times 8 \\ \hline \end{array} \quad \begin{array}{r} 30 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 40 \\ \times 1 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ \times 20 \\ \hline \end{array} \quad \begin{array}{r} 30 \\ \times 5 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \times 60 \\ \hline \end{array}$$

$$50 \times 2 = \underline{\quad\quad}, \quad 5 \times 20 = \underline{\quad\quad}, \quad 2 \times 70 = \underline{\quad\quad}, \quad 5 \times 70 = \underline{\quad\quad}$$

$$\begin{array}{r} 300 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 200 \\ \times 7 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ \times 400 \\ \hline \end{array} \quad \begin{array}{r} 500 \\ \times 6 \\ \hline \end{array} \quad \begin{array}{r} 800 \\ \times 2 \\ \hline \end{array}$$

389

Toss It!

Materials: Tagboard
Cubes or beanbags

Directions: 1. Using tagboard, make a game board with multiples of 10 or 100, as shown below.

70	40	80
60	50	20
10	90	30

700	400	800
600	500	200
100	900	300

The students will be tossing beanbags or cubes onto the game board, so be sure its squares are larger than the beanbags or cubes that will be used.

2. On three beanbags or cubes, write the numbers 1, 2, and 5.
Examples:

①

②

⑤

OR

1

2

5

3. Have each student, in turn, toss the three beanbags (cubes) onto the game board. The total score for each turn is determined by multiplying the number on each beanbag (cube) by the number in the square on which the majority of the beanbag (cube) landed and adding the three scores.
Example:

70	40 ^⑤	80
60	① 50	② 20
10	90	30

$$\begin{array}{r}
 1 \times 50 = 50 \\
 2 \times 20 = 40 \\
 5 \times 40 = 200 \\
 \hline
 290
 \end{array}$$

4. Each student should compute his/her score after each turn. The winner is the student with the highest score, lowest score, score closest to 300, etc.



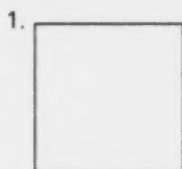
Problem Solving

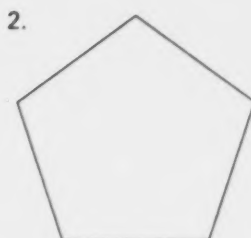
**M
U
L
T
I
P
L
E

P
E
R
I
M
E
T
E
R
S**

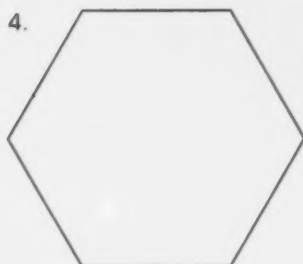
Directions: If the length of the line below represents 20 inches of string, how much string would it take to make each figure? Write the answers on the lines beneath the figures.

20 inches



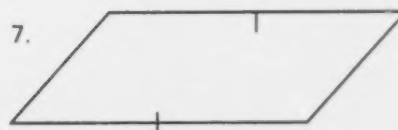














Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

**Multiplication**

Part 1. Work the following problems.

$$\begin{array}{r} \text{A. } 20 \\ \times 3 \\ \hline \end{array}$$

$$\begin{array}{r} \text{B. } 30 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{C. } 80 \\ \times 1 \\ \hline \end{array}$$

$$\begin{array}{r} \text{D. } 6 \\ \times 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{E. } 60 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{F. } 30 \\ \times 5 \\ \hline \end{array}$$

Part 2. Work the following problems.

$$\begin{array}{r} \text{A. } 20 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} \text{B. } 50 \\ \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} \text{C. } 50 \\ \times 4 \\ \hline \end{array}$$

$$\begin{array}{r} \text{D. } 5 \\ \times 20 \\ \hline \end{array}$$

$$\begin{array}{r} \text{E. } 60 \\ \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} \text{F. } 40 \\ \times 5 \\ \hline \end{array}$$

Part 3. Explain why the problems in part 1 have only one zero in their answers and the problems in part 2 have two zeros in their answers.

*TEACHER NOTE: Manipulative materials should be made available for the students who wish to use them.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

1. A cashier has seven \$20 bills in his drawer. If he exchanged them for \$1 bills, how many \$1 bills would he have? _____
2. Mary gave a clerk eight \$50 bills as a down payment on a car. How much money did she give the clerk in dollars? _____
3. A theater has rows that seat 30 people in each row. If seven rows are filled, how many people came to the show? _____
If nine rows are filled, how many people came to the show? _____
4. A theater has rows that seat 50 people in each row. If six rows are filled, how many people came to the show? _____
If eight rows are filled, how many people came to the show? _____
5. A sack of potatoes holds 40 potatoes. A clerk had six of the sacks on a cart. How many potatoes were on the cart? _____
6. A 20-lb. bag of oranges holds 33 oranges. A 20-lb. bag of grapefruit holds 20 grapefruit. If the produce clerk has two bags of oranges and three bags of grapefruit on a cart, how many pounds of fruit does he have? _____





Multiplication Pattern

1. If $3 \times 6 = 18$, then $30 \times 6 = \underline{\hspace{2cm}}$
2. If $4 \times 7 = 28$, then $7 \times 40 = \underline{\hspace{2cm}}$
3. If $3 \times 5 = 15$, then $30 \times 5 = \underline{\hspace{2cm}}$
4. If $2 \times 7 = 14$, then $2 \times 70 = \underline{\hspace{2cm}}$
5. If $1 \times 9 = 9$, then $100 \times 9 = \underline{\hspace{2cm}}$
6. If $3 \times 4 = 12$, then $300 \times 4 = \underline{\hspace{2cm}}$
7. If $6 \times 2 = 12$, then $6 \times 200 = \underline{\hspace{2cm}}$
8. If $7 \times 6 = 42$, then $700 \times 6 = \underline{\hspace{2cm}}$
9. If $3 \times 8 = 24$, then $30 \times 80 = \underline{\hspace{2cm}}$
10. If $6 \times 5 = 30$, then $600 \times 5 = \underline{\hspace{2cm}}$
11. If $8 \times 4 = 32$, then $8 \times 400 = \underline{\hspace{2cm}}$
12. If $4 \times 4 = 16$, then $40 \times 40 = \underline{\hspace{2cm}}$
13. If $2 \times 8 = 16$, then $20 \times 80 = \underline{\hspace{2cm}}$
14. If $4 \times 5 = 20$, then $400 \times 50 = \underline{\hspace{2cm}}$
15. If $2 \times 9 = 18$, then $200 \times 90 = \underline{\hspace{2cm}}$

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Multiplying two- and three-digit numbers by a one-digit number***Skill Analysis***

- A. Understands the concept of multiplication
- B. Understands the concept of place value
- C. Understands the concept of regrouping
- D. Can apply the commutative property of multiplication
- E. Can do problems using the basic facts of multiplication
- F. Can multiply by multiples of 10 and 100
- G. Can multiply by 1 and 0

**Concept Manipulative Activities*****Place Value Mat***

Materials: 9" x 12" construction paper (several pieces per student)
Tape
Crayons or markers

- Directions:**
1. Have the students make place value mats to use for modeling multiplication problems. The mats must be large enough for the students to place sets of manipulatives under the ones, tens, and hundreds columns. (If the class uses Base Ten Blocks, one 9" x 12" page should be used for each place value. If UNIFIX cubes or bean sticks are used, larger paper will be necessary.)
 2. Since the early problems will involve multiplication to only two places, have the students make their mats with columns for just the ones and tens places. Instruct them to take two sheets of paper and tape them together. Have

them label the right sheet "Ones" and the left sheet "Tens." Example:

Tens	Ones

3. The students can fold the two sheets along the tape when storing their mats.
4. When the students are ready to work problems involving hundreds, have them tape a third sheet on the left and label it "Hundreds."

Steps in Multiplication

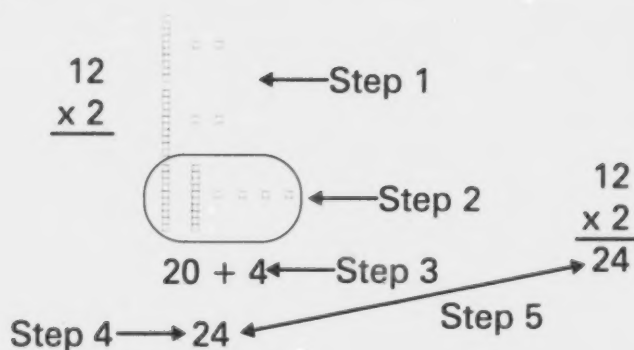
Materials: Manipulatives for modeling multiplication (bean sticks, bundles of straws, coffee stirrers, paste sticks, Base Ten Blocks, etc.)

Place value mats (one per student; directions for their construction may be found in the "Place Value Mat" activity)

- Directions:**
1. Write the problem $\begin{array}{r} 12 \\ \times 2 \\ \hline \end{array}$ on the board. Discuss how the problem is read and what it means (i.e., 2 times 12 or two groups of 12).
 2. Have the students build two groups of 12 on their place value mats by placing their manipulatives in the appropriate columns. (See step 1, below.)
 3. Then have the students combine the two sets. (See step 2, below.)
 4. Ask, "How many ones do you have?" (4) "How many tens do you have?" (2) (See step 3, below.) Ask, "How many do you have altogether?" (24) (See step 4, below.)
 5. Record this answer on the board under the problem. (See step 5, below.)

396

Example:

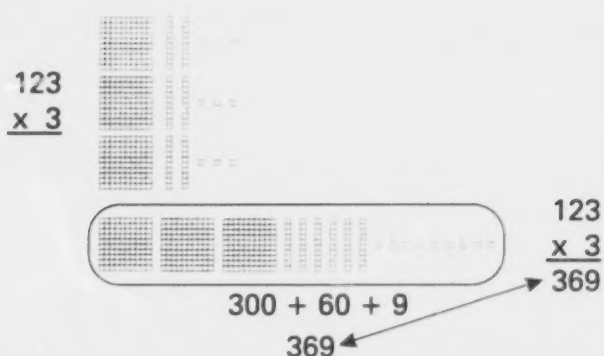


6. Repeat the activity with other two-digit times one-digit problems that do not require regrouping. Include problems

with zero (e.g., $\begin{array}{r} 30 \\ \times 3 \\ \hline \end{array}$).

7. Write the problem $\begin{array}{r} 123 \\ \times 3 \\ \hline \end{array}$ on the board. Discuss how the problem is read and what it means (i.e., 3 times 123 or three groups of 123).
8. Have the students build 123 on their place value mats by placing their manipulatives in the appropriate columns.
9. Ask, "For this problem, how many times do you need to build 123?" (3) Have the students build the remaining sets under the first sets.
10. Have the students combine the sets and state how many ones (9), tens (6), and hundreds (3) there are altogether. Ask, "How many do you have altogether?" (369)
11. Write the answer under the problem on the board.

Example:



12. Repeat the activity with other three-digit times one-digit problems that do not require regrouping. Include problems

with zero (e.g., $\begin{array}{r} 130 \\ \times 3 \end{array}$, $\begin{array}{r} 204 \\ \times 2 \end{array}$ etc.).

Pattern Multiplication

Materials: Manipulatives for multiplication (bean sticks, bundles of straws, coffee stirrers, paste sticks, Base Ten Blocks, etc.)

Place value mats (one per student; directions for their construction may be found in the "Place Value Mat" activity)

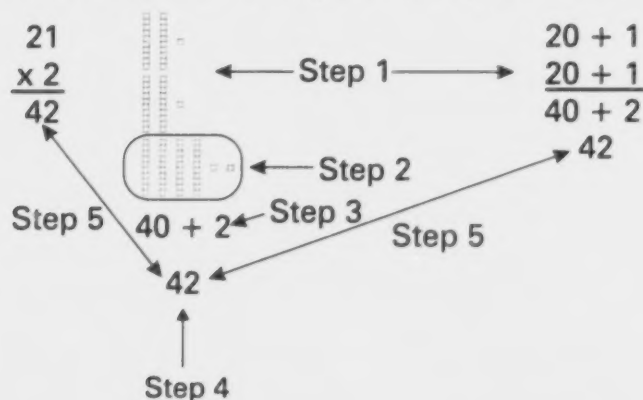
Student-made record sheets (one per student) Example:

Problem	What I See

- Directions:
1. Have the students write the problem $\begin{array}{r} 21 \\ \times 2 \end{array}$ on their record sheets. Discuss what the problem means (i.e., two groups of 21).
 2. Ask the students to build one group of 21 on their place value mats. Have them record the expanded notation for this group ($20 + 1$) on their record sheets under the heading "What I see."

398

3. Ask the students to build the second group of 21 beneath the first group on their place value mats. Have them record the expanded notation for this new group beneath the notation for the first group on their record sheets. (See step 1, below.)
4. Ask the students to combine the two groups and state how many ones and tens are in the new set. (See step 2, below.)
5. Have the students record the expanded notation ($40 + 2$) for this new set on their record sheets. (See step 3, below.)
6. Ask the students to write the answer as a standard numeral and record this numeral under the problem and under the expanded notation. (See steps 4 and 5, below.) Example:



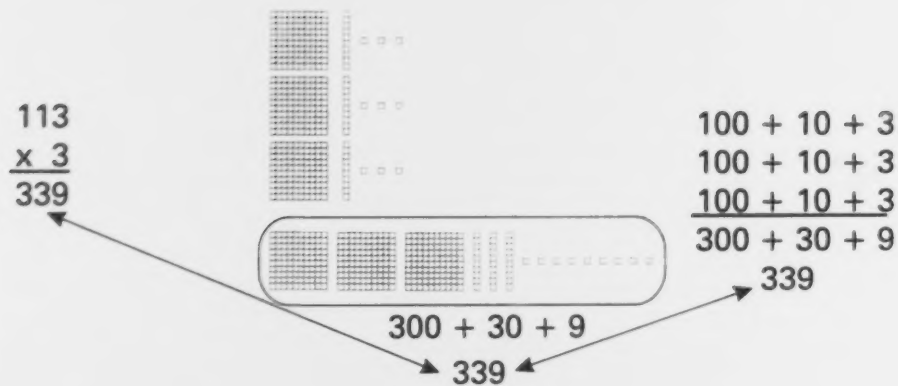
7. Repeat the activity with other two-digit times one-digit problems that do not require regrouping. Include problems

20
with zero (e.g., $\begin{array}{r} 20 \\ \times 4 \\ \hline \end{array}$).

8. Follow the same procedure and questioning techniques with three-digit times one-digit problems that do not re-

130
quire regrouping. Include problems with zero (e.g., $\begin{array}{r} 130 \\ \times 2 \\ \hline \end{array}$,
 $\begin{array}{r} 301 \\ \times 3 \\ \hline \end{array}$, etc.).

Example:



Expand and Contract

Materials: Manipulatives for multiplication (bean sticks, bundles of straws, coffee stirrers, paste sticks, Base Ten Blocks, etc.)
Place value mats (one per student; directions for their construction may be found in the "Place Value Mat" activity)

Student-made record sheets

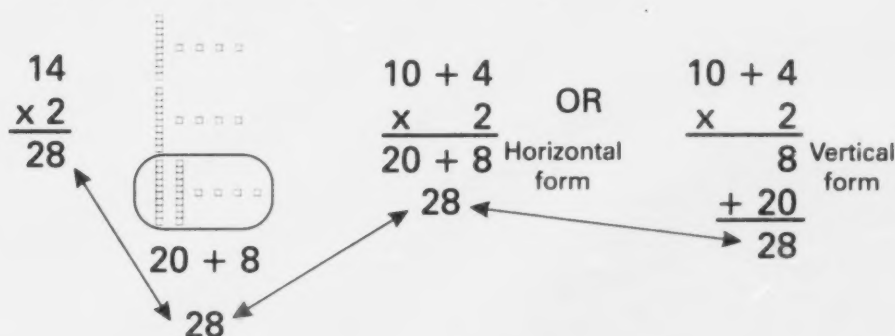
Example:

Problem	What I See

- Directions:**
1. Have the students write the problem 14×2 on their record sheets. Discuss what the problem means and how they can model it.
 2. Ask the students to build one group of 14 on their place value mats. Have them record the expanded notation for this group ($10 + 4$) on their record sheets under the heading "What I See."

400

3. Ask the students to build the second group of 14 on their place value mats. Explain that since this second group has the same expanded notation as the notation they have already recorded on their record sheets, they can record what they have done by simply writing "x 2" instead of repeating the expanded notation.
4. Ask the students to combine the groups on their place value mats and describe the new set. Ask, "What is the expanded notation for this set?" ($20 + 8$)
5. Direct the students to examine the expanded notation problem they have written on their record sheets. Ask, "What would I get if I multiplied the numbers in the problem $\begin{array}{r} 10 + 4 \\ \times \quad 2 \end{array}$?" ($20 + 8$) "How does this compare with the model you made?" (They're the same.)
6. Ask, "How can I rewrite $20 + 8$ as a standard numeral?" Have them record $20 + 8$ as the answer to their expanded notation problem. (28) Have the students write this number under their original problem and under the expanded notation problem. Example:



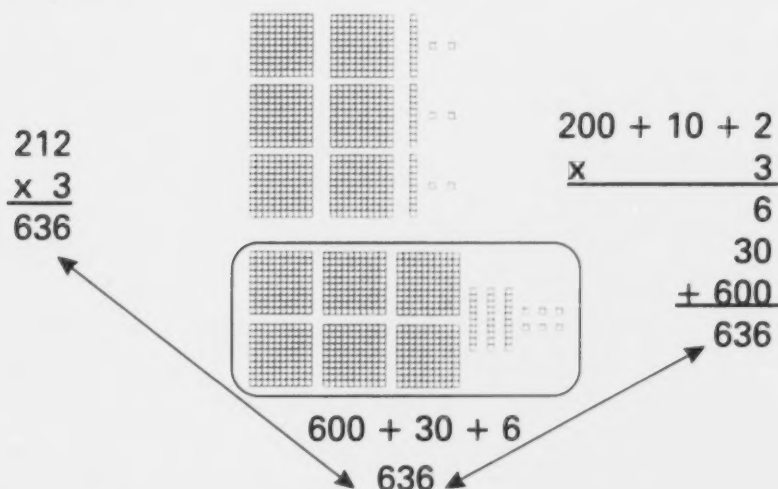
***TEACHER NOTE:** The expanded notation problem can be solved either horizontally or vertically. The horizontal form shows the direct relationship to the model. The vertical form assists the students in finding the sum as well as preparing them for future algorithmic work. Choose the form that will work better for different students.

7. Repeat the activity with additional two-digit times one-digit problems that do not require regrouping. Include problems with zero (e.g., $\begin{array}{r} 30 \\ \times 3 \end{array}$).

8. Follow the same procedure and questioning techniques with three-digit times one-digit problems that do not require regrouping. Include problems with zero (e.g.,

$$\begin{array}{r} 430 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 203 \\ \times 3 \\ \hline \end{array}, \text{ etc.).}$$

Example:



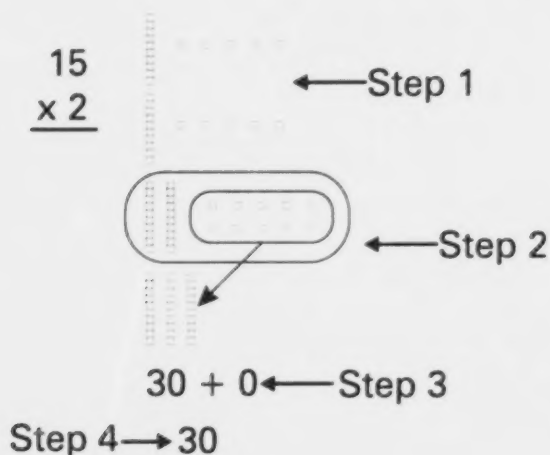
Multiplying with Regrouping

Materials: Manipulatives for multiplication (bean sticks, bundles of straws, coffee stirrers, paste sticks, Base Ten Blocks, etc.)

Place value mats (one per student, directions for their construction may be found in the "Place Value Mat" activity)

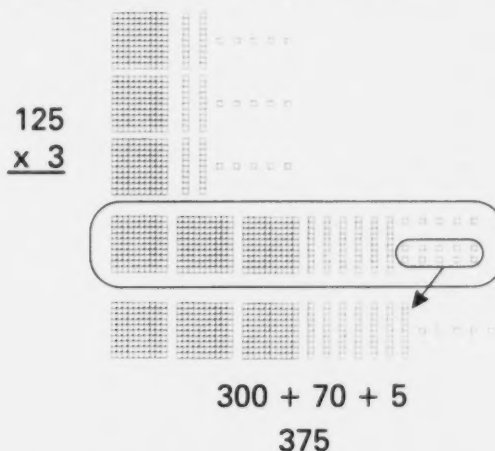
- Directions:**
1. Write the problem $\begin{array}{r} 15 \\ \times 2 \\ \hline \end{array}$ on the board. Have the students discuss the meaning of the problem and how it can be modeled.
 2. Have the students build a model of the problem on their place value mats. (See step 1, below.)
 3. Have the students combine the two sets. Ask, "How many ones and tens are in the new set?" (10 ones and 2 tens)

4. Although the students have had many experiences with regrouping with addition, they may see the concept as isolated from multiplication. Draw their attention to the set of 10 ones and ask, "What did we do with sets like this in addition problems? We'll do the same with multiplication problems."
5. Have the students exchange their 10 ones for 1 ten. (See step 2, below.)
6. Ask, "How many tens do you have now?" (3) "How much do you have altogether?" (30) (See step 3 and 4, below.)
Example:



7. Repeat the activity with other two-digit times one-digit problems that involve regrouping from the ones place to the tens place (e.g., $\begin{array}{r} 24 \\ \times 3 \end{array}$, $\begin{array}{r} 37 \\ \times 2 \end{array}$, $\begin{array}{r} 15 \\ \times 4 \end{array}$, etc.).
8. Write the problem $\begin{array}{r} 125 \\ \times 3 \end{array}$ on the board and have the students build the model for it.
9. Have the students combine the three sets. Ask, "How many ones do you have?" (15) "How many tens?" (6) "How many hundreds?" (3)
10. Ask, "What do you need to do with the ones?" Have the students exchange 10 of the ones for a ten.

11. Ask, "How many ones do you have now?" (5) "How many tens?" (7) "Can you do any more trades?" (no) "How much do you have altogether?" (375) Example:



12. Repeat the activity with other three-digit times one-digit problems that involve regrouping from the ones place to the tens place (e.g., $\begin{array}{r} 324 \\ \times 3 \end{array}$, $\begin{array}{r} 124 \\ \times 3 \end{array}$, etc.).
13. You may need to have the students work with a partner for problems involving large numbers so that there will be a sufficient amount of manipulative materials.

Recording the Regrouping

Materials: Manipulatives for multiplication (bean sticks, bundles of straws, coffee stirrers, paste sticks, Base Ten Blocks, etc.)
 Place value mats (one per student; directions for their construction may be found in the "Place Value Mat" activity)
 Overhead projector, blank transparency, Vis-A-Vis pen

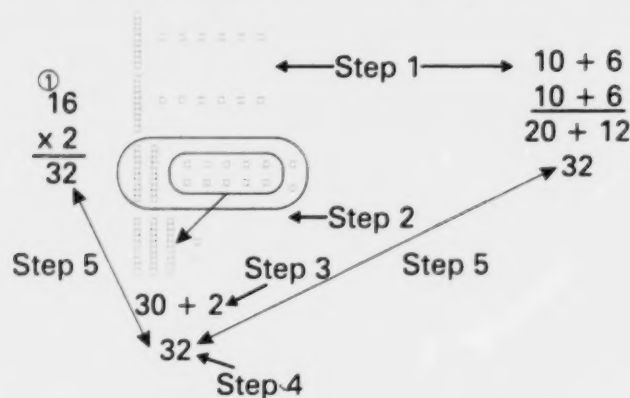
Directions: 1. Show the problem $\begin{array}{r} 16 \\ \times 2 \end{array}$ on the overhead. Discuss with the students what the problem means and how to model it.

2. Have the students build one set of 16 on their place value mats. As they are doing that, build a set of 16 on the overhead. Ask, "What is the expanded notation for this set?" Write $10 + 6$ beside the set on the transparency.
3. Have the students build the second set of 16 beneath the first set on their place value mats. Do the same on the overhead. Record the expanded notation for this set beneath the first expanded notation. (See step 1, below.)
4. Ask the students to combine the sets and describe the new set. Record the expanded notation for the new set ($20 + 12$) on the overhead. Relate the manipulatives to the expanded notation.
5. Pointing to the combined set on the overhead, ask, "How many ones do I have?" (12) "Can I make a trade?" (yes) "What should I do?" (Trade 10 ones for 1 ten.) (See step 2, below.) Place the traded ten beside the other 2 tens as shown in the example below, or place it above the other tens horizontally so that its position is related to the algorithm (e.g., $\overline{10}$).
6. Ask, "What is the expanded notation of the set we created with this trade?" ($30 + 2$) "How much do I have altogether?" (32) (See steps 3 and 4, below.)
7. Refer to the expanded notation already recorded on the transparency. Ask, "When I add $20 + 12$, what is the sum?" (32) Relate this sum to the manipulatives and the discussion just completed.

16

8. Return to the original problem, $\begin{array}{r} \times 2 \\ 62 \end{array}$, and say, "We've just recorded our work in expanded notation. Now I'll show you how to record your work when you multiply numbers in this form. We start with the ones. Two times six is twelve. What did we do with our 12 ones? We traded 10 of them for 1 ten. To record the trade, I'm going to put a 1 above the tens place and circle it so that we will remember it was a trade. I have 2 ones left, so I will record that in the ones place. I can now multiply the tens. Two times one is two,

but I also need to add in my trade. So I have 3 tens altogether. I'll record that in the tens place." Example:



9. Repeat the activity with other two- and three-digit times one-digit problems that involve regrouping from the ones

place to the tens place (e.g., $\begin{array}{r} 15 \\ \times 4 \\ \hline \end{array}$, $\begin{array}{r} 113 \\ \times 5 \\ \hline \end{array}$, $\begin{array}{r} 204 \\ \times 3 \\ \hline \end{array}$, etc.).

Regrouping and Recording

Materials: Manipulatives for multiplication (bean sticks, bundles of straws, coffee stirrers, paste sticks, Base Ten Blocks, etc.)

Place value mats (one per student; directions for their construction may be found in the "Place Value Mat" activity)

Student-made record sheets

Example:

Problem	What I See

- Directions: 1. Have the students write the problem $\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$ on their record sheets. Discuss what the problem means and how it can be modeled.

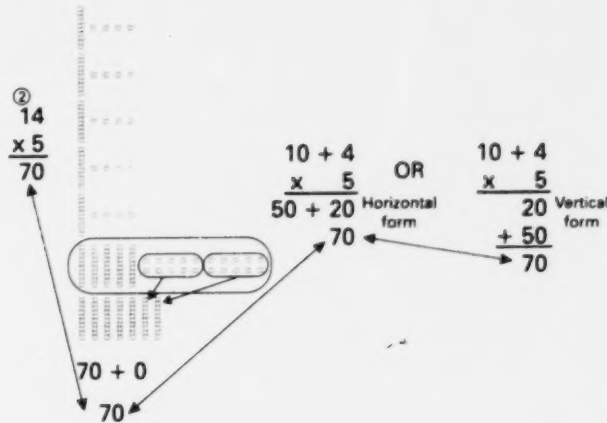
406

2. Have the students build one group of 14 on their place value mats. Have them record the expanded notation ($10 + 4$) for this group on their record sheets under the heading "What I See."
3. Have the students build the remaining four groups of 14 on their place value mats, one beneath the other. Explain that instead of writing the expanded notation five times and finding the sum, we can simply write "times 5," i.e.,

$$\begin{array}{r} 10 + 4 \\ \times \quad 5 \\ \hline \end{array}$$
4. Have the students combine the five groups on their place value mats and describe the new set. Ask, "How many ones and tens do you have altogether?" (20 ones and 5 tens)
5. Have the students multiply the numbers in the problem they recorded on their record sheets. Ask, "How does your answer compare with the model you made?" (They are the same.)
6. Ask, "If I add $50 + 20$, what do I have?" (70) Say, "Record this number on your record sheets beneath the $50 + 20$."
7. Say, "Now look at the model you made. What should you do with the ones?" (Trade 10 ones for 1 ten twice.) Ask the students to describe the new set. Point out that it's the same as the number they just wrote.
8. Direct the students' attention to the original problem,

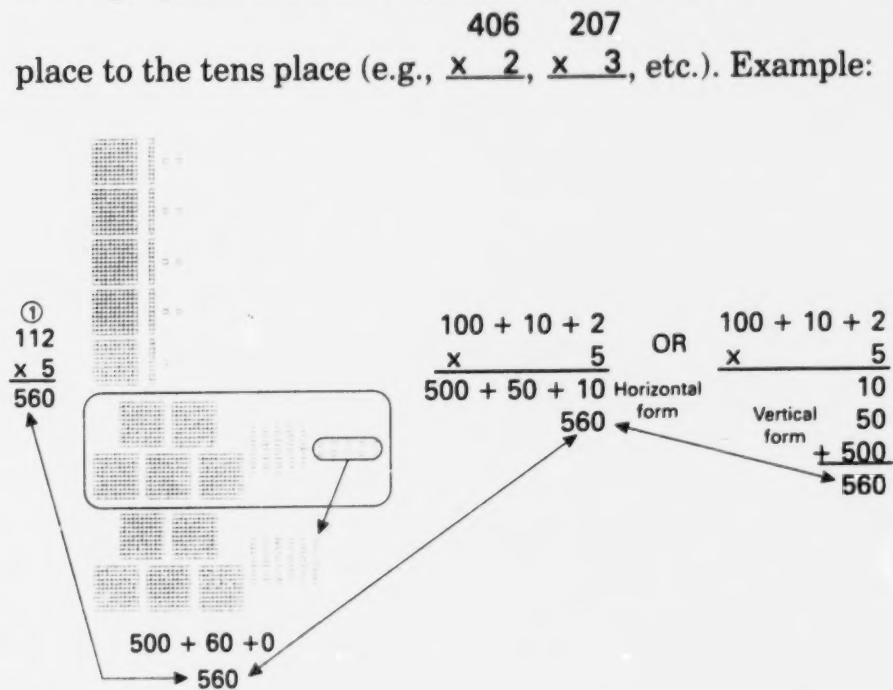
$$\begin{array}{r} 14 \\ \times 5 \\ \hline \end{array}$$
 Say, "Let me show you how to record what we've just done. Five times 4 ones is 20 ones. We traded those ones for 2 tens to record the trade. I'll put a 2 above the tens place and circle it to help me remember it was a trade. After the trade, how many ones did we have left?" (0) Say, "I'll record that number in the ones place. Now, five times 1 ten is 5 tens, but I also need to add in the 2 tens from the trade. How many tens did we have altogether?" (7) Say, "I'll record that number in the tens place."
9. Compare the original multiplication problem, the expanded notation, and the model. The students should be

able to see that they are all expressions of the same problem.



***TEACHER NOTE:** When writing the problem in the expanded notation format, the students may use either the horizontal or vertical form. Allow them to choose the form that is more understandable to them.

10. Repeat the activity with additional two-digit times one-digit problems that require regrouping from the ones place to the tens place (e.g., $\underline{406} \times \underline{2}$, $\underline{207} \times \underline{3}$, etc.). Example:



408

Regrouping Again?

Materials: Manipulatives for multiplication (bean sticks, bundles of straws, coffee stirrers, paste sticks, Base Ten Blocks, etc.)
Place value mats (one per student; directions for their construction may be found in the "Place Value Mat" activity)

- Directions:**
1. Have the students work in pairs. 161
 2. Instruct each pair to build a model for the problem $\begin{array}{r} \\ \times 2 \\ \hline \end{array}$ on their place value mat.
 3. Have the students combine the sets on their mats. Ask, "How many ones do you have?" (2) "How many tens?" (12) "How many hundreds?" (2)
 4. Ask, "Can you make any trades?" (yes) Have the students trade 10 tens for 1 hundred.
 5. Ask, "How many ones do you have now?" (2) "How many tens?" (2) "How many hundreds?" (3) "What is the total amount of your new set?" (322)
 6. Repeat the activity with other problems that require regrouping from the tens place to the hundreds place.

261 152 460

Include problems with zero (e.g., $\begin{array}{r} \\ \times 3 \\ \hline \end{array}$, $\begin{array}{r} \\ \times 4 \\ \hline \end{array}$, $\begin{array}{r} \\ \times 2 \\ \hline \end{array}$). Ask the students questions similar to those in the concept manipulative activities entitled "Recording the Regrouping" and "Regrouping and Recording."

7. Repeat the same procedures and questioning strategies for problems that involve regrouping from hundreds to

634 403 210

thousands (e.g., $\begin{array}{r} \\ \times 2 \\ \hline \end{array}$, $\begin{array}{r} \\ \times 3 \\ \hline \end{array}$, $\begin{array}{r} \\ \times 6 \\ \hline \end{array}$) and problems that

245 167

involve multiple regrouping (e.g., $\begin{array}{r} \\ \times 3 \\ \hline \end{array}$, $\begin{array}{r} \\ \times 2 \\ \hline \end{array}$).



Let's Practice

***TEACHER NOTE:** Some students may be able to visualize a model for multiplication without physically manipulating objects. Students who can demonstrate that they understand the concept should not be required to physically build the models. However, it is important to be sure that they have a good understanding of the concept. This conceptual understanding will be necessary for future success with advanced computation.

Multiply

Materials: Manipulatives for multiplication (bean sticks, graph paper, Base Ten Blocks, bundles of coffee stirrers, straws, Popsicle sticks, tongue depressors, etc.)

Cards on which you have written multiplication problems (include problems in both horizontal and vertical forms, problems with and without regrouping, and problems with zero)

Examples:

$3 \times 21 =$	$\begin{array}{r} 17 \\ \times 5 \\ \hline \end{array}$	$234 \times 2 =$	$\begin{array}{r} 120 \\ \times 6 \\ \hline \end{array}$
-----------------	---	------------------	--

- Directions:**
1. Have the students work in pairs.
 2. Explain that one student will draw a card and read the problem to the other student, who will use the models and solve it.
 3. The student who worked the problem will then select a problem card for the other student to solve. Play continues in this manner until time is called.

***TEACHER NOTE:** You can make the cards self-checking by writing the product for each problem on the reverse side of the card.

Estimating Multiplication

Materials: Manipulatives for multiplication (bean sticks, bundles of coffee stirrers, paste sticks, straws, graph paper, Base Ten Blocks, etc.)

410

Directions: 1. Review multiplying by multiples of 10 or 100. Then have

the students work several problems, such as $\begin{array}{r} 20 \\ \times 4 \\ \hline \end{array}$, $\begin{array}{r} 60 \\ \times 5 \\ \hline \end{array}$,
 $\begin{array}{r} 300 \\ \times 5 \\ \hline \end{array}$, and $\begin{array}{r} 600 \\ \times 2 \\ \hline \end{array}$. Discuss the answers and any problems that
 caused confusion. 19

2. Write the problem $\begin{array}{r} 20 \\ \times 2 \\ \hline \end{array}$ on the board. Say, "Nineteen is

close to 20. If we were to change the problem to $\begin{array}{r} 20 \\ \times 2 \\ \hline \end{array}$ what
 would the answer be?" (40) Say, "So the answer to the
 original problem should be close to 40." 19

3. Have the students determine the product of $\begin{array}{r} 19 \\ \times 2 \\ \hline \end{array}$. They
 may use manipulatives. 19

4. Ask, "What is the product of $\begin{array}{r} 19 \\ \times 2 \\ \hline \end{array}$?" (38) "Is it close to 40?"
 (yes) "When would it be helpful to know an approximate
 answer before finding an exact answer?" Allow students to
 brainstorm ideas.

5. Have the students work with a partner. Give them a variety

of problems, such as $\begin{array}{r} 21 \\ \times 3 \\ \hline \end{array}$, $\begin{array}{r} 18 \\ \times 4 \\ \hline \end{array}$, $\begin{array}{r} 32 \\ \times 5 \\ \hline \end{array}$, and $\begin{array}{r} 49 \\ \times 3 \\ \hline \end{array}$. Explain
 that, for each problem, you want one student to find the
 exact answer and the other student to find the approximate
 answer. Manipulatives should be available for students to
 use. Students should switch roles after every problem.

6. Each pair of students should discuss the two answers to
 each problem, comparing them for reasonableness.

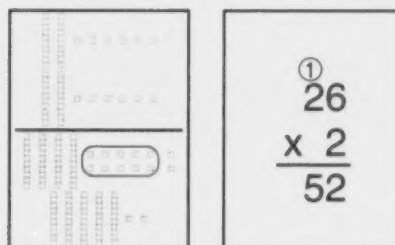
Old Maid Multiplication

Materials: Blank cards

Directions: 1. Divide the class into small groups of students.

2. Give each group a deck of cards that includes an "Old Maid"
 card and sets of two related cards, one showing a
 multiplication problem and the other showing a model for
 it. Some of the problems should involve regrouping.

Example:



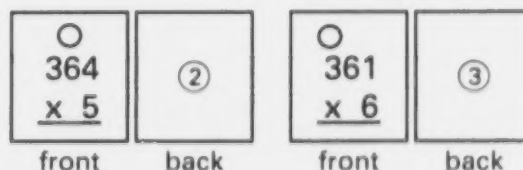
3. One student in each group deals out all of the cards.
4. Then the students take turns laying down the matching cards in their hands and explaining why each is a match.
5. Next, the student to the dealer's left draws a card from the hand of the student to his/her left. If the card that is drawn matches a card in the student's hand, he/she lays down the matching cards and explains the match.
6. Play continues to the left. The game is over when all of the pairs have been laid down and one student is the "Old Maid."
7. Variation: The cards can also be used to play "Go Fish." In this game the students ask one another for the cards needed to make a matching pair. (The "Old Maid" card is not used.)

Regrouping Number Game

Materials: Universal Game Board (one per group of students; directions for its construction may be found in the Appendix)

Self-checking game cards containing three-digit times one-digit problems that involve regrouping

Examples:



Die numbered 1, 2, 3 (one per game)

Game markers

Directions: 1. Students play this game in small groups.

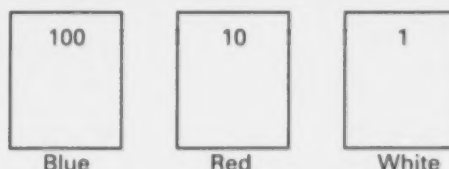
412

2. The cards are placed face up in a pile on the game board.
3. Each student, in turn, draws a card from the top of the pile and states the number that should go in the circle.
4. The student checks the answer by turning the card over.
5. If the answer is correct, the student rolls the die and moves his/her marker that many spaces on the game board.
6. The first student to the finish line is the winner.

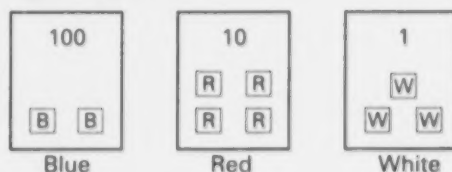
UNIFIX Multiplication

Materials: UNIFIX cubes (three different colors)
9" x 12" construction paper in the same colors as the UNIFIX cubes

Directions: 1. Have the students work in small groups. Each group should assign a value (1, 10, or 100) to the different colored cubes and write the values at the top of pieces of construction paper that correspond in color. Example:



2. Next, have the students model a variety of numbers by placing UNIFIX cubes on the construction paper. The students should use the **least** number of cubes possible for each number. Example: 243



- 123
3. Write the problem $\begin{array}{r} \times 3 \\ 123 \end{array}$ on the board. In their groups, have the students discuss what the problem means and how it should be modeled.

4. Have each group build the model on their construction paper using the UNIFIX cubes. The members of each group should discuss the problem and the model if there is disagreement about the product.
5. Repeat the activity with other problems that do not require regrouping. Include problems with zero. (Sample problems

20 31 420 201 323

include $\underline{x\ 4}$, $\underline{x\ 3}$, $\underline{x\ 2}$, $\underline{x\ 4}$, and $\underline{x\ 2}$.)

6. Write the problem $\overset{23}{\underline{x\ 4}}$ on the board. Have each group build the corresponding model on their construction paper.
7. Have the members of each group discuss the model and the product. Ask, "Have you used the **least** amount of cubes possible? If not, decide in your groups how you can change your model so that you are using the least amount of cubes."
8. Circulate among the groups to observe if the students are trading 10 ones for 1 ten. Have a student from one of the groups explain to the class the answer to the problem and what the group did so that they would be using the least amount of cubes.
9. Repeat the activity with other problems that involve regrouping from ones to tens. Expand the activity to include regrouping of tens and regrouping of both ones and tens.



Problem Solving

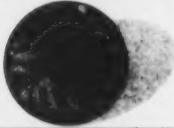
Money Trading

Materials: Play money (blackline masters may be found in the Appendix) or real money (pennies, dimes, and dollars)
 Money trading mat (one per student; blackline master follows this activity)

- Directions:
1. Give the students the problem $3 \times 26\text{¢}$ and have them write it down.
 2. Then have the students model the problem using dimes and pennies on their money trading mat.
 3. They may choose to put 26¢ on the mat three times and then combine the sets and make trades of pennies for dimes. Or, they may choose to put 26¢ on the mat, add another 26¢ , make a trade, add another 26¢ , and make a final trade.
 4. Once they have finished their trading, the students should write the answer next to the problem.
 5. Give the students several other problems to model and solve (e.g., $47\text{¢} \times 2$, $7 \times 22\text{¢}$, $64\text{¢} \times 5$, $7 \times 21\text{¢}$, $4 \times \$1.22$, $5 \times \$1.36$, $2 \times 97\text{¢}$, $37\text{¢} \times 5$, and $\$1.21 \times 6$).



Money Trading Mat

Use with "Money Trading" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Test Taking

Materials: Samples of multiplication problems from multiple choice tests
 Optional: Cards on which you have written multiplication problems with multiple choice answers
 Examples:

$$\begin{array}{r} 24 \\ \times 3 \\ \hline \end{array}$$

12 612 6 72

$$\begin{array}{r} 202 \\ \times 3 \\ \hline \end{array}$$

66 206 606 205

Directions: 1. Have each student work with a partner.

2. Write the problem $\begin{array}{r} 65 \\ \times 2 \\ \hline \end{array}$ on the board. Ask, "If I wanted to build a model for this problem, what would I need to do?" Have each pair of students discuss the model.
3. Write the following possible answers for the problem on the board and explain that the problem and answers are from a multiple choice test:
 •1,210 •130 •67 •140
4. Say, "We are not going to find the correct answer. Instead, we are going to look for answers that are impossible. These are called distractors, because they distract you from the correct answer."
5. Say, "Let's look closely at our problem and think about the model we would build. Do you see any answer on the 'test' that would be impossible? Why do you think it is impossible?"
6. Have each pair of students choose the distractors and discuss why they are impossible answers. Stress that you do **not** want them to find the exact answer and eliminate the others. You want them to locate the impossible answers and eliminate them.
7. Have the students discuss their choices and the reasons behind them as a large group.
8. Repeat the activity with several other examples. To ensure that the students have sufficient practice, you may want to give each pair several cards containing problems and

possible answers. Have them exchange cards with other groups after choosing the distractors.

9. Variation: Have each pair of students create multiplication test items with multiple choice answers. Have them include distractors for each problem. Then have the pairs exchange problems and find the distractors in the problems they've been given.

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

1. At Alice's restaurant, 317 loaves of bread are eaten each day. How many loaves of bread are eaten in 5 days?
2. Alice serves 88 pounds of butter and 45 gallons of milk to her customers in one day. How many pounds of butter will she serve in 1 week?
3. It takes 52 cows to produce the butter that Alice needs each year and 216 cows to produce the milk she needs. How many more cows are needed to produce the milk than the butter?
4. Each week Alice uses four tons of potatoes and two tons of cabbage at her restaurant. How many tons of potatoes and cabbage does Alice use in 1 year? (Hint: 52 weeks = 1 year)
5. Alice's customers use 46,000 bottles of catsup, 3,000 bottles of Alice's Secret Sauce, and 5,240 bottles of A-1 sauce in a year. How many bottles are used in all?



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

How Much Will It Cost?

Materials: Advertisements from newspapers and magazines
Blank adding machine tape
Play or real money (blackline masters for play money may be found in the Appendix)

Directions: 1. Have each student make a dollars/cents mat, like the following, from adding machine tape. Have the students paste an ad at the top of the "mat" and record how much the item costs in the columns below. Example:

paste ad here	
dollars	cents
\$4	95

2. With play or real money, have the students model how much two, three, and five of the items would cost and record the problems and answers on their dollars/cents mats. Example:

paste ad here	
dollars	cents
\$4	95
x	2

4.20

SKILL Dividing a two-digit number by a one-digit number

Skill Analysis

- A. Understands the concept of multiplication
- B. Can multiply numbers using the basic multiplication facts
- C. Understands the concept of place value
- D. Understands the concept of regrouping
- E. Understands the inverse relationship of multiplication and division
- F. Understands the concept of division
- G. Understands the “÷” and “ $\overline{)$ ” symbols and can use them to solve problems

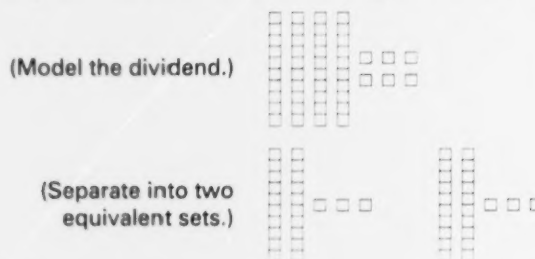


Concept Manipulative Activities

Models

Materials: Base 10 materials (beans sticks and loose beans, Base Ten Blocks, Cuisenaire rods, UNIFIX cubes, graph paper, etc.)

- Directions:**
1. Have the students model the dividend 46 using their manipulatives.
 2. Say, “Divide your model into two equal sets.” Example:



3. Say, "To keep track of what you have done with the manipulatives, you need to use symbols." Show the students how to write the symbols for the problem they have just completed. Example:

$$\begin{array}{r} 23 \\ 2 \overline{)46} \end{array}$$

We divided our model into two equivalent sets. Each equivalent set had 23 members. The number we started with was 46.

4. Repeat the activity with a problem that requires regrouping in the tens place. Example:

COMMENTS	MANIPULATIVES	SYMBOLS
a. Have students model the number 67.		$\overline{)67}$
b. Say, "Separate your model into four equivalent sets." Instruct the students to start by placing an equivalent number of tens in each of the four sets. Ask, "What should you do with the 2 tens that are left over?" If the students do not suggest regrouping them into units, ask leading questions that will guide them to this solution.		$\begin{array}{r} 1 \\ 4 \overline{)67} \\ \underline{4} \end{array}$ Separate into four equivalent sets. Four tens were used. Two tens remain.
c. Ask, "How many units do you have after trading the tens for units?" (27)		$\begin{array}{r} 1 \\ 4 \overline{)67} \\ \underline{4} \\ 27 \end{array}$ This number corresponds to the number of units the students have after regrouping.
d. Say, "Separate the 27 units into four equivalent sets." Ask, "How many units are in each set?" (6) "How many units are left over?" (3)		$\begin{array}{r} 16 \\ 4 \overline{)67} \\ \underline{4} \\ 27 \\ \underline{24} \\ 3 \end{array}$ The total number in each equivalent set. The number of units used. The number of units left over.

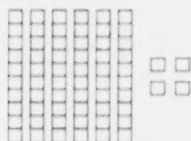
5. Repeat the activity with other problems. Include problems with and without regrouping and with and without remainders.

422

Modeling With Expanded Notation

Materials: Base 10 materials (bean sticks and loose beans, UNIFIX cubes, Base Ten Blocks, materials that can be bundled, etc.)
Overhead projector, blank transparency, Vis-A-Vis pen

Directions: 1. Model the number 64 on the overhead. Example:

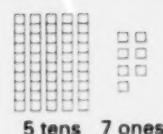


2. Say, "I'm going to divide this model into two equivalent sets. While I'm doing this, I'm going to keep a written record of my work."
3. Ask, "How many tens are there in 64?" (6) "How many ones?" (4)
4. Record
$$\begin{array}{r} \hline 6 \text{ tens} \quad 4 \text{ ones} \end{array}$$
5. Say, "If I divide the tens into two equivalent groups, how many tens will there be in each group?" (3) Model this. Ask, "Are there any tens left over?" (no)
6. Record
$$\begin{array}{r} 3 \text{ tens} \\ 2 \overline{) 6 \text{ tens} \quad 4 \text{ ones}} \end{array}$$
7. Ask, "If I divide the 4 ones into two equivalent groups, how many ones will there be in each group?" (2) Model this. Ask, "Are there any ones left over?" (no)
8. Record
$$\begin{array}{r} 3 \text{ tens} \quad 2 \text{ ones} \\ 2 \overline{) 6 \text{ tens} \quad 4 \text{ ones}} \end{array}$$
9. Repeat the activity using other examples, such as $3\overline{)56}$, $5\overline{)28}$, $5\overline{)96}$, and $4\overline{)63}$. Include some problems that require regrouping and some that do not. After going over a few examples, have the students manipulate the models and do the necessary record keeping themselves.

Moving From Expanded Notation to Division Algorithm

Materials: Base 10 materials (bean sticks, UNIFIX cubes, Base Ten Blocks, materials that can be bundled, etc.)
Overhead projector, blank transparency, Vis-A-Vis pen

- Directions:**
1. Model the number 57 on the overhead with manipulatives. Have the students do the same at their desks.
 2. Ask one of the students to write the number 57 in expanded notation (i.e., 5 tens 7 ones) on the transparency.
 3. Say, "We're going to divide our models into three equivalent groups and keep a written record of our work."
 4. Write the problem as $3 \overline{) 5 \text{ tens } 7 \text{ ones}}$. Make sure the students understand the meaning of the expanded notation. For students having difficulty, relate the expanded form to the model. Example:



5. Say, "If we partition the 5 tens into three equivalent sets, how many tens will be in each set?" (1) Here, and throughout the activity, the students should manipulate their models to find the answers to your questions. Say, "We write '1 ten' in the quotient since each set has 1 ten. Because 3 tens have been used, we write '3 tens' below the '5 tens' in the dividend. Then we subtract to see how many tens remain. Example:

$$\begin{array}{r}
 \begin{array}{c} \text{Number of} \\ \text{equivalent sets} \end{array} \nearrow 3 \overline{) \begin{array}{c} 1 \text{ ten} \\ 5 \text{ tens } 7 \text{ ones} \\ - 3 \text{ tens} \\ \hline 2 \text{ tens} \end{array}} \\
 \begin{array}{l} \text{Number of tens in each set} \leftarrow 1 \text{ ten} \\ \text{Number of tens used altogether} \leftarrow 3 \text{ tens} \\ \text{Number of tens remaining} \leftarrow 2 \text{ tens} \end{array}
 \end{array}$$

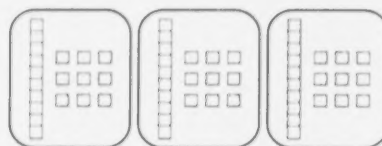
6. Say, "We need to regroup the 2 tens that remain into ones. When we join them with the original 7 ones, we have a set of 27 units." Ask, "If we partition the 27 units equally among the three sets, how many additional units will there be in each set?" (9) Say, "We write '9 ones' in the quotient since each set has 9 ones. Because three sets of nine is 27, we

424

write '27 ones' below the 27 in the problem and subtract." Ask, "How many ones are left over?" (0) Students should discuss the fact that at times there are no remainders. Example:

$$\begin{array}{r} \text{1 ten} \quad \text{9 ones} \\ 3 \overline{) 5 \text{ tens} \quad 7 \text{ ones}} \\ \underline{- 3 \text{ tens}} \\ 2 \text{ tens} \quad 7 \text{ ones} \end{array}$$

$$\begin{array}{r} (27 \text{ ones}) \\ - 27 \text{ ones} \\ \hline 0 \end{array}$$



7. The students' models should now consist of three groups of 1 ten and 9 ones. To help the students see the relationship between multiplication and division, have them combine the three sets following the rule "If you can make a set of 10, you have to." They will see that three sets of 19 can be combined to make one set of 57.
8. Relate the expanded notation form for the problem the students just worked to the standard algorithm.

$$\text{a. } \begin{array}{r} 3 \overline{) 5 \text{ tens} \quad 7 \text{ ones}} \end{array} \quad 3 \overline{) 57}$$

$$\text{b. } \begin{array}{r} \text{1} \\ 3 \overline{) 5 \text{ tens} \quad 7 \text{ ones}} \\ \underline{- 3 \text{ tens}} \\ 2 \text{ tens} \end{array} \quad \begin{array}{r} \text{1} \\ 3 \overline{) 57} \\ \underline{- 3} \\ 2 \end{array}$$

$$\text{c. } \begin{array}{r} \text{1} \\ 3 \overline{) 5 \text{ tens} \quad 7 \text{ ones}} \\ \underline{- 3 \text{ tens}} \quad \downarrow \\ 2 \quad 7 \text{ ones} \end{array} \quad \begin{array}{r} \text{1} \\ 3 \overline{) 57} \\ \underline{- 3\downarrow} \\ 27 \end{array}$$

$$\text{d. } \begin{array}{r} \text{1} \quad \text{9} \\ 3 \overline{) 5 \text{ tens} \quad 7 \text{ ones}} \\ \underline{- 3 \text{ tens}} \\ 2 \text{ tens} \quad 7 \text{ ones} \\ \underline{- 2 \text{ tens} \quad 7 \text{ ones}} \\ 0 \end{array} \quad \begin{array}{r} \text{19} \\ 3 \overline{) 57} \\ \underline{- 3} \\ 27 \\ \underline{- 27} \\ 0 \end{array}$$

9. Repeat the activity using problems with and without remainders.



Let's Practice

Universal Division

Materials: Universal Game Board (one per group of students; directions for its construction may be found in the Appendix)
 Division game cards (see step 3)
 Game die (one per game)
 Game markers
 Base 10 materials (Base Ten Blocks, UNIFIX cubes, bean sticks, etc.)

- Directions:**
1. This game is played by small groups of students.
 2. Students roll the die to determine who will go first. It can be either the high or the low roller.
 3. Cards with the following problems written on them are placed in a pile on the game board, problem side up. (Answers are written on the back of each card to make the game self-checking.)

$2\overline{)9}$	$63 \div 9$	$7\overline{)49}$	$33 \div 4$	$57 \div 9$
$3\overline{)22}$	$33 \div 5$	$7\overline{)28}$	$7\overline{)48}$	$39 \div 9$
$2\overline{)16}$	$19 \div 8$	$5\overline{)22}$	$10 \div 5$	$36 \div 6$
$7\overline{)52}$	$65 \div 7$	$8\overline{)73}$	$66 \div 8$	$49 \div 9$
$3\overline{)26}$	$31 \div 6$	$7\overline{)9}$	$45 \div 6$	$39 \div 4$
$3\overline{)16}$	$6\overline{)42}$	$47 \div 9$	$42 \div 5$	
$2\overline{)11}$	$24 \div 4$	$46 \div 8$	$5\overline{)23}$	
$6\overline{)39}$	$26 \div 3$	$17 \div 2$	$15 \div 3$	
$56 \div 8$	$7\overline{)59}$	$9\overline{)75}$	$68 \div 8$	
$9\overline{)87}$	$7\overline{)39}$	$18 \div 5$	$23 \div 6$	

4. The students, in turn, draw the top card and solve the problem. They may use manipulatives to model the problem if they want to. The students check the answer by looking at the back of the card. If a student's answer is correct, the student rolls the die and moves his/her game marker forward that number of spaces on the game board.
5. The first student to reach the finish line is the winner.
6. Variation: Rather than moving the number of spaces indicated by a roll of the die, the student moves the number of spaces that corresponds to the remainder of the problem he/she worked. If the remainder is zero, the student moves forward the number of spaces named by the quotient.

Division Puzzle

Materials: Picture (from a newspaper, magazine, etc.)
Plain paper (cut to the size of the picture)
Glue
Scissors
Base 10 manipulatives

- Directions:
1. Cut a relatively square picture from a newspaper, magazine, or other source.
 2. Divide a sheet of paper that is the same size as the picture into 4 x 4 sections and copy the following or similar problems and answers onto it. Make sure each problem and its answer are on opposite sides of the lines.

Example:

$7 \overline{)92}$	$8 \overline{)91}$	$3 \overline{)38}$	$5 \overline{)24}$
19 $58 \div 3$	19 R1 $75 \div 6$	12 R3 $88 \div 3$	29 R1 $48 \div 7$
$15 \overline{)82}$	$11 \overline{)84}$	14	12
$3 \overline{)47}$	$8 \overline{)92}$	$6 \overline{)84}$	$2 \overline{)24}$
6 $84 \div 6$	14 $59 \div 3$	19 R2 $65 \div 7$	9 R2 $81 \div 6$
$17 \overline{)81}$	11	9	$22 \overline{)82}$
$5 \overline{)86}$	$9 \overline{)99}$	$1 \overline{)9}$	$3 \overline{)68}$
9 R1 $52 \div 2$	26 $68 \div 4$	17 $45 \div 4$	11 R1 $58 \div 4$
$12 \overline{)81}$	$23 \overline{)81}$	$24 \overline{)81}$	$15 \overline{)81}$
$7 \overline{)85}$	$2 \overline{)47}$	$4 \overline{)97}$	$5 \overline{)76}$
11 R1 $68 \div 7$	9 R5 $89 \div 3$	29 R2 $96 \div 5$	19 R1 $25 \div 2$
10	8 R5	2	7 R6

3. Glue the picture to the back of the paper. Cut the paper apart on the lines.
4. Have the students, individually, work the problems and fit the puzzle pieces together so that the problems and their solutions are touching. They may use manipulatives if they want to.
5. The students can check their work by turning the paper over. If all of the problems are matched correctly, the picture will be intact.

Division Duo

Materials: Base 10 materials (UNIFIX cubes, bean sticks, Ziploc bags and loose beans, etc.)

- Directions:**
1. Have the students work in pairs.
 2. One student in each pair should make up a division problem in which the **dividend** is less than 100 and the **divisor** is less than 10.

428

3. The student should read the problem to his/her partner, who should solve it using manipulatives.
4. The students should switch roles after each problem, continuing to solve problems until time is called.
5. You may want to have the students keep a record of each problem they work. Example:

37 divided into 5 groups =
7 groups with 2 left over.

OR

7 with 2 left over.
 $5\overline{)37}$



Problem Solving

Divvy Up the Money

Materials: Play money (dimes and pennies; blackline master may be found in the Appendix)

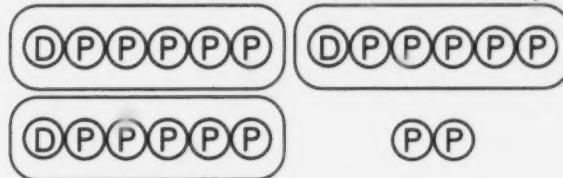
Problem cards (blackline master follows this activity; copy and cut apart)

- Directions:**
1. The students may work individually or in pairs for this activity.
 2. Place the problem cards in a pile.
 3. Have the students choose a card, divide the money amount as indicated, and record their results. They may use play money to help them with regrouping.

Examples:

47¢
 3 equivalent
 groups

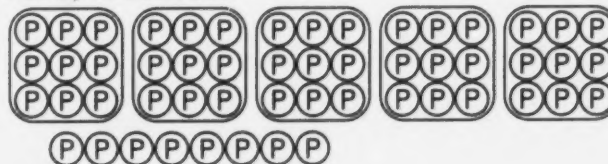
- a. Make 47¢ with 4 dimes and 7 pennies.
- b. Divide this amount into 3 equivalent sets.
- c. One dime will have to be traded for 10 pennies.



- d. Two pennies are left over.

\$.53
 9 cents in
 each set

- a. Trade the 5 dimes for 50 pennies.
- b. Put 9¢ in each set.



- c. Eight pennies are left over.

Record Sheet

Total Amount of Money	Number of Sets	Amount in Each Set
47¢	3	15¢, 2¢ left over
53¢	5	9¢, 8¢ left over

436



Problem Cards

69¢ 5 equivalent sets	52¢ 8¢ in each set	86¢ 6 equivalent sets	31¢ 9 cents in each set
\$.12 5 cents in each set	\$.29 3 equivalent sets	\$.49 7¢ in each set	\$.74 7 equivalent sets
95¢ 2 equivalent sets	17¢ 4 cents in each set	24¢ 4 equivalent sets	37¢ 6¢ in each set
\$.42 1¢ in each set	\$.57 7 equivalent sets	\$.62 3 cents in each set	\$.78 6 equivalent sets
83¢ 2 equivalent sets	90¢ 9¢ in each set	14¢ 8 equivalent sets	21¢ 2 cents in each set
\$.34 6 cents in each set	\$.46 3 equivalent sets	\$.54 8¢ in each set	\$.65 9 equivalent sets
70¢ 8 equivalent sets	89¢ 5 cents in each set	98¢ 4 equivalent sets	33¢ 7¢ in each set
46¢ 3¢ in each set	52¢ 9 equivalent sets	\$.78 4 cents in each set	\$.60 5 equivalent sets

Use with "Divvy Up the Money" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Quotient Estimates

Materials: Calculators (one per pair of students)

- Directions:
1. The purpose of this activity is for students to have an opportunity to make **reasonable** estimates in division. They will estimate the answers to division problems and then see how close their estimates were by finding the actual answers with a calculator.
 2. Have the students work in pairs.
 3. If your students are not familiar with how to do division problems on a calculator, provide instruction. For example, show which number should be entered first on the calculator. Examples:

$$\begin{array}{cc} 46 \div 7 & 7 \overline{)46} \\ \uparrow & \uparrow \\ \text{enter} & \text{enter} \\ \text{first} & \text{first} \end{array}$$

Also explain that both the $\div$ and $\overline{)}$ symbols are represented by $\div$ on the calculator.

4. Write division problems on the board. Have the students estimate what they think the answers will be and then find the actual answers using their calculators. Have them keep a record of the problems, the estimates, and the actual answers. Example:

Problem	Estimate	Calculator Answer
$37 \div 8$	5	4.625
$9 \overline{)53}$	6	5.8888888

5. Have the students reenter problems if a very large discrepancy exists between their estimate and the calculator answer. This situation can provide an excellent learning opportunity, helping students see that we can make mistakes when entering numbers on a calculator.

*TEACHER NOTE: The students have been writing remainders as whole numbers. However, when using a calculator, remainders are shown in decimal form. Even if

432

your students have not been formally introduced to decimals, they should be able to understand that a decimal in the answer means there is a remainder. Solving a few problems with and without remainders, both with paper and pencil and with a calculator, should help the students make this connection. Some calculators can be set for "0" decimals and will round off automatically.

**S
T
O
R
Y

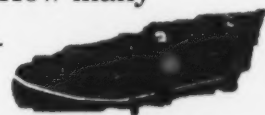
P
R
O
B
L
E
M
S**

1. The P.E. teacher wants to organize the balls in the gym so that there will be 10 balls on each shelf in the closet. The teacher found 15 balls in the closet, 14 balls in the ball bag, and 20 balls in the storage room. How many shelves will the teacher need to use for the balls? _____ Will every shelf have 10 balls? _____



2. If oranges are on sale at six for 89¢, what do you think you would be charged for only one orange? _____ Explain your answer.

3. Ms. Berry bought 84 erasers for \$2.35. She put them in packages of 3 each to give away as prizes. How many packages of erasers did she make? _____ Were any erasers left over? _____

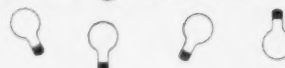


4. The Johnsons went on vacation for 32 days. How many full weeks were they gone? _____ How many days short were they of being gone one more full week of vacation? _____

- 5a. Mary bought five packages of lightbulbs. Three packages held four lightbulbs each. Two packages held six lightbulbs each. How many lightbulbs did Mary buy? _____



- b. If the lightbulbs had all been packaged in boxes of four, how many packages would equal the amount of lightbulbs Mary bought? _____



6. Make up a word problem of your own.



***TEACHER NOTE:** Students should be allowed to use manipulatives to model these problems.

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Multiplying by multiples of 10 and 100

Skill Analysis

- A. Recalls the basic multiplication facts
- B. Understands the zero property of multiplication
- C. Understands how to multiply by 10 and 100
- D. Understands the concept of place value
- E. Can apply the commutative property of multiplication



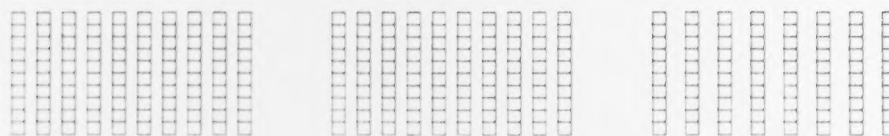
Concept Manipulative Activities

Multiplying by 10 or 100

Materials: Base Ten Blocks

Graph paper (blackline masters may be found in the Appendix)

- Directions:
1. Have the students model problems such as 7×10 and 7×100 using Base Ten Blocks.
 2. Then give the students the problem 28×10 .
 3. Ask, "What does 28×10 mean?" (It means 28 tens or that you multiply 28 ten times.)
 4. Have the students model 28 tens. Example:



5. Ask, "What does 28 tens equal?" (280)
6. Repeat the questioning and modeling for several other examples involving multiplying by 10.

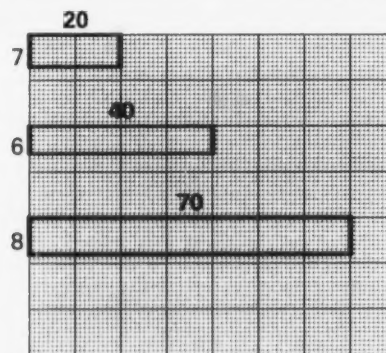
7. Ask, "What pattern do you see? What relationship exists between the problem and the answer?"
8. Repeat the activity with problems that involve multiplying by 100 (e.g., 28×100).



Graph Paper Model

Materials: Base 10 graph paper (blackline master may be found in the Appendix)

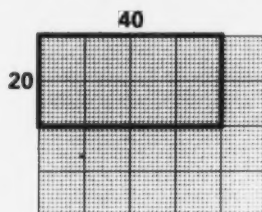
- Directions:**
1. Review with the students how to use arrays to model multiplication.
 2. Distribute base 10 graph paper to each student or to small groups of students.
 3. Have the students model the problem 7×20 on the graph paper. Then have them model the problems 6×40 , 8×70 , and other problems that have a multiple of 10 as a factor.



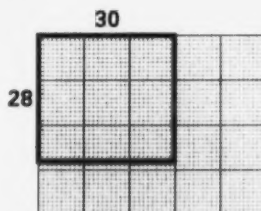
4. Have the students explain how to find the products. Several methods are possible. For 7×20 , for example, they could count by 10s, count by 20s, or count 7 rows of 2 tens, which is 14 tens, which equals 140, etc.

436

5. Write the problems with the answers on the board: $\begin{array}{r} 20 \\ \times 7 \\ \hline 140 \end{array}$, $\begin{array}{r} 40 \\ \times 6 \\ \hline 240 \end{array}$, $\begin{array}{r} 70 \\ \times 8 \\ \hline 560 \end{array}$. Ask the students what pattern they see.
6. Use the associative property to show why the pattern works: $7 \times 20 = 7 \times (2 \times 10) = (7 \times 2) \times 10 = 14 \times 10 = 140$.
7. Ask the students to model 20×40 , 70×80 , 30×60 , and other similar problems on the base 10 graph paper.



8. Write the problems with the answers on the board: $\begin{array}{r} 20 \\ \times 40 \\ \hline 800 \end{array}$, $\begin{array}{r} 30 \\ \times 60 \\ \hline 1,800 \end{array}$, $\begin{array}{r} 70 \\ \times 80 \\ \hline 5,600 \end{array}$. Ask the students what pattern they see.
9. Use the commutative and associative properties to show why the pattern works: $20 \times 40 = (2 \times 10) \times (4 \times 10) = (2 \times 4) \times (10 \times 10) = 8 \times 100 = 800$.
10. Have the students model the problem 28×30 and determine the answer.



11. Show them that they can get the answer using the same pattern: $28 \times 30 = 28 \times (3 \times 10) = (28 \times 3) \times 10 = 84 \times 10 = 840$, which is the same as

$$\begin{array}{r} 28 \\ \times 3 \\ \hline 84 \end{array} \times 10 = 840$$

12. Give the students similar problems to solve in the same way.

Pairs of Products

Materials: None

Directions: 1. Write the following pairs of problems on the chalkboard:

$$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ \times 30 \\ \hline \end{array} \quad \begin{array}{r} 13 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 13 \\ \times 40 \\ \hline \end{array} \quad \begin{array}{r} 26 \\ \times 5 \\ \hline \end{array} \quad \begin{array}{r} 26 \\ \times 50 \\ \hline \end{array}$$

2. Say, "Compare the first pair of problems. How are the second factors related?" (The second factor in the second example is 10 times the second factor in the first example.) Ask, "How do you think the products are related?"
3. Have the students find the products and compare them. Ask again, "How are the products related?" (The product for the second example is 10 times the product for the first example.)
4. Review with the other pairs of problems in the same way.
5. Ask a student volunteer to write the next logical problem for each set (i.e., 8×300 , 13×400 , and 26×500). Ask, "How are the factors and products in these problems related to the previous ones? What kind of pattern is occurring?"
6. Have the students experiment with other related problems to better see the pattern.
7. Having the students organize their related problems in a triangular fashion may help them to see the pattern emerging. Example:

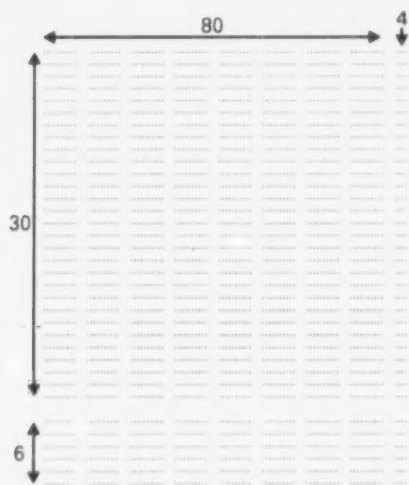
$$\begin{array}{l} 6 \times 3 = 18 \\ 60 \times 3 = 180 \\ 600 \times 3 = 1,800 \end{array}$$

Estimating Products

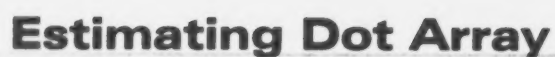
Materials: "Estimating Dot Array" handout (one per student; blackline master follows this activity)

433

- Directions:
1. Pose the following problem to the class: "There are 67 rows of apple trees on the A-Apple Tree Farm. In each row there are 31 trees. About how many trees are there on this farm?"
 2. Have students estimate the number of trees and share the process they used.
 3. Discuss how rounding to multiples of 10 or 100 simplifies the estimation process.
 4. Explain that in the apple tree farm problem both 67 and 31 may be rounded to the nearest 10. Therefore, 67 would be rounded to 70 and 31 would be rounded to 30. These rounded off numbers can be easily multiplied: $70 \times 30 = 2,100$. Have the students multiply 67×31 to see how close both this estimate and the one they derived were to the actual answer.
 5. Give the students other problem-solving situations and ask them to estimate the answers. Then have them discuss their procedures.
 6. Have the students estimate the number of dots on the "Estimating Dot Array" handout.
 7. Have them discuss the process they used for estimating. Then have them compare their estimates to the actual answer.
 8. Explain how they may have broken up or divided the array into groups of ones, tens, and hundreds to simplify the estimation process.



430

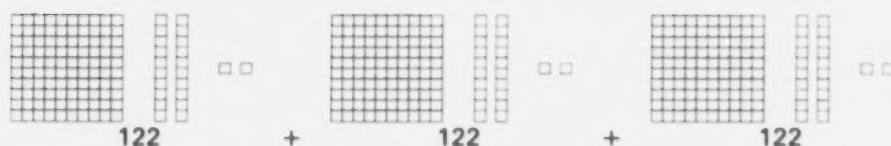
A 20x20 grid of small, dark, irregular shapes, possibly representing a microscopic view of a material or a digital noise pattern. The shapes are scattered across the grid, with some appearing more densely than others. The overall pattern is somewhat random but shows some local clustering.

4.10

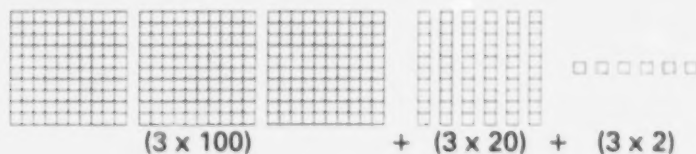
Distributive Model

Materials: Base Ten Blocks or centimeter graph paper (two sheets per student; blackline master may be found in the Appendix)
Scissors

- Directions:**
1. Have the students work in small groups or with a partner.
 2. If the students are using graph paper, have them each cut 4 hundred squares, 15 ten strips, and 20 ones from two sheets of graph paper.
 3. Have the students use the Base Ten Blocks or graph paper sections to model two- and three-digit numbers. Examples:
 $122 = 100 + 20 + 2$
 $145 = 100 + 40 + 5$
 $63 = 60 + 3$
 4. Have the students construct a model for 3×122 . Ask, "What does 3×122 mean? How can you use your materials to show what this means?" Allow the students to explain their model.



5. Ask, "If I asked you to combine the different parts of your model, what would you do?" Have the students explain how the model should be combined. (By combining the pieces for the same place value, the students can easily solve the problem.)



6. Discuss how the final model can be recorded without making a drawing of all of the materials. Be sure the students understand that $3 \times 122 = (3 \times 100) + (3 \times 20) + (3 \times 2) = 300 + 60 + 6 = 366$.

7. Have the students build models for other problems and record the problems on a record sheet, as shown in step 6.
Examples: 2×342 ; 3×213 ; 4×23 ; 5×114 ; 8×24 ; etc.



Let's Practice

Computing

Materials: Set of cards numbered 1-9
Set of cards numbered by tens, 0-90
Calculator
For variation: Set of cards numbered by hundreds, 0-900

- Directions:**
1. Two to four students may play this game.
 2. Have the students, in turn, draw a card from the ones set and a card from the tens set, multiply the two numbers, and state the product.
 3. The other students should check the answer with a calculator.
 4. If the student is correct and computed the problem mentally, he/she receives 3 points; if the student used paper/pencil, he/she receives 2 points; and if the student used a calculator, he/she receives 1 point.
 5. When time is called, all of the students add up their points. The player with the highest score wins.
 6. Variation: As students gain speed in calculating the answers, change from ones and tens cards to tens and hundreds cards.

Dice Practice

Materials: Die with faces labeled 2, 3, 5, 7, 8, 9
Die with faces labeled 40, 70, 80, 100, 200, 600

- Directions:** 1. Two to four students may play this game.

412

2. Have the students take turns rolling the two dice and naming the product of the numbers rolled.
3. If the product is less than 400, the student scores 2 points.
4. If the product is 400 or more, the student scores 1 point.
5. The winner is the first student to score 15 points or more.
6. Variation: Label one die 800, 500, 30, 400, 600, 70 and label the other die 20, 90, 60, 70, 40, 80. Have the students take turns rolling the two dice and naming the product of the numbers rolled. After a predetermined number of rolls, the player who has the greatest **sum** of products wins the game.

Calculator or Mentally—Which Is Faster?

Materials: Calculators (one per student)

- Directions:
1. Have the students use calculators to find the products of the following problems:
 - a. 26×10
 - b. 10×383
 - c. 30×23
 - d. 12×300
 - e. 30×600
 - f. 500×11
 2. Ask the students which of the problems could be solved faster with a calculator and which could be solved faster mentally. Have the students justify their choices. If students disagree about the preferred method, allow them to show that their method is faster.
 3. Discuss when a calculator is helpful or needed and when solving a problem mentally is advantageous.
 4. Have the students generate problems involving multiples of 10 or 100 that they believe are easier to solve mentally than with a calculator. Ask, "Why is it faster to solve these problems mentally?"
 5. Then have the students generate problems involving multiples of 10 or 100 that they believe are easier to solve

with a calculator. Ask, "Why is it faster to use a calculator for these problems?"

6. Discuss the patterns emerging from the first set of problems and then from the second set of problems. Compare the two sets. Allow for extended discussion and justification.

Estimating Money

Materials: Calculators (one per group of students)

- Directions:
1. Present the students with story problems in which they are asked to estimate different products. Have them work in small groups to estimate the products.
 2. Examples of problems:
 - a. If I bought 14 candy bars at 39¢ each, about how much did I spend?
 - b. I need 22 oranges for a punch recipe. They cost 21¢ each. What is the approximate total cost?
 - c. Mrs. Thomas purchased 36 pencils at 11¢ each. About how much did she spend?
 - d. Acme Sports Store bought 196 marbles at 8¢ each. About how much did the marbles cost?
 3. Have the students share how they estimated the products. (Several different strategies could be used.) Then have them compute the actual cost of the items using their calculators to see how close their estimates were.
 4. Ask, "What is the advantage to knowing how to estimate to find approximate products? Did you use zeros in your factors? Why? What are the advantages to multiplying with zeros?"



Problem Solving

Dream Vacation

Materials: Newspapers (possibly the Sunday edition vacation section)
 Travel magazines
 Poster paper
 Map of the world

- Directions:**
1. Explain that you want the students to calculate the cost of a 2-week dream vacation to a location of their choice and to create a display for their vacation showing recreation scenes as well as the total cost for the vacation.
 2. Using newspaper or travel magazine advertisements, have the students determine the cost of roundtrip airfare to their destination.
 3. Have them use \$60 per day for their hotel budget and \$30 per day for their food budget.
 4. Instruct them to make a chart like the following to summarize the expenses.

Destination	Hotel Cost	Food Cost	Airfare	Total Cost

5. Have them use pictures from newspapers and magazines to create their displays. Post the displays on a bulletin board to share them with the class.
6. Have the students locate and label their destinations on a world map.

Advertisements

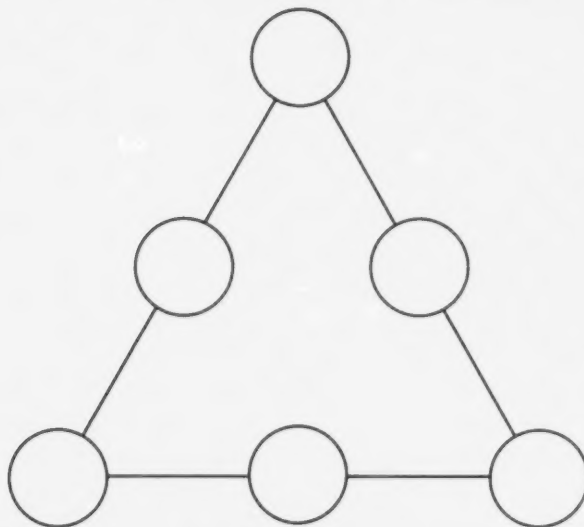
Materials: None

- Directions:
1. Have each student write a short advertisement for his/her favorite food or TV program.
 2. Then have the students calculate the cost for printing their ads based on the price of 20¢ (or any other designated multiple of 10) for each word.
 3. Have the students share their advertisements and their costs.
 4. Ask, "Who has the least expensive/most expensive advertisement? Who do you think said the most in the smallest amount of words?"
 5. Have the students tape-record their advertisements as if they were to be aired on TV. As they do so, have them use a stopwatch or a clock with a second hand to time the length of the advertisement. Then have them calculate the cost of broadcasting the advertisement based on the price of \$1,000 for each second of "TV time."

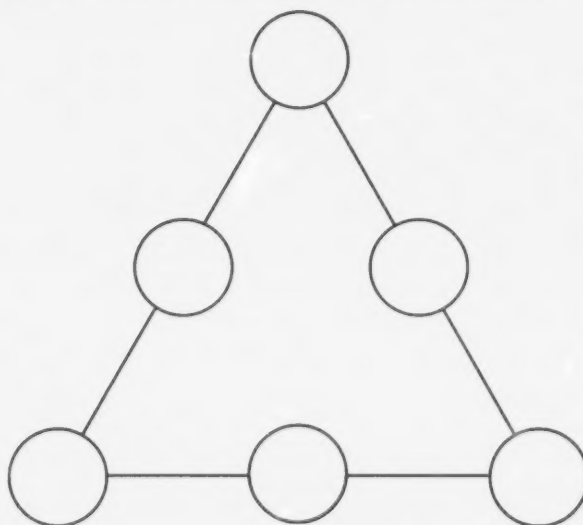
S
A
M
E

A
N
S
W
E
R

Directions: Using the numbers 1, 4, 5, 6, 20, and 30, fill in the circles so that the product of each side of the triangle is 600. Each number should be used one time.



Make up a similar problem using, as above, some numbers that are multiples of 10 or 100. Have a friend solve your triangle.



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Multiplying two-digit numbers by two-digit numbers

Skill Analysis

- A. Understands the concept of multiplication
- B. Recalls the basic multiplication facts
- C. Can apply the distributive property of multiplication
- D. Understands the concept of regrouping
- E. Understands the concept of place value
- F. Can multiply by multiples of 10 and 100

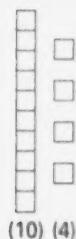


Concept Manipulative Activities

Growing

Materials: UNIFIX cubes

- Directions:
1. Distribute UNIFIX cubes to pairs of students.
 2. Tell the students that you will give them numbers to model with their UNIFIX cubes. Each time they have 10 cubes, they must build a tower. Single cubes must remain single; they cannot be joined in a tower. For example, the number 14 would be modeled as a tower of 10 cubes and 4 single cubes.



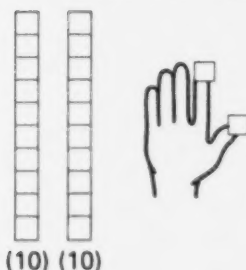
***TEACHER NOTE:** A good way for students to keep track of single cubes is to have them place them on their fingers. Example:



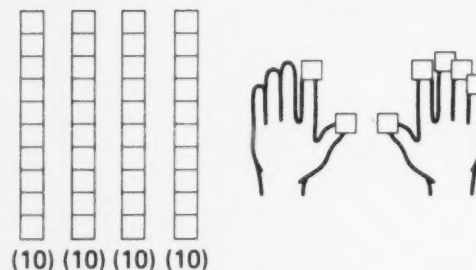
When all 10 fingers are covered, a tower must be built.

3. Give the pairs of students numbers to model. You can also have students give their partners numbers to build. Examples:

“Show me 22.”



“Show me 47.”



4. Have the students practice modeling numbers up to 99.

More Growing

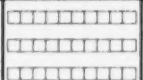
Materials: UNIFIX cubes

Place value mats (one per student; directions for their construction may be found in the “Place Value Mat” activity in Section 3.6-2)

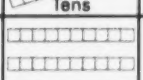
Hundreds	Tens	Ones

Directions: 1. Review with students the concept of multiplication. Ask, “What does 3×14 mean?” Allow students to describe in their own words their understanding of this concept.

2. Ask, "How would you build a model for this problem using UNIFIX cubes?" Have the students build a model for this problem on their place value mats.

Hundreds	Tens	Ones
		

3. Ask, "Using the distributive property, what do you know about 3×14 ?" ($3 \times 14 = (3 \times 10) + (3 \times 4)$) "If you combine the tens and ones in your model, what do you have?" (3 tens and 12 ones)
4. Say, "Look at your model again. What rule do we have about 10 or more ones?" (If I have 10 cubes, they have to be placed in a tower.)
5. Say, "Model that with the UNIFIX cubes on your place value mats. Put the new tower above the mat so you will remember it is special."

Hundreds	Tens	Ones
		

6. Ask, "How many towers of 10 cubes do you have now?" (4) "How many single cubes do you have?" (2) "How many do you have altogether?" (42)
7. Ask, "How does this number compare with the answer you got using the distributive property?" (They are the same.)
8. Say, "Let's record what happened in this problem. Let's

write the problem vertically."
$$\begin{array}{r} 14 \\ \times 3 \\ \hline \end{array}$$
 "Now let's write it in expanded notation."

$$\begin{array}{r} (10 + 4) \\ \times 3 \\ \hline \end{array}$$

Say, "We had three sets of 4 single cubes, or ones. How many ones did we have in all?" (12) "We will record that under the ones."

$$\begin{array}{r} 10 + 4 \\ \times \quad 3 \\ \hline 12 \end{array}$$

9. Ask, "But what did we do with the 12 ones?" (We made a tower of 10 with 2 left.) "Where did we put the special tower?" (Over the tens place on our place value mat.) Show the students how to record what they did.

$$\begin{array}{r} \textcircled{1} \\ 10 + 4 \\ \times \quad 3 \\ \hline 2 \end{array}$$

10. Say, "Our record now shows 3×10 **and** an extra 10 above. Since $3 \times 10 = 30$ and an extra 10 is 40, we write 40 under the tens on our record."

$$\begin{array}{r} \textcircled{1} \\ 10 + 4 \\ \times \quad 3 \\ \hline 40 + 2 \end{array}$$

11. Ask, "How many cubes did we use in all?" (42) Have the students write 42 beneath the " $40 + 2$ " on their record.
12. Have the students model other problems on their place value mats and record their work. Examples:

$$\begin{array}{r} 23 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 25 \\ \times 2 \\ \hline \end{array} \quad \begin{array}{r} 41 \\ \times 4 \\ \hline \end{array} \quad \begin{array}{r} 17 \\ \times 5 \\ \hline \end{array} \quad \begin{array}{r} 19 \\ \times 6 \\ \hline \end{array}$$

13. When the students are comfortable with this process, allow them to reduce the number of steps they record. This will lead them to use the more standard form of the multiplication algorithm. Example:

$$\begin{array}{r} \textcircled{1} \\ 14 \\ \times 3 \\ \hline 42 \end{array}$$

***TEACHER NOTE:** While students are modeling these problems and recording their work, ask, "What will your answer be close to? How can you tell? How can you estimate before you begin your work? Why would you want to estimate?" Have the students share their strategies and why it is important to estimate.

Base Ten Block Models

Materials: Base Ten Blocks

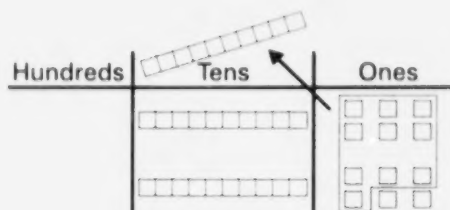
Place value mats (one per student; directions for their construction may be found in the "Place Value Mat" activity in Section 3.6-2)

Hundreds	Tens	Ones

- Directions:
1. Have each student work with a partner.
 2. Ask the students to define what 12×16 means. Ask, "How would I model this problem using Base Ten Blocks?" Have the students to share their ideas.
 3. Say, "If we model 12 as 1 ten and 2 ones, we can write this problem as $12 \times 16 = (10 + 2) \times 16 = (10 \times 16) + (2 \times 16)$."
 4. Say, "Let's all write this problem vertically before we work it out with Base Ten Blocks." Write the following on the board or overhead.

$$\begin{array}{r} 16 \\ \times 12 \\ \hline \end{array} = \begin{array}{r} 16 \\ \times 10 \\ \hline \end{array} + \begin{array}{r} 16 \\ \times 2 \\ \hline \end{array}$$

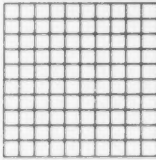
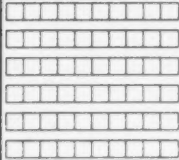
5. Say, "Working with your partner, model 2×16 on one of your place value mats." Ask, "What happens when we have 10 ones?" (We trade for 1 ten.) Have the students make the trade, placing the new ten above the tens place on their mat.



6. Say, "Let's record what we have done so far."

$$\begin{array}{r} \textcircled{1} \\ 16 \\ \times 2 \\ \hline 32 \end{array}$$

7. Say, "Now I'd like you to model the second part of the problem (10×16) on your other mat. How many cubes will you need to model this part of the problem?" (10 sets of 16)
8. Ask, "Before you make the model for 10×16 , what do you think your final outcome will be?" (160) "What will it look like?" (1 hundred, 6 tens, 0 ones) Have the students build the model for 10 sets of 16 and make any necessary trades.

Hundreds	Tens	Ones
		

9. Say, "Let's record this answer under the problem $\begin{array}{r} 16 \\ \times 10 \\ \hline \end{array}$. If we add the answers to both parts of the problem, what do we have?" (192)

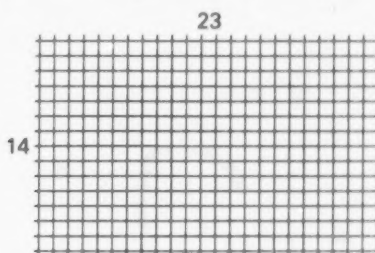
$$\begin{array}{r} 16 \\ \times 12 \\ \hline \end{array} = \begin{array}{r} 16 \\ \times 10 \\ \hline 160 \end{array} + \begin{array}{r} 16 \\ \times 2 \\ \hline 32 \end{array} = 192$$

10. Ask, "If you combine the blocks from both of your mats, what do you have?" (192) "How does this answer compare to the one on your record?" (They are the same.)
11. Repeat the activity with other two-digit times two-digit problems that can be modeled with Base Ten Blocks.

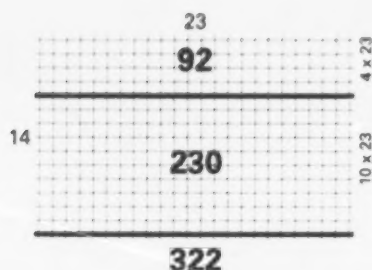
Graph Paper Multiplication

Materials: 1/4" graph paper (blackline master may be found in the Appendix)
 Transparency made from the graph paper blackline master
 Vis-A-Vis pen
 Overhead projector
 Glue
 9" x 12" construction paper

- Directions:
1. Write the problem 23×14 on the board or overhead.
 2. Outline the problem on the graph paper transparency. Have the students count the squares on the sides of the outline to verify your diagram. Example:



3. Have the students brainstorm ways to determine how many squares are within the outline. (Note: Various ways of "breaking down" the diagram may be suggested. Suggest that the diagram be broken down with as few lines as possible, and reinforce the idea of using multiples of 10.)
4. Explain that one way to "break down" the diagram would be to "break" 14 into $10 + 4$ by drawing a line across the diagram under the tenth row of squares. Explain that the problem now becomes $(10 + 4) \times 23$. Using the distributive property, that becomes $(10 \times 23) + (4 \times 23)$.
5. Have a student volunteer calculate each part of the problem on the overhead and write the products to the parts within the corresponding regions. Have the student write the total of the two products below the diagram.

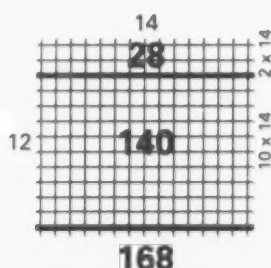


6. Repeat the activity with other problems. Call on student volunteers to demonstrate their work on the overhead.
7. Have the students use graph paper to make their own record sheets of problems. Have them cut out the graph paper section, glue it to construction paper, draw lines to break down the section, and label each part with the corresponding numbers.

Short Cut

Materials: 1/4" graph paper (blackline master may be found in the Appendix)

Directions: 1. Have the students construct a graph paper model for 12×14 as demonstrated in the previous activity, "Graph Paper Multiplication." Have the students label the parts of the model and calculate the product. Review their work with them.



2. Say, "This method works, but it uses a lot of paper. I'm going to show you how you can think your way through the problems instead of making the graph paper model."

3. Say, "Let's look again at the problem $\begin{array}{r} 14 \\ \times 12 \\ \hline \end{array}$. How can we 'think' our way through this?"

$$\begin{array}{r} 14 \\ \times 12 \\ \hline \end{array} \quad \text{Think} \quad \begin{array}{r} 14 \\ \times 10 \\ \hline \end{array} + \begin{array}{r} 14 \\ \times 2 \\ \hline \end{array}$$

4. Say, "Use the graph paper model you made to help you make a record of the problem."

$$\begin{array}{r} 14 \\ \times 12 \\ \hline 28 \\ + 140 \\ \hline 168 \end{array} \quad \begin{array}{c} \text{Think} \\ \begin{array}{r} 14 \\ \times 2 \\ \hline \end{array} \\ \begin{array}{r} 14 \\ \times 10 \\ \hline \end{array} \end{array}$$

5. Go over this "thinking" process with students. As a group, "think" through several problems with no modeling or recording. Examples:

$$\begin{array}{r} 17 \\ \times 23 \\ \hline \end{array} \quad \begin{array}{r} 45 \\ \times 26 \\ \hline \end{array} \quad \begin{array}{r} 19 \\ \times 12 \\ \hline \end{array} \quad \begin{array}{r} 38 \\ \times 37 \\ \hline \end{array}$$

6. After the students are comfortable with this "thinking" process, have them "think" their way through the problem $\begin{array}{r} 32 \\ \times 13 \\ \hline \end{array}$. Have students record what they are thinking and the answers at each step through this process. Example:

$$\begin{array}{r} 32 \\ \times 13 \\ \hline 96 \\ + 320 \\ \hline 416 \end{array} \quad \begin{array}{c} \text{Think} \\ \begin{array}{r} 32 \\ \times 3 \\ \hline \end{array} \\ \begin{array}{r} 32 \\ \times 10 \\ \hline \end{array} \end{array}$$

*TEACHER NOTE: Some students may need to continue to use graph paper models as they are learning this thinking process. If students are having difficulty aligning their work, have them solve the problems on graph paper, writing one number per square, or on lined paper turned sideways.

7. Give the students several problems that do not require regrouping to record and solve in the same way. Expand into problems that require regrouping.

8. Throughout the activity, stress that students use estimation strategies so that they have an approximation before beginning the algorithmic work.



Let's Practice

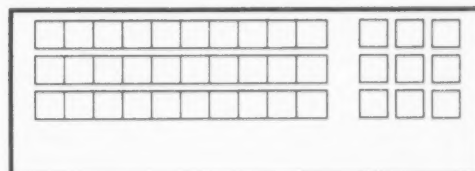
Activity Cards

Materials: 3" x 5" cards
 Base Ten Stamps or base 10 patterns (blackline master may be found in the Appendix)
 5-1/2" x 8-1/2" strips of unlined paper
 Scissors
 Glue
 Paper clips

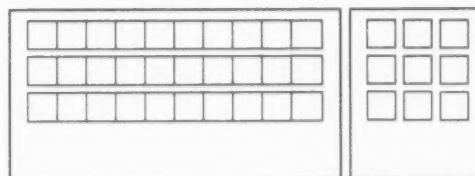
Directions: 1. Write multiplication problems like the following on 3" x 5" cards, laminate the cards, and place them in an independent learning center. Examples:

3×13	2×142	5×31	8×223
---------------	----------------	---------------	----------------

2. Have the students select a card and construct a model for the problem written on it by gluing Base Ten Stamps or base 10 patterns to one of the strips of unlined paper.
 Example: 3×13

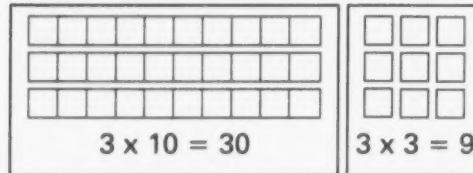


3. Have the students cut the piece of paper so that the "tens" are together and the "ones" are together.



457

4. Then have them record the multiplication problem modeled on each section.



5. Have the students record the final answer on the back of one of their pieces of paper and clip their models to the activity card. The cards with the models attached can be used in several of the other activities in this topic.

Multiplication Application

Materials: 3" x 5" index cards

- Directions:**
1. Have the students work in teams of two or three.
 2. As a class, brainstorm places where multiplication might be used in the school. Examples:
 - a. Cafeteria—27 hot lunches, 12 french fries on each plate
 - b. Cafeteria—18 tables, 25 chairs at each table
 - c. Workroom—a packet of 23 pages, need to make 32 copies of the packet
 3. List the places that come up in the discussion on the chalkboard.
 4. Have each team investigate a location and write down five multiplication problems that are applicable to that location.
 5. Have them write the problems on 3" x 5" index cards, one problem per card, with the answers on the back.
 6. Have the teams compare their problems for similarities and differences.

***TEACHER NOTE:** If students will be questioning adults in the school, obtain the permission of those adults before the students begin step 4.

Ins and Outs

Materials: "Ins and Outs" handout (one per group of students; blackline master follows this activity)

Calculators (one per group of students)

- Directions:
1. Have the students work in groups of three.
 2. Discuss the directions on the handout with the students. Ask, "By looking at the first sets of numbers, can you come up with an idea of what 'consecutive' means?" Allow the students to discuss their interpretation of the directions.
 3. Work the first problem as a class. Inside factors: $7 \times 8 = 56$; outside factors: $6 \times 9 = 54$.
 4. Have the students complete the worksheet in their groups. The students should rotate responsibilities for each problem, with one student working the inside factors, another working the outside factors, and the third using the calculator to check their work.
 5. When the students have completed the worksheet, have them state a generalization about the problems and the results.
 6. Have them try other problems to see if their generalization holds true.
 7. Variation: Have the students explore answers for consecutive even numbers, consecutive odd numbers, or consecutive multiples of a given number (e.g., $3 = 6, 9, 12, 15$). Have them look for patterns for their "Inside Answers" and "Outside Answers" for each of the variation ideas.



Ins and Outs

Names _____

Write four consecutive numbers on each line in the first column. Multiply the inside factors. Multiply the outside factors. Write your answers in the spaces provided.

	Numbers I Picked	Inside Answer	Outside Answer
1.	6, 7, 8, 9	_____	_____
2.	12, 13, 14, 15	_____	_____
3.	_____	_____	_____
4.	_____	_____	_____
5.	_____	_____	_____
6.	_____	_____	_____
7.	_____	_____	_____
8.	_____	_____	_____
9.	_____	_____	_____
10.	_____	_____	_____

Explain the relationship between the inside answers and the outside answers:

Use with "Ins and Outs" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

The Greatest

Materials: Dice (four per group of students)
Lined paper

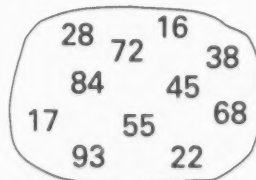
- Directions:
1. Players take turns rolling the four dice and recording the four numbers rolled.
 2. After each player has rolled the dice, he/she uses the numbers rolled to make a two-digit times two-digit multiplication problem.
 3. The players find the products to their problems.
 4. At the end of each round, the player or players with the largest product get 1 point.
 5. The winner is the player with the most points at the end of the game.
 6. Variation: The player with the **smallest** product gets 1 point.

Calculator Practice

Materials: Calculators (one per student)
"Three in a Row" handout (one per pair of students; blackline master follows this activity)
Student-made record sheet
Example:

Numbers Selected	Estimated answer	Calculator answer
------------------	------------------	-------------------

- Directions:
1. Put the following numbers on the chalkboard or on a transparency.



2. Have a student volunteer select two numbers from the group and estimate their product. Have the student explain

461

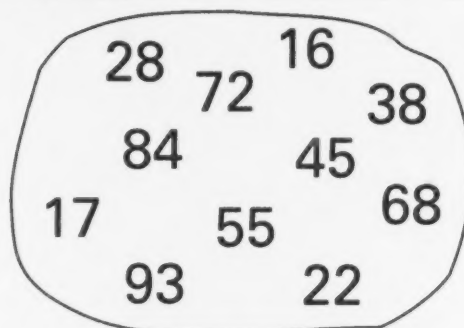
his/her estimation strategy. Then have all of the students work the problem on their calculators. Example:

Numbers Selected	Estimated answer	Calculator answer
84, 22	1,600	1,848

3. Discuss the relationship of the estimated answer and the calculator answer.
4. Select two other numbers from the group. Have each student record the numbers on his/her record sheet, record an estimated product, and record a calculator product. Discuss the results and the estimation strategies the students are using.
5. Have the students continue the activity, selecting pairs of numbers and completing their record sheets.
6. Say, "I want to choose two numbers from the group that, when multiplied, have a product that is close to 3,000. Which numbers should I choose?"
7. Allow the students to discuss which two numbers should be chosen and why. Have the students find the products of the numbers they chose. Discuss the results.
8. Give each pair of students a copy of the "Three in a Row" handout. Have the partners play the game, selecting numbers, multiplying them, and attempting to get three squares in a row.
9. The game can be played several times to reinforce students' estimation strategies.



Three in a Row



Directions: Working with a partner, take turns choosing two numbers from the set above, finding the product on the game board, and initialing or covering that square. The first player to get three in a row (vertically, horizontally, or diagonally) wins.

Game Board

2,736	720	3,780	6,324	1,224	1,496	1,581
880	1,540	1,488	1,344	1,904	2,046	1,156
7,812	272	2,584	3,960	374	1,210	646
3,740	5,712	1,584	448	5,115	3,534	2,090
616	2,604	935	1,088	1,428	836	2,016
1,260	476	1,152	6,696	4,620	1,710	990
2,475	608	4,185	3,192	1,848	3,240	2,352

Use with "Calculator Practice" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

403



Problem Solving

Shortcuts

Materials: None

Directions: 1. Have the students work a multiplication problem using the distributive property. Example:

$$3 \times 48 = 3 \times (40 + 8) = (3 \times 40) + (3 \times 8) = 120 + 24 = 144 \text{ or}$$

$$\begin{array}{r} 48 \\ \times 3 \\ \hline \end{array} = \begin{array}{r} 40 + 8 \\ \times 3 \\ \hline \end{array} = \begin{array}{r} 40 \\ \times 3 \\ \hline 120 \end{array} + \begin{array}{r} 8 \\ \times 3 \\ \hline 24 \end{array} = 144$$

2. Discuss the problem and the steps the students used to calculate the answer.
3. Have the students work the problem 8×24 using the distributive property but eliminating as many steps as they can.
4. After the students have worked the problem, have them discuss the processes they used, as well as the steps they eliminated.
5. Have the partners work together on the problem 5×47 . Have them discuss how they could solve the problem most easily, without using a calculator, and then have them use that method to solve the problem.
6. Have the partners form groups of four and discuss the strategies they are using to simplify these problems.
7. Have the groups of four students work the problem 6×56 and discuss the strategies they used.
8. Have each group write down their method and strategies as a set of directions to be shared with another group.
9. Have the groups exchange their directions and use the new set to solve the problem 8×55 .
10. Discuss the directions that each group followed and the similarities and differences among them.

How Close!

Materials: Calculators (one per student)

- Directions:
1. Write the number 3,717 on the chalkboard. Have the students use their calculators to find two 2-digit numbers whose product is close to 3,717.
 2. As the students come up with their 2-digit numbers, record some of the results on the board (e.g., $78 \times 50 = 3,900$). Ask, "Can someone get closer?" ($90 \times 41 = 3,690$) "Can someone get closer than that?"
 3. Allow the students to experiment for a day or several days to see who can get the actual number or the closest product.
 4. Discuss some of the strategies the students used.
Examples:
 - a. $78 \times 50 = 3,900$. This was too large, so I tried 78×44 . That equaled 3,432. This was too small, so I tried 78×47 . That equaled 3,666. . . .
 - b. I looked for two numbers that, when multiplied together, had a product that was close to 37. Since $4 \times 9 = 36$, I tried $4 \quad \times 9 \quad$. Since $7 \times 5 = 35$, I also tried $7 \quad \times 5 \quad$.
 - c. I just picked two numbers and multiplied them. Then I used larger or smaller numbers until I came close to 3,717. For example, I tried 81×49 . That came to 3,969. So I used smaller numbers next.
 5. Variation: Instead of using 3,717, use 247; 899; 1,517; 345; 4,982; 5,063; 2,706; 253; or 6,097.

465



Word Problems

Work space

1. The school play will be held in the gym in 3 days. The janitor has set up the chairs. There are 29 rows with 22 chairs in each row. How many people will be able to attend?
2. You are planning a hike. You want to give each person two bags of candy to take along as snacks. Seventy-two people are going on the hike, and you want each bag to contain 18 pieces of candy. How many pieces of candy will you need to buy?
3. Everyone in your class is going to keep a journal. There are 29 students in your class. Each journal will have 63 pages. How many total pages will be needed so that everyone will have a full set?
4. About how many students are in your school? The classes average 27 students and there are 23 classes.
5. The students at your school are attending a concert. Thirteen students rode in each of the 25 minibuses. How many students are attending the concert?
6. Ramen soup sells at 3 bags for 77¢. How much will 12 bags cost?

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Dividing two- and three-digit numbers by a one-digit number

Skill Analysis

- A. Understands the concept of multiplication
- B. Can multiply problems using the basic multiplication facts
- C. Understands the concept of place value
- D. Understands the concept of regrouping
- E. Understands the inverse relationship of multiplication and division
- F. Understands the concept of division
- G. Understands the symbols " $\overline{) }$ " and " $\div$ " and can use them to solve problems



Concept Manipulative Activities

Partitioning Sets

Materials: Base 10 materials (bean sticks, Base Ten Blocks, UNIFIX cubes, Popsicle sticks or coffee stirrers rubber-banded in sets of 10, etc.)

Place value mats made with 12" x 18" construction paper (one per student; directions for their construction may be found in the "Place Value Mat" activity in Section 3.6-2)

Hundreds	Tens	Ones

Small paper plates

- Directions:
1. Say, "Use your base 10 materials to show me a model for 24." (Students should make 2 tens and 4 ones.) Have the students share their model.
 2. Ask, "If you wanted to share the materials you used in your model with a friend so that you each had the same amount, how many tens would you each get?" (1) Say, "Place the tens on two different paper plates so that we can see what each person will get."
 3. Ask, "Do you have any tens left over?" (no) "How many ones do you have?" (4) "How many ones would each person get?" (2) Say, "Place the ones on the paper plates. Altogether, counting tens and ones, how much does each person get?" (1 ten and 2 ones, or 12)
 4. Have the students practice with several other problems involving 2- and 3-digit dividends with no regrouping. For each problem, ask the questions in steps 2 and 3, above.
 5. Have the students report how they were able to divide the materials by stating the steps and the questions asked.
 6. Give the students problems that involve remainders. Have the students discuss how these will be handled.
 7. Include problems with zeros in the dividend or quotient. By relating such problems to the concrete materials and the questions, the students will find these problems no more difficult than the others.
 8. Say, "Use your base 10 materials to show me a model for 56. I'm going to have you divide the materials you used in your model into four equal groups. You'll use your paper plates, as before, to show me these groups."
 9. Ask, "How many tens do you have?" (5) "How many tens will go on each plate?" (1) Have the students place the tens on the plates. Ask, "How many tens did you use?" (4) "How many do you have left?" (1) "What can you do with this 1 ten?" (Make a trade for 10 ones.)
 10. Have the students trade their 1 ten for 10 ones and combine them with the 6 ones on their place value mats.

11. Ask, "How many ones do you have now?" (16) "How many ones will go on each plate?" (4) Have the students place the ones on the plates. Ask, "How many ones did you use?" (16) "How many ones do you have left?" (0)
12. Repeat the activity with additional problems that involve regrouping from tens to ones and with problems that involve regrouping from hundreds to tens.
13. Have the students explain how they modeled the numbers and the processes they used to partition the sets.

Place Value Division—Without Regrouping

Materials: Place value mats made with 12" x 18" construction paper (one per student; directions for their construction may be found in the "Place Value Mat" activity in Section 3.6-2)

Hundreds	Tens	Ones

Base 10 materials
Small paper plates

- Directions:**
1. Say, "Imagine that 36 students are going on a field trip. Three minibuses will be used. Let's figure out how many students will be on each bus."
 2. Have the students build a model of the number 36 on their place value mats.

H	T	O

3. Write the problem on the chalkboard as follows:

$$\begin{array}{r} 3 \overline{) 3 \text{ tens } 6 \text{ ones}} \end{array}$$
4. Say, "Use your small paper plates to represent the minibuses. How many of the tens would go on each bus if

460

each bus is to have the same amount?" (1 ten) "Divide your tens evenly among the three minibuses (paper plates)."



5. Say, "To record the division, think, 'How many tens does each minibus have?'" (1) "How many tens did I use in all?" (3) "How many tens do I have left over?" (0) While the students are responding, record their work on the chalkboard.

$$\begin{array}{r}
 1 \text{ ten} \quad \leftarrow \text{each minibus} \\
 3 \overline{) 3 \text{ tens } 6 \text{ ones}} \\
 \underline{3 \text{ tens}} \quad \leftarrow \text{used} \\
 0 \text{ tens} \quad \leftarrow \text{left}
 \end{array}$$

6. Ask, "How many ones do you have on your place value mat?" (6) Say, "We will record that before we begin our division work."

$$\begin{array}{r}
 1 \text{ ten} \\
 3 \overline{) 3 \text{ tens } 6 \text{ ones}} \\
 \underline{3 \text{ tens}} \\
 0 \text{ tens } 6 \text{ ones}
 \end{array}$$

7. Say, "Divide your ones evenly among the three minibuses (paper plates). How many ones does each minibus have?" (2) "How many ones did you use in all?" (6) "How many ones are left over?" (0) While the students are responding, record their work on the chalkboard.



$$\begin{array}{r}
 1 \text{ ten } 2 \text{ ones} \quad \leftarrow \text{each minibus} \\
 3 \overline{) 3 \text{ tens } 6 \text{ ones}} \\
 \underline{3 \text{ tens}} \\
 0 \text{ tens } 6 \text{ ones} \\
 \underline{6 \text{ ones}} \quad \leftarrow \text{used} \\
 0 \text{ ones} \quad \leftarrow \text{left}
 \end{array}$$

470

8. Discuss the record-keeping process with the students. Be sure that they see the correlation between the written record and the manipulation of the materials.
9. Give the students several other division problems. Walk them through the modeling and record keeping. As the students become comfortable with the record-keeping process, they may model and record the problems on their own.
10. Repeat the activity with problems that include zeros in the dividend or quotient. By following the same questioning process as earlier, the students will find these problems no more difficult than the others.
11. Encourage the students to describe a story that corresponds to division problems you give them. (For $48 \div 4$, for example, a student may say, "Mary had 48 pencils that she wanted to give to four of her friends. How many pencils will each friend get?")

Place Value Division—With Regrouping

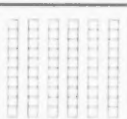
Materials: Place value mats made with 12" x 18" construction paper (one per student; directions for their construction may be found in the "Place Value Mat" activity in Section 3.6-2)

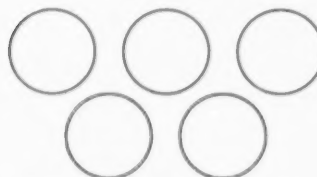
Hundreds	Tens	Ones

Base 10 materials
Small paper plates

- Directions:**
1. Say, "James has 65 keys that need to go on key rings. He has five key rings. Let's figure out how many keys will go on each ring."
 2. Have the students build a model for the number 65 on their place value mats. Explain that they will use their paper plates to represent the key rings.

471

H	T	O
		



3. Write the problem on the chalkboard as follows:
- $$\begin{array}{r} 5 \overline{) 6 \text{ tens } 5 \text{ ones}} \end{array}$$
4. Ask, "How many tens do you have on your place value mat?" (6) Say, "Divide the tens evenly onto the key rings (paper plates). How many tens does each ring get?" (1) "How many tens did you use in all?" (5) "How many tens are left over?" (1) While the students are responding to your questions, record their answers on the chalkboard.

$$\begin{array}{r} 1 \text{ ten} \\ 5 \overline{) 6 \text{ tens } 5 \text{ ones}} \\ \underline{5 \text{ tens}} \\ 1 \text{ ten} \end{array} \quad \begin{array}{l} \leftarrow \text{each key ring} \\ \leftarrow \text{used} \\ \leftarrow \text{left} \end{array}$$

5. Ask, "What should you do with the 1 ten that was left over?" (Trade it for 10 ones.) Have the students trade their 1 ten for 10 ones and combine them with the 5 ones on their place value mats. Record their trade on the chalkboard as follows:

$$\begin{array}{r} 1 \text{ ten} \\ 5 \overline{) 6 \text{ tens } 5 \text{ ones}} \\ \underline{5 \text{ tens}} \\ 1 \text{ ten } 5 \text{ ones} \\ 15 \text{ ones} \end{array}$$

6. Ask, "How many ones do you have now?" (15) Say, "Divide the ones evenly onto the key rings (paper plates). How many ones did you put on each key ring?" (3) "How many ones did you use in all?" (15) "How many ones are left?" (0) While the students are responding to your questions, record their work on the chalkboard.

$$\begin{array}{r} 1 \text{ ten } 3 \text{ ones} \\ 5 \overline{) 6 \text{ tens } 5 \text{ ones}} \\ \underline{5 \text{ tens}} \\ 1 \text{ ten } 5 \text{ ones} \\ 15 \text{ ones} \\ \underline{15 \text{ ones}} \\ 0 \text{ ones} \end{array}$$

7. Discuss the various parts of the record-keeping process with the students.
8. Give the students several other division problems. Walk them through the modeling and record keeping. As the students become comfortable with the record-keeping process, have them work in pairs taking turns being the "record keeper/questioner" and the "modeler."

Fold for Division

Materials: Lined paper
 Transparency of lined paper
 Overhead projector
 Vis-A-Vis pen
 Base 10 materials

- Directions:
1. Have the students turn a sheet of lined paper sideways so that the lines are vertical. Here, and throughout the activity, model what you want the students to do on the overhead.
 2. Have the students draw a division bar on their paper, leaving space for the quotient. (See Figure 1.)



Figure 1

3. Working from left to right, have the students write in the place value names and plus signs beneath the division bar, leaving spaces for numbers. (See Figure 2.) The students may abbreviate the place value names if they want (e.g., Hundreds = H, Tens = T, and Ones = O).

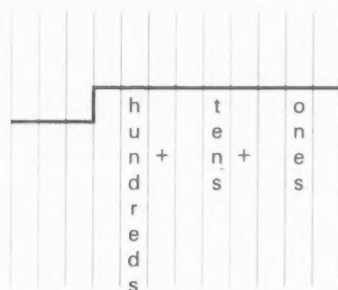


Figure 2

4. Have the students make a crease at the line to the left of each plus sign and at the line just after the word "ones." (See Figure 3.)

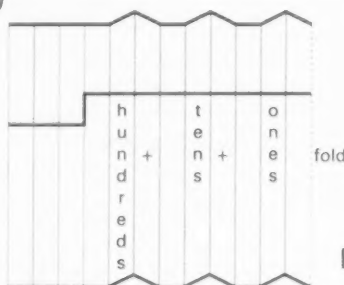


Figure 3

5. Have the students fill in the appropriate columns for the problem $447 \div 3$. (See Figure 4.)

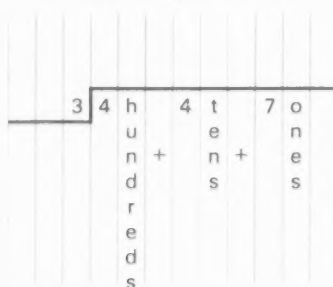


Figure 4

6. Go over the problem as follows:
- Say, "We have 4 hundreds to be divided into three equal groups. How many hundreds will each group get?" (1) "Write 1 in the number column for the hundreds answer. How many hundreds will we use?" (3) "Write 'minus 3' in the work space for hundreds. How many hundreds do we have left?" (1) "Write 1 as the difference of 4 and 3." (See Figure 5.)

474

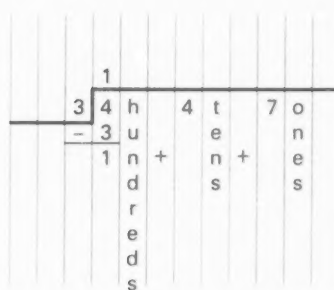


Figure 5

- b. Say, "We now have 1 hundred + 4 tens + 7 ones. We need to trade the 1 hundred for 10 tens and combine them with the 4 tens. Fold your paper to show 14 tens and 7 ones. The 4 is "too high," so we "bring it down" beside the 1 to make it easier to read." (See Figure 6.)

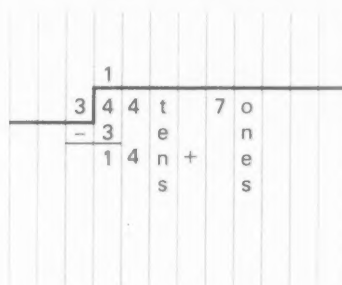


Figure 6

- c. Say, "We now have 14 tens to be divided into three equal groups. How many tens will each group get?" (4) "Write 4 in the number column for the tens answer. How many tens will we use?" (12) "Write 'minus 12' in the work space for tens. How many tens do we have left over?" (2) "Write 2 as the difference of 14 and 12." (See Figure 7.)

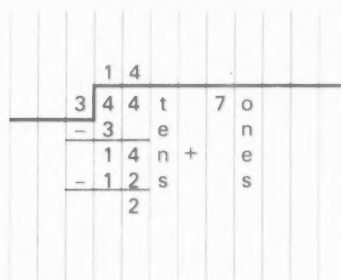


Figure 7

- d. Say, "We now have 2 tens + 7 ones. We need to trade the 2 tens for 20 ones and combine them with the 7 ones. Fold your paper to show 27 ones. The 7 is "too high" so we "bring it down" beside the 2 to make it easier to read." (See Figure 8.)

475

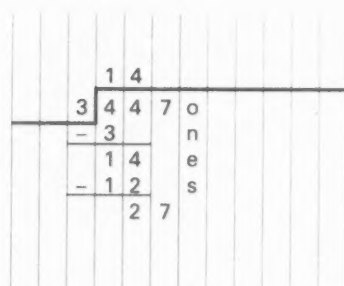


Figure 8

- e. Say, "We now have 27 ones to be divided into three equal groups. How many ones will each group get?" (9) "Write 9 in the number column for the ones answer. How many ones will we use?" (27) "Write 'minus 27' in the work space for ones. How many ones do we have left over?" (0) "Write 0 as the difference of 27 and 27." (See Figure 9.) Say, "Fold your paper to hide the word 'ones.' What you have showing is the standard way to write the problem and its answer."

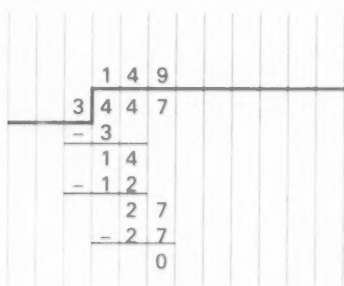


Figure 9

Vertical Dividing

Materials: UNIFIX cubes

- Directions:
1. Say, "I have 81 candy bars. If 3 candy bars fit in one bag, how many bags will I need to hold all of the candy bars? Let's work this problem together."
 2. Write the problem $3 \overline{)81}$ on the chalkboard. The students can use UNIFIX cubes to model the problem as you go over it.

3. Relate each step of the story to the algorithm as you present it. Point out the estimating being done. Explain that it is not necessary to select the largest multiple of the divisor in order to find the quotient.

Example:

$ \begin{array}{r} 7 \leftarrow \\ 10 \leftarrow \\ 10 \leftarrow \\ 3 \overline{)81} \\ \underline{-30} \rightarrow 10 \\ 51 \\ \underline{-30} \rightarrow 10 \\ 21 \\ \underline{-21} \rightarrow 7 \\ 00 \end{array} $	Let's start by filling 10 bags. That would use 30 candy bars. There are 51 left. Let's fill 10 more bags using 30 more candy bars. Now there are 21 left. They would fill 7 bags. There are 0 candy bars left. 27 bags were used.
---	---

4. Ask, "Could we have started by filling 20 bags?" (Yes, 20 bags would have used 60 candy bars.) "Could we have started by filling 30 bags?" (No, 30 bags would use 90 candy bars, and we have only 81.)
5. Give the students other division problems to solve in the same way. Include some that have remainders. For each problem, encourage the students to create a story that corresponds to the numbers.



Let's Practice

How Much?

Materials: Base 10 materials
Calculator

- Directions:
1. Work with 12-16 students.
 2. Divide the students into three groups.
 3. Write the problem $2\overline{)78}$ on the chalkboard.
 4. Give directions to Group 1, "The Modelers." Say, "You are responsible for constructing a model of this problem using

the base ten materials. You must also create a story that corresponds to the problem and your model. Be prepared to present your model and your story to the class."

5. Give directions to Group 2, "The Brainstormers." Say, "Without computing the answer, brainstorm everything you know about this problem. Record your ideas to share with the class." The students' list might include:
 - a. 78 is the dividend.
 - b. 78 is the whole.
 - c. 2 is the divisor.
 - d. We are dealing with groups of two, or two equal groups.
 - e. We can count by 2s to solve the problem.
 - f. The answer is called the quotient.
 - g. An estimated answer is 40.
 - h. The answer will not have a remainder.
6. Give instructions to Group 3, "The Computers." Say, "Compute the quotient using any method we have discussed. Be prepared to give your answer and explain your method to the class. You may use a calculator to check your answer."
7. Have each group complete its task and share its work with the class.
8. Give the class other problems. Rotate the tasks for each group so that everyone has an opportunity to complete each task.

Tic-Tac-Toe

Materials: Tic-Tac-Toe Game Board (one per pair of students; blackline master follows this activity)
Calculators (one per pair of students)

- Directions:**
1. Have each student work with a partner.
 2. Call out the numbers from the sample game board below, one at a time. Have the partners write the numbers randomly on their game board.

478

Tic-Tac-Toe			
63	102	84	126
249	76	57	118
59	177	136	83
114	166	51	38

3. Write the following lists on the chalkboard.

List A

354
408
228
504
498

List B

8
4
3
2
6

4. In each group, the first player should choose a three-digit dividend from List A and a one-digit divisor from List B. He/she should use a calculator to find the quotient (e.g., $504 \div 4$).

*TEACHER NOTE: This activity will reinforce the order of entering numbers into a calculator and the use of the $\div$ symbol.

- If the quotient appears on the game board, the student should cover it with his/her mark (X or O) or with a square of colored paper. If the quotient is not on the game board or is already covered, the student loses his/her turn.
- Play progresses with the students taking turns selecting numbers and solving the problems.
- The first player to cover four squares in a row, horizontally, vertically, or diagonally, wins.
- Reinforce with students how important it is to estimate the quotients before actually choosing their numbers. Discuss estimation strategies the students can use.



Tic-Tac-Toe Game Board

Tic-Tac-Toe			

Use with "Tic-Tac-Toe" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

459

Estimate and Divide

Materials: Variety of counters (buttons, beans, bread tags, cubes, etc.)
Six large containers for the counters
Small containers
"Estimate and Divide" handout (one per student; blackline master follows this activity)
Six sets of cards numbered 1-9
Timer

- Directions:**
1. Place a large container of counters at each of six stations. Use a different type and quantity of counters at each station.
 2. Put a set of number cards (face down) and four to five small containers at each station.
 3. Number each station with a large numeral.
 4. Give each student an "Estimate and Divide" handout. Divide the students into groups of four or five. Explain that each group will start at a different station, complete the task described, and move on to the next station when the timer goes off. Explain that you will set the timers for (5-8) minutes and that each group should go to every station.
 5. Place the following, or a similar, set of directions at each station:
 - a. On your handout, write the name of what you are counting in the first column beside the number of this station.
 - b. Take a handful of counters and place them in a small container.
 - c. Estimate how many counters are in your container. Write your estimate at the top of the box in the second column.
 - d. Count your objects. Write the actual number of counters **under** your prediction. Circle this number.
 - e. Draw a card from the deck.
 - f. Write a division problem in the third column using the **actual** number of counters as the dividend and the number on the card as the divisor.
 - g. Work the division problem in the space provided on your handout.

**Estimate and Divide**

What I Counted	My Prediction/Actual Count	Division Problem
1.		
2.		
3.		
4.		
5.		
6.		

Use with "Estimate and Divide" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Problem Solving

The Bigger, the Better

Materials: Nine 1" ceramic tiles or square pieces of paper numbered 1–9
(two sets)

Unlined paper (one sheet per team)

- Directions:**
1. This game is played by two teams of two students each.
 2. Place one set of numbered tiles face down at the center of the playing area between the teams. Place the other set of tiles, face up, off to the side as a “draw pile.”
 3. Have each team draw a division symbol ($\square$) on its paper that is large enough to use with the number tiles.
 4. Explain that the teams will be drawing tiles to use in a division problem of the form $\square \overline{\square \square \square}$. The general goal is to make a problem with the largest possible quotient.
 5. Team A, starts the game by drawing a tile from the center pile. Team B chooses the same tile from the “draw pile.” Each team decides where it wants to place this tile in the division problem. (A barrier can be placed between the teams to keep tile placement secret.)
 6. Next, team B draws a second tile from the center pile, and team A chooses the same tile from the “draw pile.” Each team places the second tile in the division problem.
 7. Draws continue until all four tiles have been placed.
 8. Each team then solves its division problem. The team with the largest quotient scores 1 point.
 9. Tiles are returned to the piles and the game continues.
 10. The team with the most points at the end of a set amount of time is the winner.
 11. Variation: Have the teams score points for producing the smallest possible quotient.

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

1. Ninety-six fourth grade students are going on a field trip to the planetarium. There are eight rows of seats at the planetarium. How many students will sit in each row?
2. The Court Club is having a tennis tournament for its 16 members. The club will need to buy 74 balls for the tournament. If the balls are packaged 3 in a can, how many cans will the club need to buy?
3. Six people delivered advertising flyers to a neighborhood for a day. If the group was paid \$246 and each person received the same amount, how much did each person earn?
4. A seven-member Boy Scout troop stopped at a restaurant. Each member ordered three items (a hamburger, french fries, and a drink). If the total bill was \$21.98, how much did each member owe?
5. The Booster Club had a candy sale. It divided the selling area into nine regions. If the members sold 432 cases of candy with an equal number sold in each region, how many cases were sold in each region?

If the sale lasted 7 days, about how much candy was sold each day?



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

484

Division Race

Materials: "Division Race" handout (blackline master follows this activity)

Calculators (one per team)

- Directions:
1. This game is played by two teams.
 2. Each team chooses two numbers from the number box at the top of the handout.
 3. The teams use the calculators to divide the two numbers. The team members decide which number will be the dividend and which will be the divisor.
 4. If a team's quotient is on the game board, the team puts its mark (X or O) on that space.
 5. The first team to get a path of answers connecting its two sides of the game board wins.
 6. Emphasize the importance of estimating in this game.



Division Race

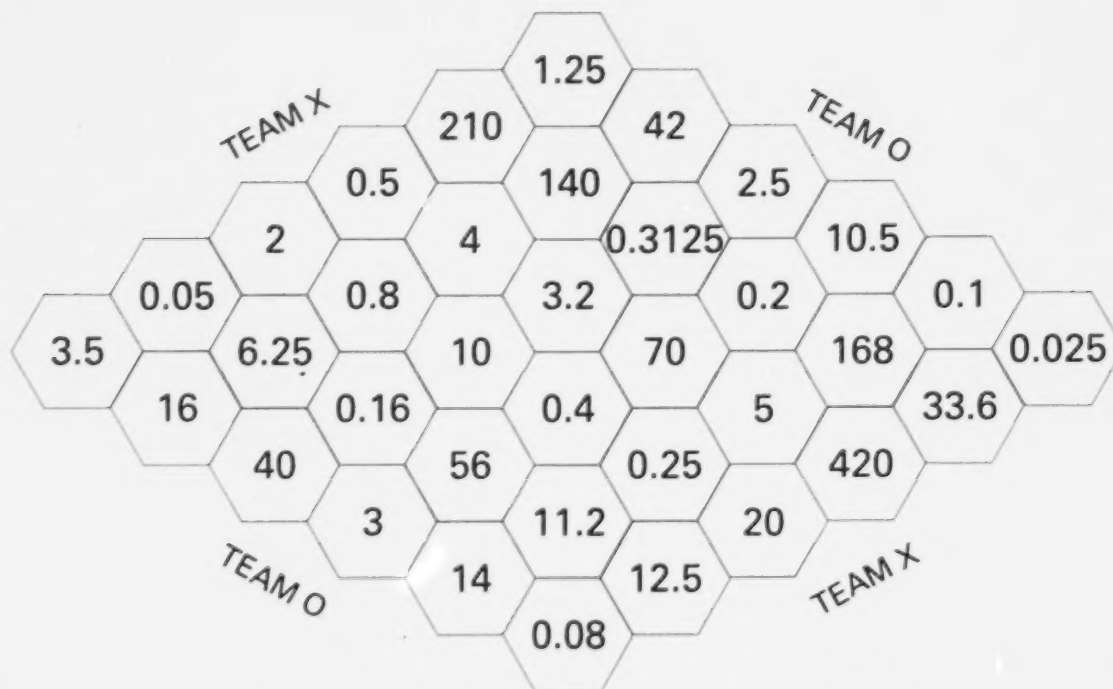
Team Members:

Team O:

Team X:

Number Box

2	5	25	280
4	20	80	840



Use with "Division Race" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Multiplying four-digit numbers by two-digit numbers***Skill Analysis***

- A. Understands the concept of multiplication
- B. Understands the concept of place value
- C. Recalls the basic multiplication facts
- D. Can multiply two-digit numbers by two-digit numbers

**Concept Manipulative Activities*****Block Models***

Materials: Base Ten Blocks

- Directions:
1. Distribute Base Ten Blocks to each student.
 2. Have the students represent the number 124 with their blocks.
 3. Ask, "What would you do to show two times this amount?" If students need more blocks, have them combine blocks with other students to show the answer.
 4. Have the students combine their blocks and record the answer.
 5. Ask the students to separate their blocks and then have each student model 124 again.
 6. Say, "Now I'd like you to combine your blocks to model 3×124 ." After they do so, have the students discuss their model and any necessary trades that were made.
 7. Have the students record their model, the trading, and the final answer on paper. Discuss the record-keeping process.

8. Repeat the activity with other combinations of students' blocks including problems with two-digit multipliers (e.g., 11×124).
9. For practice, have the students model and record various other problems (e.g., 5×247 ; 7×389 ; 6×561 ; 12×230 ; 21×177).

Graph Models

Materials: 1/4" graph paper (blackline master may be found in the Appendix)

- Directions:**
1. Have the students build graph paper models (by shading in squares on the graph paper) for multiplication problems in which one of the factors is a multiple of 10 or 100. For each problem, have the students determine the largest place value of the **answer** (e.g., $5 \times 50 = 100\text{s place}$, $6 \times 70 = 100\text{s place}$, $8 \times 500 = 1,000\text{s place}$).
 2. Progress into two-digit times three-digit problems (e.g., 20×400). Discuss with students the applicability of this concept and how its use can assist in determining the reasonableness of the products of larger multiplication problems.

**TEACHER NOTE: This modeling activity becomes cumbersome quickly, so discussion of large quantities should be ongoing.*

3. Ask students to shade 100 squares on their graph paper. Then ask them what 1,000 shaded squares would look like.
4. Working in pairs, have students shade 1,000 squares. Encourage students to decide the best way to model 1,000.
5. Discuss how the class could show 10,000, 20,000, etc.

Zero Pattern Multiplication

Materials: Calculators (one per student)

- Directions:**
1. Write problems on the chalkboard in which one or both factors are multiples of 10 or 100 (e.g., 5×10 ; 100×37 ; $40 \times 1,000$; $30 \times 5,000$).

2. Using calculators, have the students compute the answers to the problems.
3. In groups of three or four, have the students generate similar problems, compute their answers using a calculator, and write down the answers.
4. Looking at the problems and answers, have the students discuss any patterns discovered.
5. Have the students record their problems horizontally and vertically. Discuss **how** the problems should be recorded vertically. Ask, "Does the way problems are recorded vertically **really** make a difference?"
6. Have each student think about the ways problems can be written and share with the class their thoughts on how problems are best recorded.

Estimating

Materials: Calculators (one per student)

- Directions:**
1. Say, "Suppose you didn't have paper, pencil, or a calculator and you needed to solve a multiplication problem. How could you solve it?" The students may have many suggestions. Discuss their suggestions, emphasizing estimation.
 2. Say, "Pretend that the school is going to have a pizza dinner for students, parents, and staff. The staff determined that 288 pizzas will be needed, and they cost \$12 each. What is the estimated cost of the pizzas?"
 3. Have the students work in pairs to decide on an estimated cost. Have different groups share their estimated amounts and the procedures they used. The answers will vary, but they should be reasonable.
 4. Have the students write similar situations for the class to solve using estimation. After determining an estimate, the students should use calculators to check for the reasonableness of their results.

More Graph Models

Materials: 1/4" graph paper (blackline master may be found in the Appendix)

Optional: Calculators

- Directions:
1. Have the students shade 125 squares on their graph paper.
 2. Ask, "Without shading any more squares on your graph paper, how could you model 4×125 ?"
 3. Have the students discuss how they could model this problem.
 4. Next, discuss the different ways this problem could be recorded. Have a student volunteer record the problem on the board as "repeated addition" and another student record it using the "standard algorithm." Example:

$$\begin{array}{r}
 \overset{12}{125} \quad \overset{12}{125} \\
 125 \quad \times 4 \\
 125 \quad \hline
 +125 \\
 \hline
 500
 \end{array}$$

5. Have the class compare the two records and discuss their similarities and differences.
6. Repeat the activity with other problems, such as 6×125 , 7×125 , and 9×125 .
7. Have the students record the repeated addition problem for 10×125 on a piece of paper. Discuss how the final answer relates to the standard algorithm record for the same problem.
8. Have 11 students put their graph paper models for the number 125 together to show 11×125 . Discuss with students different ways to solve this problem. For example, since $11 = 6 + 5$, we could multiply 6×125 and 5×125 and then add the answers together.
9. Discuss with the students which of the possible models would be easiest to use. Allow the students to generate the idea of using multiples of ten (i.e., $10 \times 125 + 1 \times 125$).

490

10. Have the students divide their model of 11×125 into 10 groups of 125 and 1 group of 125. Record the multiplication problem on the board using the distributive property. Example:

$$\begin{array}{r} 125 \\ \times 11 \\ \hline \end{array} \quad \rightarrow \quad \begin{array}{r} 125 \\ \times 10 \\ \hline 1,250 \end{array} + \begin{array}{r} 125 \\ \times 1 \\ \hline 125 \end{array} = 1,375$$

11. Discuss how this problem can be "shortened" by using the standard algorithm.

$$\begin{array}{r} 125 \\ \times 11 \\ \hline 125 \rightarrow (1 \times 125) \\ 1,250 \rightarrow (10 \times 125) \\ \hline 1,375 \end{array}$$

12. Repeat the activity with other problems involving the students' graph paper models for 125. The final problem involving the models for 125 should consist of combining all of the students' models and obtaining the product.
13. You might want to give the students practice working with larger numbers. If their model for a product becomes too cumbersome, have them use a combination of models already constructed. For example, they could use the model for 11×125 (1,375) to compute $15 \times 1,375$.



Let's Practice

Estimate the Place Value

Materials: Calculators (one per student)

- Directions: 1. Write a multiplication problem on the board. Use blanks to represent the **possible** place value of the answer. In the following example, the number of blanks is intentionally wrong:

491

$$\begin{array}{r} 22 \\ \times 14 \\ \hline \end{array}$$

2. Have the students copy the problem onto paper and estimate the number of places needed for the answer. Have them circle what they estimate is the proper number of places.

$$\begin{array}{r} 22 \\ \times 14 \\ \hline \end{array}$$

3. Next, have the students work the problem on their calculators and insert the answer in the blank spaces.
Example:

$$\begin{array}{r} 22 \\ \times 14 \\ \hline 308 \end{array}$$

4. If the students' estimates were correct, the answer should be within the circled region with all the spaces used.
5. Have the students work in pairs to generate similar problems for their partners to solve.

Telephone Directory Count

Materials: Old telephone directories
Scissors

- Directions:**
1. Have each student carefully rip a page out of an old telephone directory and cut apart the columns of the page. Then have the students fold each column into fourths.
 2. From the information given in one fourth of a column, have the students determine the greatest number of listings and the least number of listings on the page.
 3. Using old telephone directories, have the students determine approximately how many people are listed in one alphabetical section.

4. Have them determine approximately how many people are listed in the white pages.

Heartbeat

Materials: Calculators (one per student)

- Directions:**
1. Have the students take their pulse for 1 minute to determine the number of times their hearts beat per minute.
 2. Challenge the students to use this information to determine the number of times their hearts beat per hour, day, year, in their lifetime.

Calculated Estimation

Materials: Calculator

Twenty cards on which you have written four-digit times two-digit problems, one problem per card

- Directions:**
1. This game is played by two students.
 2. Player A draws a card, estimates the answer, and explains his/her method of estimation.
 3. Player B works the problem on the calculator.
 4. If the estimated answer is reasonably close to the exact answer, player A keeps the card. If it is not reasonably close, player A places the card at the bottom of the deck.
 5. Play continues with players exchanging roles after each problem.
 6. The winner is the player with the most cards when the entire deck has been used.
 7. Variation: Have the students move one space forward on a Universal Game Board for each correct answer.

Multiplication Assembly Lines

Materials: Calculators (one per group of students)

- Directions:
1. The students work in groups of four and sit in a circle.
 2. Each student writes a four-digit times two-digit problem on his/her paper, estimates the answer for the problem, and writes the estimate to the right of the problem.
 3. The students pass their papers to the student on their right and complete the first step of the problem they've been given.
 4. The papers are passed to the right again, and the students complete the second step of the problem they've been given.
 5. The papers are passed to the right a third time, and the students calculate the answer to the problem.
 6. The papers are passed to the right a fourth time, returning them to their owner. He/she uses a calculator to determine the correct answer to the problem and records this number under the problem. Example:

$$\begin{array}{r}
 2,308\text{—Student 1} \quad \boxed{100,000} \text{ Estimate} \\
 \times 45 \\
 \hline
 11,540\text{—Student 2} \\
 92,320\text{—Student 3} \\
 \hline
 103,860\text{—Student 4} \\
 \text{Calculator } \boxed{103,860}
 \end{array}$$

7. If the calculator answer does not agree with the "student answer," the students should pass the paper around the circle again and each student should use a calculator for his/her section of the problem.
8. Variation: The students can determine a score for each round by calculating the difference between the final answer for their problem and the answer they estimated. The student with the closest estimate for each round would score one point. The student with the most points when time is called would be the winner. Alternatively, the students could keep a running total of the differences for their problems. In this variation, the student with the **lowest** sum when time is called would be the winner.



Problem Solving

Product Paces

Materials: Stopwatches or a clock with a second hand
“Paces” handout (one per student; blackline master follows this activity)
Meter stick or measuring tape
Calculators (one per group of students)

- Directions:**
1. Explain to the students that a “pace” is one normal stride or step and that pace size varies from person to person.
 2. Have the students work in groups of two.
 3. Have each student count the number of paces he/she takes in 10 seconds while his/her partner measures the time. Be sure the students walk at a leisurely rate. (Most people take between 10 and 20 paces in 10 seconds.)
 4. Have each student complete the “Paces” handout.
 5. When the students are finished, have them discuss their results.



Paces

Directions: A pace is a single step. To count your paces in 10 seconds, walk at your normal rate while a friend measures the time. Count every time either foot hits the ground.

I can walk _____ paces in 10 seconds.

Use a calculator to help you answer the following.

- How many paces could you walk in 1 minute? _____
In 30 minutes? _____ In 1 hour? _____ In 24 hours? _____
- Select three different distances to walk between two given points. Measure the distances in paces. After pacing off the distances, estimate the time it would take you to walk the distances. Check your estimates.

		Paces	Time Estimate
_____ to _____	_____	_____	_____
_____ to _____	_____	_____	_____
_____ to _____	_____	_____	_____

- Explain how you could determine your average pace distance (in centimeters).

- Compute the distance of your average pace in centimeters. _____
- Calculate the distances you selected in #2 in centimeters based on the number of paces you wrote down.

Location 1 _____ Location 2 _____ Location 3 _____

Use with "Product Paces" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Calculator Products

Materials: Calculators (one per pair of students)

Directions: 1. Have each student work with a partner on all or some of the following activities.

2. **Largest product:** What is the largest product possible when multiplying: (1) a two-digit number times a two-digit number? ($99 \times 99 = 9,801$); (2) a three-digit times a two-digit number? ($999 \times 99 = 98,901$); (3) a four-digit number times a two-digit number? ($9,999 \times 99 = 989,901$). What is the smallest possible product in each case? What numbers in each case can be multiplied to get closest to 1,000; 2,000; 10,000; etc.
3. **Product estimation:** Give the students a product (e.g., 2,200). Have one student enter a possible factor on the calculator. Have the second student enter the multiplication function, the second factor, and the equals sign. The student chooses a factor that he/she feels will, when multiplied with the first factor, come close to the product. The activity is repeated with player 2 entering different numbers until the calculator answer is very close to the product. Example:

Player 1 enters "35" and player 2 enters " $\times 80 =$." (2,800—Too high)

Player 1 enters "35" and player 2 enters " $\times 60 =$." (2,100—Too low)

Player 1 enters "35" and player 2 enters " $\times 62 =$." (2,170—Closer)

Player 1 enters "35" and player 2 enters " $\times 63 =$." (2,205—Closest)

Have the students continue play by reversing roles and choosing their own target numbers. Have the students attempt to complete each problem in the fewest number of turns.

4. **Number pairs:** Make a list of four-number series (e.g., 52, 53, 54, 55) on the board. Have each pair of students write down one of the series and predict how close any two products of numbers in that series will be (e.g., 52×53 and 54×55 or 52×55 and 53×54 or 52×54 and 53×55). Have the students try at least five different four-number series.

Ask them what patterns they discover and if these patterns would “work” for all four-number series.

**P
A
T
T
E
R
N
S**

Directions: Work with a partner. Solve a few problems in each set using a calculator. Find a pattern for each set and fill in the remaining answers using that pattern. Then add to the problems, following the same pattern, and fill in the answers. Be prepared to discuss the patterns with the class.

1.

$$37 \times 101 = \underline{\hspace{2cm}}$$

$$46 \times 101 = \underline{\hspace{2cm}}$$

$$53 \times 101 = \underline{\hspace{2cm}}$$

$$79 \times 101 = \underline{\hspace{2cm}}$$

$$97 \times 101 = \underline{\hspace{2cm}}$$

$$\underline{\hspace{1cm}} \times 101 = \underline{\hspace{2cm}}$$

$$\underline{\hspace{1cm}} \times 101 = \underline{\hspace{2cm}}$$

$$\underline{\hspace{1cm}} \times 101 = \underline{\hspace{2cm}}$$

2.

$$37 \times 3 = \underline{\hspace{2cm}}$$

$$37 \times 6 = \underline{\hspace{2cm}}$$

$$37 \times 9 = \underline{\hspace{2cm}}$$

$$37 \times 12 = \underline{\hspace{2cm}}$$

$$37 \times 15 = \underline{\hspace{2cm}}$$

$$37 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$37 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$37 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

3.

$$9 \times 2,222 = \underline{\hspace{2cm}}$$

$$9 \times 3,333 = \underline{\hspace{2cm}}$$

$$9 \times 4,444 = \underline{\hspace{2cm}}$$

$$9 \times 5,555 = \underline{\hspace{2cm}}$$

$$9 \times 6,666 = \underline{\hspace{2cm}}$$

$$9 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$9 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$9 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

4.

$$6 \times 7 = \underline{\hspace{2cm}}$$

$$66 \times 67 = \underline{\hspace{2cm}}$$

$$666 \times 667 = \underline{\hspace{2cm}}$$

$$6,666 \times 6,667 = \underline{\hspace{2cm}}$$

$$66,666 \times 66,667 = \underline{\hspace{2cm}}$$

$$666,666 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$6,666,666 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$

$$66,666,666 \times \underline{\hspace{1cm}} = \underline{\hspace{2cm}}$$



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

**S
T
O
R
Y**

**P
R
O
B
L
E
M
S**

Directions: Discuss and solve each of the following problems with a partner. Use the work space provided to the right of the problems. Use a calculator to check your work.

Work Space

1. How many baskets of berries are there in 30 full crates if each crate holds 48 baskets?
2. On the first day of school, 22 students in Mrs. Smith's class brought packages of 100 sheets of notebook paper. The rest of the students in the class brought packages of 500 sheets of paper. How many total sheets of paper did the students bring if there were 30 students in the class?
3. The market had a sale on oranges. In just two days it was able to sell 31 dozen oranges. How many total oranges did it sell?
4. A daughter is 12 years old. Her mother is three times as old. What is the product of their two ages?
5. Each of the Scout camp's vans holds 14 students and one driver. If 11 full vans were used to transport the Scouts to camp, how many people were in the vans?



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

500

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

Work Space

6. Mary read a book that was 99 pages long. There were about 28 lines per page and about 15 words per line. About how many words were in the book?
7. Bill earned \$270 at his summer job. He spent \$120. If he continued at this rate, how many summers would it take him to save \$600?
8. A truck is loaded with 56 boxes. Each box weighs 111 pounds. When the truck is empty it weighs 4,000 pounds. It is approaching a bridge with a weight limit of 12,000 pounds. Can the truck safely cross the bridge? Explain your answer.
9. Jill delivers a weekly newspaper to 31 customers. Her profit is 7¢ per copy. How much will she earn in a year?
10. If the average 11-year-old student spends 7 hours per day at school, how many hours will he/she spend at school in a 184-day school year?



Missing Numbers

Materials: "Find the Missing Digit" handout (one per student; blackline master follows this activity)

Calculators (one per student)

1/2" graph paper (blackline master may be found in the Appendix)

- Directions:
1. Distribute copies of the "Find the Missing Digit" handout and calculators. Have the students complete the handout.
 2. When the students are finished, discuss the answers with the group. Ask, "What strategies did you use to help you find the missing digits?"
 3. Have the students write six of their own two-digit times four-digit problems on graph paper (one number per square) in pencil.
 4. Have them solve the problems showing their steps. They may use a calculator to check the final answers.
 5. When they are finished, have them erase selected digits of the problems and answers and ask a partner to complete their worksheet. (Encourage the students to not erase too many numbers.)
 6. Discuss the strategies and procedures the students used to recreate the problems and answers.



Find the Missing Digit

Use a calculator to help you.

$$\begin{array}{r} 1. \quad \quad \quad 2 \_ 0 \\ \times \quad \quad \quad \_ \\ \hline \quad \quad 1,80 \_ \end{array}$$

$$\begin{array}{r} 2. \quad \quad \quad 7 \_ 2 \_ \\ \times \quad \quad \quad \_ 2 \_ 6 \\ \hline \quad \quad 42126 \\ \quad 140 \_ 2 \\ \hline 1 \_ 2,546 \end{array}$$

$$\begin{array}{r} 3. \quad \quad \quad 628 \\ \times \quad \quad \quad 4 \_ \\ \hline \quad \_ \_ \_ 2 \\ \quad \_ \_ \_ 2 \\ \hline 30,7 \_ \_ \end{array}$$

$$\begin{array}{r} 4. \quad \quad \quad 6331 \\ \times \quad \quad \quad 8 \_ \\ \hline \quad \quad 5064 \_ \\ \quad \quad 64 \_ \\ \hline 55,7128 \end{array}$$

$$\begin{array}{r} 5. \quad \quad \quad \_ \_ \\ \times \quad \quad \quad 24 \\ \hline \quad \_ \_ 4 \\ \quad \_ \_ 2 \\ \hline 62 \_ \end{array}$$

$$\begin{array}{r} 6. \quad \quad \quad 509 \\ \times \quad \quad \quad \_ \_ \\ \hline \quad \quad 2545 \\ \quad \quad 3563 \\ \hline \_ \_ \_ , \_ \_ \_ \end{array}$$

Answers: Cover before copying.

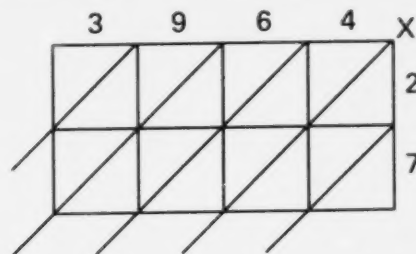
1. 200	2. 7,021	3. 628	4. 6331	5. 26	6. 509
$\begin{array}{r} \times 9 \\ \hline 1,800 \end{array}$	$\begin{array}{r} \times 26 \\ \hline 42,126 \\ 14,042 \\ \hline 182,546 \end{array}$	$\begin{array}{r} \times 49 \\ \hline 5,652 \\ 2,512 \\ \hline 30,772 \end{array}$	$\begin{array}{r} \times 88 \\ \hline 50,648 \\ 50,648 \\ \hline 557,128 \end{array}$	$\begin{array}{r} \times 24 \\ \hline 104 \\ 52 \\ \hline 624 \end{array}$	$\begin{array}{r} \times 75 \\ \hline 2,545 \\ 3,563 \\ \hline 38,175 \end{array}$

Use with "Missing Numbers" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

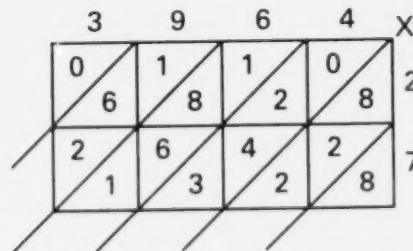
"Lattice" Multiply

Materials: Colored chalk or overhead projector, blank transparency, and Vis-A-Vis pens

- Directions:**
1. Discuss the definition of "lattice" with the students. Ask if they have seen any examples of latticework.
 2. Say, "A lattice can also be used to help us multiply." Draw a four-digit times two-digit lattice on the board or overhead. Draw the diagonals with a different color of chalk or ink than the vertical and horizontal lines. Example:



3. Explain that the product for each part of the multiplication problem is written in the box where the horizontal and vertical digits intersect and that the diagonals divide the ones and tens places. If a product contains a numeral in the ones place only, a zero is written in the space for the tens place. Example:



594

4. Explain that the next step is to add the numbers within each of the diagonal sections and place the answer for that diagonal "outside" the lattice. Demonstrate how to regroup into the next diagonal when regrouping is necessary. Example:

	3	9	6	4	X
	0 ¹	1 ¹	1 ¹	0 ¹	
	6	8	2	8	2
1	2	6	4	2	
	1	3	2	8	7
0	7	0	2	8	

5. Explain that the product is read from the top left down to the bottom right. Therefore, for the problem $27 \times 3,964$, the product is 107,028.
6. Have the students generate their own two-digit times four-digit multiplication problems and solve them using the lattice method.

SKILL Dividing two- to four-digit numbers by one-digit numbers***Skill Analysis***

- A. Understands the concept of division
- B. Knows the basic multiplication and division facts
- C. Can multiply one-digit numbers by four-digit numbers
- D. Understands the concept of place value and the use of zero as a place holder
- E. Understands the terms "quotient," "divisor," and "dividend"

**Concept Manipulative Activities*****Modeling Division—No Regrouping***

Materials: Base 10 materials
Nine containers (e.g., margarine tubs) per pair of students

- Directions:**
1. Have each student work with a partner.
 2. Give each pair of students 4 tens and 8 ones from the base 10 materials.
 3. Say, "You and your partner are going to share the materials I've given you. Place two containers between you to hold what each of you will receive."
 4. Ask, "How many tens do you have altogether?" (4)
 5. Say, "Give each person the same amount of tens by placing them in the containers."
 6. Ask, "How many tens did each person receive?" (2) "How many tens did you use?" (4) "How many tens are left?" (0)

7. Ask, "How many ones do you have altogether?" (8)
8. Say, "Give each person the same amount of ones by placing them in the containers."
9. Ask, "How many ones did each person receive?" (4) "How many ones did you use?" (8) "How many ones are left?" (0)
10. Continue with other examples of problems with two- to four-digit dividends and one-digit divisors that do not require regrouping. Include some problems with zero(s) in the dividend and/or quotient as well as problems with remainders. By modeling these problems like the others, the students will not see them as special cases.
11. For each problem, have the students build the model using the containers and base 10 materials and answer the questions orally. Do **not** introduce the algorithm at this time.

Modeling Division—With Regrouping

Materials: Base 10 materials
Nine containers (e.g., margarine tubs) per pair of students

- Directions:**
1. Have each student work with a partner.
 2. Give each pair of students 3 tens and 2 ones from the base 10 materials.
 3. Say, "You and your partner are going to share the materials I've given you. Place two containers between you to hold what each of you will receive."
 4. Ask, "How many tens do you have altogether?" (3)
 5. Say, "Give each person the same amount of tens by placing them in the containers."
 6. Ask, "How many tens did each person receive?" (1) "How many tens did you use?" (2) "How many tens are left?" (1)
 7. Ask, "What can we do with this left-over ten?" (Trade it for 10 ones.) Have the students make the trade.
 8. Ask, "How many ones do you have now?" (12)

9. Say, "Give each person the same amount of ones by placing them in the containers."
10. Ask, "How many ones did each person receive?" (6) "How many ones did you use?" (12) "How many ones are left?" (0)
11. Continue with other examples of problems with two- to four-digit dividends and one-digit divisors that require regrouping. Include some problems with zero(s) in the dividend and/or quotient as well as problems with remainders. By modeling these problems like the others, the students will not see them as special cases but as problems that can be solved following the same questioning strategies.
12. For each problem, have the students build the model using the containers and base 10 materials and answer the questions. Do **not** introduce the algorithm at this time.
13. You may want to have the partners work as a team, where one student is the "questioner" and the other student is the "modeler." The students should exchange roles after each problem so that they become comfortable with both the modeling and the questioning.

Division With Record Keeping

Materials: Base 10 materials
 Nine containers (e.g., margarine tubs) per pair of students
 "Transition Steps for Division" handout (one per student; blackline master follows this activity)
 Overhead projector, transparency of "Transition Steps for Division," blank transparency, and Vis-A-Vis pen

- Directions:**
1. Have each student work with a partner.
 2. Give each pair of students 2 tens and 4 ones from the base 10 materials.
 3. Have the partners divide the base 10 materials into two equal sets as you ask them the questions used in the "Modeling Division—No Regrouping" and "Modeling Division—With Regrouping" activities.

4. As the students respond to the questions you ask, record their answers on the chalkboard or overhead within the division problem, as shown on the "Transition Steps for Division" handout. The completed record will replicate the "standard" division algorithm.
5. Repeat the activity with several other problems. As before, record each problem on the chalkboard or overhead while the students model it.
6. Write a completed division problem on the chalkboard. Have the students describe what would happen with the model for each step of the algorithm. (If needed, the students may use manipulatives to model the problem.)
7. Distribute copies of the "Transition Steps for Division" handout and display the transparency on the overhead. Discuss the steps with the students.
8. Have the students work with their partners to solve the problem $3\overline{)96}$. Have one student be the "modeler" and the other student the "record keeper."
9. After the students complete the problem, have one pair of students explain their model and record.
10. Give the students other problems with two- to four-digit dividends and one-digit divisors to model and record in the same way. Include some problems with zero(s) in the dividend or quotient as well as problems with remainders. Have the partners exchange roles after each problem.
11. You may want to have the students work in groups of four (two sets of partners). After both sets of partners complete each problem, the group can compare and discuss their work.



Transition Steps for Division

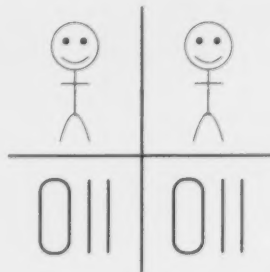
★ Give out tens!

1. How many tens for each? → 1 ten 2 ones ←
2. How many tens were used? → 2 | 2 tens 4 ones
3. How many tens are left? → 2
4. How many ones altogether? → 0 4

★ Give out ones!

5. How many ones for each? → 4
6. How many ones were used? → 0
7. How many ones are left?

Example:



Remember: This example assumes two people and 24 items to be divided, and it answers the question, "How many items will each person receive?"

Use with "Division With Record Keeping" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Division Trade

Materials: Chip Trading Kits
Place value mats

- Directions:
1. Have the students work in groups of four. Give each group nine place value mats.
 2. Explain that, when modeling division problems with Chip Trading Kits, you use chips to represent the dividend and divide them evenly among the number of place value mats that represents the divisor. Explain that blue chips represent tens and yellow chips represent ones.
 3. Say, "For practice, let's model the problem $96 \div 3$. Everyone will need 3 place value mats, 9 blue chips, and 6 yellow chips. How would you divide the chips evenly among the mats?" (Put 3 blue chips and 2 yellow chips on each mat.)
 4. Give the students other problems to model that have two- to four-digit dividends and one-digit divisors. Select problems that do not require regrouping. However, include problems with remainders as well as problems with zero(s) in the dividend and/or quotient.
 5. Next, give the students the problem $78 \div 3$. Discuss how chips are traded with Chip Trading Kits (e.g., 1 blue chip = 10 yellow chips).
 6. Have the students model the problem. They should divide the chips evenly among 3 place value mats. When they are finished, each mat should have 2 blue chips and 6 yellow chips. (One blue chip was traded for 10 yellow chips making 18 yellow chips in all.)
 7. Give the students other problems to model that have two- to four-digit dividends and one-digit divisors and involve regrouping.

8. Following the steps outlined in the "Division With Record Keeping" activity, have the groups record their work. One or two students can be the "modelers" while the other students are the "record keepers." Roles can be exchanged after each problem.

*TEACHER NOTE: Chip trading involves a more abstract concept than the other base 10 materials because of the trading factor of **1** object equaling **10** (e.g., 1 blue chip equals 10 yellow chips).

Estimation in Division

Materials: Calculators (one per student)

- Directions:
1. Discuss with the students **why** we estimate.
 2. Then discuss **when** estimation is used with division (e.g., when ordering pizzas for the soccer team).
 3. Solicit ideas from the students about **who** estimates division problems in their jobs (e.g., truck and bus drivers, painters, window washers, custodians, real estate agents, teachers).
 4. Discuss strategies for estimating division problems. After the students have come up with some ideas, say, "One way to estimate a division problem involves front-end rounding. In the problem $8 \overline{)1,784}$, for example, we would 'front-end round' 1,784 to 1,700. Why would we use 1,700 instead of 1,000?"
 5. Allow the students to discuss the reasonableness of an answer if we round too far away.
 6. Say, "How many digits would be in the quotient of our problem?" (It must be a three-digit answer.)
 7. Ask, "How can you tell?" Allow the students to justify their answer.
 8. Say, "We are only going to start the division problem. Remember, all we want is an estimate."
 9. Ask, "How many eights are in 17?" (2)

10. Say, "Record the 2 where it belongs in the quotient. Without doing any more division, what is an estimate for this problem?" (200) "Where did you get the zeros?"
11. Say, "Two hundred is our estimate for this problem. Do you think the actual answer will be greater or less than 200? Discuss your thoughts with a partner."
12. Have the students share their conclusions and justifications.
13. Have the students use a calculator to find the exact answer to the problem $8\overline{)1,784}$.
14. Discuss the result and its proximity to their estimate of 200.



Let's Practice

Home Base

Materials: Home Base Game Board (blackline master follows this activity)

Game markers

Deck of cards or small squares of tagboard containing the numerals 2 through 9 (one numeral per card)

Optional: Math Explorer calculator

- Directions:
1. This game is played by two to four students.
 2. Each player starts with his/her marker in the lower right-hand corner of the game board.
 3. In turn, each player draws a numeral card, which becomes the divisor for the problem the student will solve. The dividend is the number on which his/her marker is resting.
 4. The player divides the problem and states the remainder.
 5. The player moves his/her marker that many spaces clockwise on the game board.
 6. Play continues until one player is able to move on to "Home Base" with the exact number.

7. Variation: Have the students use Math Explorer calculators to solve the problems. By pressing the "Int $\div$ " key on the Math Explorer calculator, the quotient to a division problem is stated in terms of the quotient and a remainder.



Home Base

14	37	86	41	24
63	70	44	89	72
95	34	Home Base	58	57
23	17	66	84	9
49	31	56	77	92

Start on this
number.

Use with "Home Base" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Tic-Tac-Toe Division

Materials: Calculator

Colored squares of paper to serve as game markers (one color per player)

- Directions:
1. This game is played by two players.
 2. Have the students make a 4 x 4 grid on a piece of paper.
 3. Read aloud the numbers on the sample grid, below, and have the students randomly fill in their grid with those numbers.

Sample Grid			
1,126	659	401	344
371	834	1,318	2,252
563	1,668	2,636	556
692	301	417	173

4. Write List A and List B on the chalkboard. Explain that the numbers in List A are dividends and those in List B are divisors.

List A	List B
2,408	7
1,384	4
5,272	6
3,336	2
4,504	8

5. The game is played as follows: In turn, each student chooses a number from List A and a number from List B and uses a calculator to compute the quotient of the division problem. If the quotient appears on the game board, the player covers the number with his/her game marker. If the quotient is not on the game board, or is

already covered, the player loses his/her turn. The first player to get four in a row, horizontally, vertically, or diagonally, is the winner.

6. Discuss the importance of using estimating strategies before making a final selection of the numbers to be used.

Short Division Game

Materials: Short Division Game Board (blackline master follows this activity; you may wish to copy the game board on tagboard and laminated it)

Eight 3" x 5" index cards numbered 2-9 (Divisor cards)

Game markers

- Directions:**
1. This game is played by two to four students.
 2. Shuffle the eight divisor cards and place them in a pile, face down, at the center of the playing area, next to the game board.
 3. The game is played as follows: Each player, in turn, places his/her game marker on the "Start" square and chooses a divisor card from the deck.
 4. The player uses the numeral on the divisor card as the divisor and the numeral on the game board square as the dividend and computes the division problem.
 5. The player moves his/her game marker the number of spaces that corresponds to the remainder of the problem. If there is no remainder, the player does not move his/her marker.
 6. The first player to reach "Home" is the winner.



Short Division Game Board

465	5,839	610	Go Back Two Spaces	157				
2,953				1,000		2,950		
34				3,254		7,042		
6,209				498		4,730		
Skip Your Turn				6,666		Double the Remainder; Move Backward 8,476		
894				Double the Remainder; Move Forward 515		7,521		
2,716				73		2,849		
56				4,895		86		
751				3,741		9,323		
170				400		742		
6,732	356	443	START	9,562	3,810	Take Another Turn	5,001	5,969

Use with "Short Division Game" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Estimate Division

Materials: Nine 1" x 1" ceramic tiles numbered 1–9 per student
Calculator

- Directions:
1. This game is played by two students.
 2. Place the tiles face down in the center of the playing area.
 3. Have each player draw a large division problem on a piece of paper to indicate a one-digit divisor and a four-digit dividend. Example: $\begin{array}{r} \overline{} \end{array}$
 4. Explain that the object of the game is to create a division problem with the number tiles that will result in the largest possible quotient.
 5. The game is played as follows: Each player, in turn, draws a number tile and decides where to place the number in his/her problem. Once a tile has been placed it may not be moved. (A barrier may be placed between the players to keep their number placements secret.) The players continue taking turns drawing tiles until all five digits have been placed.
 6. Each player then estimates the quotient for his/her problem, records the estimate next to the problem, and uses a calculator to find the actual quotient.
 7. The player whose problem has the larger quotient wins the round and is awarded points equal to his/her estimate.
 8. The player with the most points after a predetermined number of problems is the winner.



Problem Solving

Averages

Materials: Paper plates
Counters

Directions: 1. Display 3 paper plates and 12 counters. Place a random number of counters on each plate. Ask what addition sentence can be written for your models. Example:



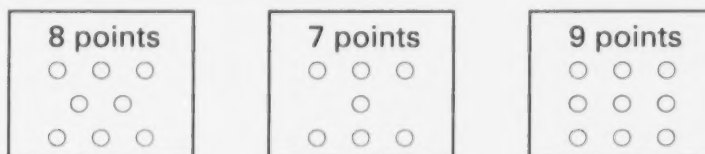
Addition sentence: $6 + 2 + 4 = 12$

Have a student record the sentence on the board.

2. Ask, "Have I divided these counters equally among the plates?" (no) "What if I wanted to evenly distribute them? How many would I put on each plate? How do you know?" (Four, because there are 12 counters and 3 plates to divide them among, so $12 \div 3 = 4$.) Have a student record the division sentence under the addition sentence on the board.
3. Say, "Four is the number of counters that would be on each plate." Have a student rearrange the counters into equal groups. Say, "For this problem we would say four is the average number of counters on a plate."
4. Repeat the activity with other groups of counters. Be sure the number of counters is a multiple of the number of plates being used (e.g., 15 counters on 3 plates, 10 counters on 5 plates, or 8 counters on 4 plates).
5. Have the students practice using 5 plates and 20 counters. Have one student arrange the counters equally on the plates and then have another student rearrange the counters to form unequal groups. Discuss how no matter

how the counters are rearranged the **total** is constant (5 plates/20 counters) and the **average** is also the same.

6. Say, "Mary is a very good volleyball player. In her most recent game she scored 8 points. In the game before that, she scored 7 points. In the game before that, 9 points. How could Mary find the average number of points she has scored in the three games?"
7. Have the students model the problem using counters and plates. You might also have them draw a pictorial representation on a piece of paper.



8. Give the students other problems to solve involving real-world examples of averaging.

*TEACHER NOTE: This activity could be used simply as an application for division rather than for learning the concept of averaging. In such case, eliminate the discussion of averages.

Averages II

Materials: "Average Team Scores" handout (one per student; blackline master follows this activity)
Calculators (one per student)

Directions: *TEACHER NOTE: Students should have some experience calculating averages before attempting this activity.

1. Explain to the students that athletic teams like to/need to average their scores for the season to determine how well they did overall.
2. Explain that the procedure for averaging is the same no matter how large the numbers or how many numbers. Review the algorithm for averaging (add the numbers; divide by the number of items).
3. Pass out the handouts and review how the table should be read. Ask, "How many teams are there?" (5) "How many

games did each team play?" (4) "If I wanted to know how many points a particular **team** scored for all four games, how would I read the table?" (horizontally) "If I wanted to know how many points all the teams scored for a particular **game**, how would I read the table?" (vertically)

4. Have the students complete the worksheet, beginning with problem 1. When they are finished, have them check their answers with others in the group.
5. Optional: Have the students find their own average scores for a sport in which they participate.



Average Team Scores

	Scores				W	X
Teams	Game 1	Game 2	Game 3	Game 4	Totals	Averages
Panthers	65	76	98	85		
Bears	89	72	87	92		
Mustangs	75	78	82	93		
Colts	55	67	95	87		
Saints	81	82	88	93		
Y Totals						
Z Averages						

Use a calculator to solve these problems.

- What was the total number of points the Panthers scored in all four games? _____ What was their average number of points scored per game?

$$\frac{\text{Total Points Scored}}{\text{Number of Games Played}} = \frac{\text{Average Number of Points Scored Per Game}}$$

- Find the total and average number of points scored for the other teams. Fill in **columns** W and X on the table. Then write the averages in the spaces below.

Bears _____ Colts _____ Mustangs _____ Saints _____

- What was the total number of points all five teams scored in Game 1? _____ What was the average number of points scored per team?

$$\frac{\text{Total Points Scored for Game 1}}{\text{Number of Teams}} = \frac{\text{Average Number of Points Scored Per Team}}$$

- Find the total and average number of points scored by all of the teams for each of the other games played. Fill in **rows** Y and Z on the table. Then write the averages in the spaces below.

Game 2 _____ Game 3 _____ Game 4 _____

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

523

5. Find the following scoring averages.

a. Panthers in Games 1, 3, and 4:

$$\frac{\text{Total Points Scored}}{\text{Number of Games Played}} = \frac{\text{Average Number of Points Scored Per Game}}$$

b. Mustangs in Games 2 and 3: _____ ÷ _____ = _____

c. Game 1—Panthers, Mustangs, and Colts = _____

d. Game 4—Panthers, Bears, and Mustangs = _____

e. Games 2 and 3—Saints and Mustangs = _____

Use with "Averages II" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

**S
T
O
R
Y

P
R
O
B
L
E
M
S**

Directions: Solve these problems using division. **Think**, "What does the problem tell me?" Then organize the numbers. Show your work in the work space provided.

Work Space

1. Marge has a part-time job. She worked 2 hours on Monday, 3 on Wednesday, and 3 on Friday. Her total paycheck was \$40. How much did she earn per hour?
2. Jon bought three Nintendo games and two calculators as gifts for a total of \$160. Each item cost the same amount of money. How much did each item cost?
3. Jennifer and Rebecca rode their bikes 245 miles last week. They rode the same distance each day, including Saturday and Sunday. How far did they ride each day?
4. Mrs. Raylor bought each of her seven children new tennis shoes. She paid the same price for each pair. Her total bill was \$182. How much was each pair?
5. George and Phil made 85 chocolate chip cookies for their Boy Scout troop. They had enough to give each boy 5 cookies. How many boys are in the troop?



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

525

**S
T
O
R
Y**

**P
R
O
B
L
E
M
S**

Work Space

- | | |
|--|----|
| 6. During their last basketball game, the Bears scored a total of 56 points. There are five girls and three boys on the team. If each player scored the same number of points, how many points did each score? | 6. |
| 7. Using the information from problem 6, how many baskets did the team make if each basket was worth 2 points? | 7. |
| 8. Using problem 6, how many baskets did each team member make? | 8. |

Each of the following problems is missing some information. Add a sentence or two to turn these into division word problems and then solve them.

- | | |
|--|-----|
| 9. The Oliver triplets weigh a total of _____ pounds. (Hint: each child) | 9. |
| 10. Molly gets an allowance of \$24 per month. (Hint: week) _____ | 10. |
| 11. There are seven fish tanks at our local pet store. (Hint: how many fish) _____ | 11. |



Guided Maze

Directions: Use the "Guided Maze" handout with this worksheet. Find "Start" on the left side of the maze. Enter the maze there. Draw a line until you reach the numeral 1. Solve problem 1, below, and check to see which column (at the bottom of this page) the answer is in. If it is in the "left" column, make a left turn in the maze. If it is in the "right" column, make a right turn. Follow the maze until you reach the numeral 2. Solve problem 2 and continue as before. Work your way through the entire maze in this manner. You may use a calculator to double-check your answers.

1. $9 \overline{)873}$

2. $\begin{array}{r} 892 \\ \times 7 \\ \hline \end{array}$

3. $6 \overline{)714}$

4. $5 \overline{)7,590}$

5. $\begin{array}{r} 9,123 \\ \times 4 \\ \hline \end{array}$

6. $4 \overline{)1,523}$

7. $3 \overline{)5,432}$

8. $\begin{array}{r} 1,755 \\ \times 8 \\ \hline \end{array}$

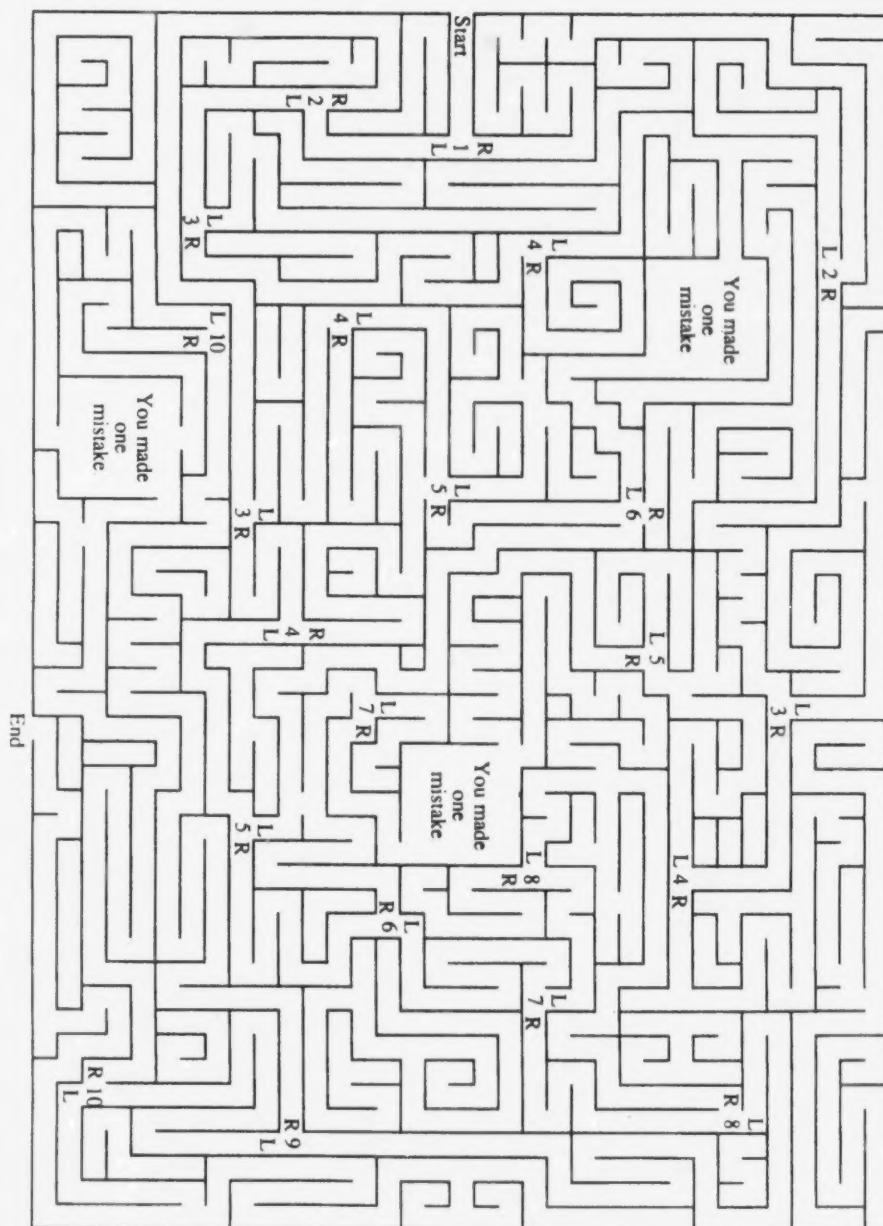
9. $9 \overline{)4,191}$

10. $7 \overline{)5,825}$

Left	Right
532	5
3	1,815
1,518	6,244
6,442	96
191	119
97	14,400
12,040	1,810 R2
4	523 R6
832 R1	832
380 R3	36,492
523 R7	



Guided Maze



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

SKILL Dividing two- to four-digit numbers by two-digit numbers***Skill Analysis***

- A. Knows the basic multiplication facts
- B. Understands the concept of division
- C. Understands the concept of place value
- D. Understands subtraction with regrouping
- E. Can multiply by multiples of 10 and 100

**Concept Manipulative Activities*****Let's Share a Process***

Materials: Base 10 materials

- Directions:
1. Discuss with the students the concept of division. Ask, "When would you want to divide? Let's list examples on the chalkboard."
 2. Allow the students to discuss examples of real-world applications of division. Be sure that they list examples of both partitioning sets and repeated subtraction.
 3. Say, "We have modeled division problems and kept written records when the divisor was a one-digit number. In the problems we're going to go over now, we will use the same process for two-digit divisors."
 4. Say, "Imagine that you have 65 books that you want to divide evenly among 12 people. How can you model 65? Build the model at your desk."

5. Have the students build a model of 65 with their base 10 materials. Ask, "How can you model $65 \div 12$ with these materials?"
6. Allow the students to share a process they would use for such a problem. The amount of direction they will require will depend upon their previous experiences.
7. Ask, "How many tens are in 65?" (6) "Can you divide the tens evenly among 12 people?" (no) "What must you do?" (Regroup—trade the 6 tens for 60 ones.) Have the students make the trade.
8. Ask, "How many ones do you have now?" (65) "Can you divide these evenly among 12 people?" (no) "How many ones will each person receive?" (5) "How many ones will you use in all?" (60) "How many ones will you have left?" (5) Have the students divide the ones into 12 groups.
9. Have the students draw their model on paper and record the division process.



65



12 people

$$\begin{array}{r} 5 \text{ R}5 \\ 12 \overline{)65} \\ \underline{60} \\ 5 \end{array}$$

10. Repeat the activity with other problems that have two-digit dividends and two-digit divisors.
11. Ask, "If I want to set up a model for $135 \div 12$, what do I need?" Have the students explain what materials would be necessary to model this problem. Then have the students make a model for 135.



135

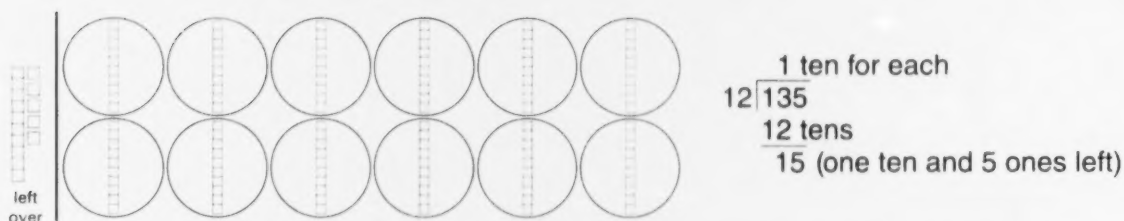


12 people

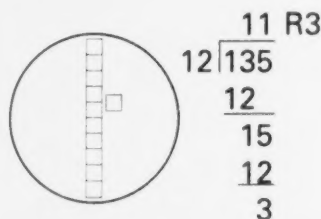
12. Ask, "What does your model for 135 show?" (1 hundred, 3 tens, 5 ones)

530

13. Ask, "Can you divide the 1 hundred evenly among 12 people?" (no) "What must you do?" (Trade the 1 hundred for 10 tens.) Have the students make the trade and then ask, "How many tens do you have now?" (13)
14. Ask, "How many tens should each person get?" (1) "How many tens will you use?" (12) "How many tens will you have left?" (1)
15. Have the students draw the model on paper and record the division process.



16. Ask, "If you trade the 1 ten for 10 ones, how many ones will you have in all?" (15) "How many ones should each person get?" (1) "How many ones will you use?" (12) "How many ones will be left?" (3)



17. Repeat the activity with other problems that have three-digit dividends and two-digit divisors. Include problems with and without remainders as well as problems with zero(s) in the dividend and/or quotient. By making the connection between these problems and the model, the students will not think of them as “special cases.”
18. Expand to problems that include four-digit dividends.

Divisor Table

Materials: Calculators (one per student)

- Directions:**
1. Write the problem $43 \overline{)2,924}$ on the chalkboard.
 2. Say, "Sometimes we won't have enough materials to model a division problem. For those problems, we have to try to "SEE" what our model would look like. To show you what I mean, we'll work together to solve the problem I just wrote on the board without using counters." (If this proves difficult for some students, they should be allowed to use concrete materials.)
 3. Say, "Make a list of multiples of the divisor, 43. Include multiples from 43×1 to 43×9 . You may use a calculator to find the products." Example:

$43 \times 1 = 43$	$43 \times 4 = 172$	$43 \times 7 = 301$
$43 \times 2 = 86$	$43 \times 5 = 215$	$43 \times 8 = 344$
$43 \times 3 = 129$	$43 \times 6 = 258$	$43 \times 9 = 387$
 4. Say, "I'll show you how to use this list to help you with the division problem."
 5. Ask, "If I were to build a model for 2,924, what would it look like?" (2 thousands, 9 hundreds, 2 tens, and 4 ones)
 6. Ask, "Can 2 thousands be divided evenly among 43 people?" (no) Say, "Think about regrouping the thousands as hundreds. Can 29 hundreds be divided evenly among 43 people?" (no) "Think about regrouping the hundreds as tens. Can 292 tens be divided evenly among 43 people?" (no)
 7. Say, "Look at your list of multiples of 43. If I have 292 tens to be divided among 43 people, how many tens would each person get?" (Six, because 258 is close to 292 but 301 is too much.)
 8. Say, "Write $43 \overline{)2,924}$ on a piece of paper and record 6 in the quotient." Ask, "If I gave 6 tens to each person, how many tens would I use in all?" (258) "How many tens would be left?" (34) Show the students how to record their work.

532

$$\begin{array}{r}
 6 \text{ tens for each} \\
 43 \overline{) 2,924} \\
 \underline{258} \text{ tens in all} \\
 34 \text{ tens left}
 \end{array}$$

9. Ask, "Can 34 tens be divided evenly among 43 people?" (no)
Say, "Again, think about regrouping. After regrouping, how many ones would I have?" (344)
10. Say, "Look at your list again. If I have 344 ones to be divided among 43 people, how many ones would each person get?" (8)
11. Say, "Record 8 in the quotient." Ask, "If I gave 8 ones to each person, how many ones would I use in all?" (344)
"How many ones would I have left?" (0) Show the students how to complete their written record.

$$\begin{array}{r}
 68 \text{ ones for each} \\
 43 \overline{) 2,924} \\
 \underline{258} \\
 344 \\
 \underline{344} \text{ ones in all} \\
 0 \text{ ones left}
 \end{array}$$

12. Repeat the activity for other problems with four-digit dividends and two-digit divisors.

Casting Out 10s and 100s

Materials: Egg carton (for a dozen eggs)

- Directions:**
1. Say, "Mary Jones has a chicken farm. She supplies eggs to the local grocery store. Yesterday her hens laid 2,892 eggs. How many dozen did they lay?"
 2. Display several egg cartons and discuss the number of eggs in a dozen.
 3. Ask, "How can we determine how many dozen eggs Mary's chickens laid?" Have the students discuss ways of calculating the answer and share them with the class.
 4. Say, "The grocery store asks Mary to ship the eggs in special crates. Not only are the eggs put in cartons that hold a dozen eggs each, but the cartons are put in crates that hold 10 dozen or 100 dozen eggs."

5. Ask, "If Mary puts 10-dozen eggs in a '10-crate,' how many eggs has she packed?" (120) "How many has she packed if she fills a '100-crate'?" (1,200)
6. Say, "Mary uses a special method to calculate how many dozen eggs are in each of her shipments."
7. Write the following problem on the board.

$$12 \overline{)2,892}$$

8. Say, "First, Mary determines how many 100-crates (or 100-dozen eggs) she can pack. If she packs **one** 100-crate, how many eggs will she pack?" (1,200) "This is the beginning of Mary's record. What does it mean?"

$$\begin{array}{r} 12 \overline{)2,892} \quad 100 \\ \underline{1,200} \end{array}$$

9. Allow the students to explain what they think this model means.
10. Ask, "Can Mary pack another 100-crate? How do you know?"

$$\begin{array}{r} 12 \overline{)2,892} \quad 100 \\ \underline{1,200} \\ 1,692 \quad 100 \\ \underline{1,200} \end{array}$$

11. Ask, "What do you think will happen next?" Allow the students to discuss what they think will be the next step. Have them justify their answers.
12. Have the students complete the problem using "Mary's" method.

$$\begin{array}{r}
 241 \\
 12 \overline{) 2,892} 100 \\
 \underline{1,200} \\
 1,692 100 \\
 \underline{1,200} \\
 492 10 \\
 \underline{120} \\
 372 10 \\
 \underline{120} \\
 252 10 \\
 \underline{120} \\
 132 10 \\
 \underline{120} \\
 12 1 \\
 \underline{12} 241 \\
 0
 \end{array}$$

13. Ask, "How many dozen eggs did Mary send to the grocery store?" (241) Have the students discuss the completed process.
14. Ask, "Could you use Mary's method with other problems?" Allow the students to explain the method and how it could be used.
15. Have the students use this method for other problems with four-digit dividends and two-digit divisors.

A Subtraction Method of Division

Materials: Calculators (one per student)

- Directions:**
1. Some students may find the following method interesting to explore. Allow them to use calculators, since this method is not easily modeled.
 2. Write the problem $20 \overline{) 1,940}$ on the board. Ask the following questions, completing the written record as shown below based on the students' answers.

535

$$\begin{array}{r}
 \textcircled{97} \\
 7 \\
 40 \\
 50 \\
 20 \overline{)1,940} \\
 \underline{1,000} \\
 940 \\
 \underline{800} \\
 140 \\
 \underline{140} \\
 0
 \end{array}$$

- a. "How many 20s in 1,000?" (50) Subtract 1,000 from 1,940. (940)
 - b. "How many 20s in 900?" (40) Subtract 800 from 940. (140)
 - c. "How many 20s in 140?" (7) Subtract 140 from 140. (0)
 - d. "What do you get when you add the partial quotients?"
 (50 + 40 + 7 = 97) "Therefore, $20 \overline{)1,940} = \textcircled{97}.$ "
3. Work through another example with the students.

$$\begin{array}{r}
 \boxed{225} \\
 5 \times 35 = 175 \\
 20 \times 35 = 700 \\
 200 \times 35 = 7,000 \\
 35 \overline{)7,875} \\
 \underline{7,000} \\
 875 \\
 \underline{700} \\
 175 \\
 \underline{175} \\
 0
 \end{array}$$

Expand With Zeros

Materials: None

Directions: 1. Write the problem $3 \overline{)36}$ on the chalkboard. Have the students solve the problem.

526

2. Ask, "If I add some zeros to the dividend, what do you think will happen to the answer?"
3. Allow the students to discuss their thoughts. Then write the problems $3\overline{)360}$ and $3\overline{)3,600}$ on the chalkboard.
4. Ask, "How has the problem been changed? What do you think will happen to the answer?"
5. Ask, "What happens when zeros are added to the divisor (e.g., $30\overline{)360}$ and $30\overline{)3,600}$)?"
6. Discuss the patterns that the students see.
7. Have the students write and solve their own problems to display what happens when zeros are added to dividends, divisors, or both. Example: $5\overline{)30}$, $5\overline{)300}$, $5\overline{)3,000}$, $50\overline{)300}$, $50\overline{)3,000}$

Estimation in Division

Materials: Calculators (one per student)

- Directions:
1. Say, "A huge department store uses many supplies. In 26 days, the store uses 8,239 folding boxes. The store manager wants to find out about how many boxes the store uses every day."
 2. Explain to the students that for many real-life situations, like the one you just described, you don't need an exact answer in division. Most often, estimates are all you need.
 3. Say, "For the problem I just described, we need to find an **estimate** of the number of boxes used every day."
 4. Say, "We can begin by dividing the hundreds. Think: $26\overline{)8,200}$. The quotient will begin in the hundreds place because $82 \div 26$ will be 3 with some left over."
 5. Write $26\overline{)8,239}$ on the chalkboard.
 6. Say, "Since we only want an estimate for this problem, I will write zeros for the other digits in the quotient." Write 300
 $26\overline{)8,239}$ on the chalkboard.

537

7. Say, "We can say that the store uses about 300 boxes every day."
8. Ask, "Is this estimate greater or less than the exact answer?" Allow the students to justify their responses.
9. Have the students use a calculator to find the exact answer. Discuss the estimate and the exact answer.



Let's Practice

Tic-Tac-Toe

Materials: Calculators (one per pair of students)

- Directions:**
1. This game is played by two students.
 2. Each player writes down nine division problems, each with a four-digit dividend and a two-digit divisor, and numbers the problems 1 to 9.
 3. The players then draw a tic-tac-toe grid (one per pair of students) and number the sections 1 to 9.
 4. The game is played like tic-tac-toe, but in order for a player to put his/her mark (X or O) in a square, he/she must correctly solve the corresponding problem from his/her opponent's problem list.
 5. The students check the answer given for each problem with a calculator.
 6. The winner is the first player to place three X's or three O's in a row.

Create a Problem

Materials: Round cards, with a two-digit number written on each
 Square cards, with a three- or four-digit number written on each
 Calculators (one per student)

- Directions:**
1. Mix each pile of cards separately.

2. Have the students pick a card from each pile and write a division problem in the form $\div \bigcirc = \underline{\hspace{2cm}}$.
3. Have the students work the problem on paper and check their work with a calculator.

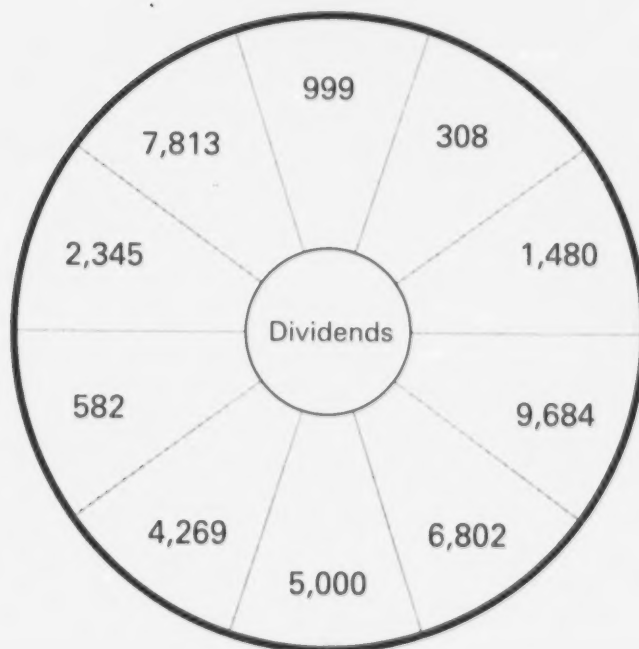
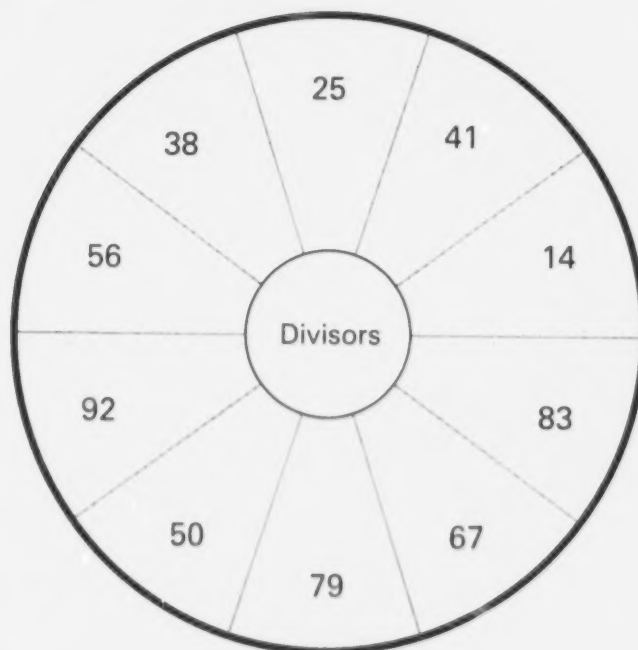
Spin, Divide, and Check

Materials: Calculators (one per student)
"Spinners" handout (one per group of students; blackline master follows this activity)
Paper clips

- Directions:
1. Divide the students into small groups. Give each group a "Spinners" handout and a paper clip.
 2. Demonstrate how to use a paper clip as a spinner. (Place it at the center of a circle, hold it in place with a pencil point, and then spin it around the pencil point.)
 3. Have the students, in turn, spin a paper clip around each circle on the handout and create a division problem with the numbers on which the paper clip lands. All of the students in the group should solve the problem and check their work with a calculator.
 4. Variation: Have the students fill in divisors and dividends on blank spinners.
 5. Variation: Have the students write two-digit numerals on two blank spinners. Have them spin the paper clip on each spinner and use a calculator to multiply the numbers. Then have them divide the numbers on paper to check the calculator answers.



Spinners



Use with "Spin, Divide, and Check" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Game Time

Materials: Universal Game Board (directions for its construction may be found in the Appendix)
3" x 5" cards
Calculator
Game markers
Die numbered 1, 2, 3, 1, 2, 3

- Directions:**
1. This game is played by a small group of students.
 2. Have the students design and make a Universal Game Board.
 3. Then have the students make game cards using 3" x 5" cards. Explain that they should write a division problem with a four-digit dividend and a two-digit divisor on the front of each card and the answer on the back. (To find the answer, they should divide each problem using paper and pencil and then check their work with a calculator by multiplying the quotient times the divisor and adding the remainder.)
 4. The game is played as follows: The first student rolls the die, draws a problem card, and works the problem. If the student's answer is correct, the student moves his/her game marker the number of spaces indicated on the die. If the answer is incorrect, the game marker is not moved.
 5. The first player to reach "Finish" is the winner.



Problem Solving

Phone Home: Calling on Your Division Skills

Materials: "Phone Home" handout (one per student; blackline master follows this activity)
Optional: Push-button phone

- Directions:
1. Using a push-button phone or the chart on the "Phone Home" handout, show the students how each number corresponds to different letters on a phone pad.
 2. Then have the students read the directions on the handout and complete the problems.
 3. Discuss examples of words that **cannot** be made using the letters found on a push-button phone. (There is no Q or Z on a push-button phone.)
 4. Have the students make up their own codes and problems.
 5. Have the students divide the last four digits of their phone number by their age.
 6. Variation: Use a transparency of the handout and have the class solve the problems in teams.

512



Phone Home

Directions: For the problems below, replace the letters with the corresponding numbers and solve the division problems. Show your work.

1	ABC 2	DEF 3
GHI 4	JKL 5	MNO 6
PRS 7	TUV 8	WXY 9
*	Operator 0	#

Example: $CALL \div NO = 34 R11$

$$\begin{array}{r}
 34 R11 \\
 66 \overline{)2255} \\
 \underline{-198} \\
 275 \\
 \underline{-264} \\
 11
 \end{array}$$

1. $HOME \div GO =$ _____

6. $RACE \div ON =$ _____

2. $RUN \div WE =$ _____

7. $GOOD \div BY =$ _____

3. $TOP \div IT =$ _____

8. $DEAR \div ME =$ _____

4. $KID \div EM =$ _____

9. $RATED \div PG =$ _____

5. $BLEW \div UP =$ _____

10. $BIKES \div GO =$ _____

Now, on the back of this handout, make up five problems of your own and solve them.

Use with "Phone Home" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Four in a Row

Materials: "Four in a Row" handout (one per pair of students; blackline master follows this activity)
Calculators (one per pair of students)
Game markers (different color for each partner)

- Directions:
1. Divide the class into pairs of students.
 2. In turn, each partner will choose a divisor and dividend from the charts at the top of the handout, write the problem down, and work it on the calculator. If the answer is on the quotient table, the student will cover the number with his/her game marker.
 3. Explain that the object of the game is to get four squares in a row, either diagonally, vertically, or horizontally. Emphasize the importance of using estimation strategies.
 4. As the students are playing, walk around the room and ask them what estimation strategies they are using.
 5. Have the students write down all of the problems used to avoid repetitions.



Four in a Row

Divisors

12	15	18	21
28	36	45	63

Dividends

1,260	3,024	8,316
2,268	6,615	1,764

Quotients

35	36	252	189	70
462	60	48	144	315
84	147	168	126	45
108	297	105	693	63
396	28	441	98	231

Use with "Four in a Row" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

**S
T
O
R
Y**

**P
R
O
B
L
E
M
S**

1. The pet shop had 115 birds, which it divided evenly among 23 birdcages. How many birds were put into each cage?

2. Parakeets cost \$13.00 each. How many parakeets can Ryan buy with \$50.00?

How much money will he have left?

Betty has \$50.00. She buys a birdcage for \$15.00 and birdseed for \$3.00. How many parakeets can she buy with the money she has left?

How much money will she have left over?

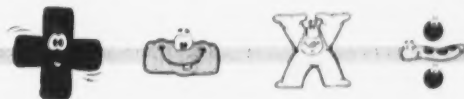
3. *Apollo 10* was the first lunar module to orbit the moon. The mission lasted for 192 hours. How many days did it last?

4. One day 1,254 cars passed through a tollbooth. About how many cars passed through the tollbooth each hour, assuming the traffic flow was steady?

5. A driver averages 55 miles per hour. About how many hours will it take him to drive 3,960 miles?

How many days will it take him to drive the 3,960 miles if he drives 10 hours a day?

6. A restaurant uses eight dozen eggs a day. How long will it take the restaurant to use 1,248 eggs?



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Plan a Trip

Materials: Highway maps for your state (one per student)
Automobile advertisements (stating approximate gas mileage)
Optional: "Trip Planning Sheet" (one per student; blackline master follows this activity)

- Directions:**
1. Explain that you'd like each student to plan a trip or vacation within his/her state.
 2. Give each student a map. Point out the distance scale and have students explain how they would use it. Have them calculate the distance between various places they plan to visit.
 3. Have the students look at the automobile advertisements and decide what kind of car they will be driving. Have them use the estimated miles per gallon stated in the ad to calculate the amount of gasoline necessary for their trip.
 4. Discuss the average current price of gasoline per gallon in your area. Have the students compute how much money they will need for gas.
 5. Discuss speed limits on highways. What is the average speed traveled on interstates versus other roads in your state? Have the students use these average speeds to compute the driving time for their trip.
 6. Variation: Discuss the cost of lodging or camping, the cost of meals, etc. You could also have the students research the various places they plan to visit.

***TEACHER NOTE:** The "Trip Planning Sheet" handout can help the students keep track of information as they plan their trip. Or, the sheet can be used as a checklist **after** some attempt has been made by the student. You might want to have the students gain more practice by planning a similar trip using another mode of transportation.



Trip Planning Sheet

Information: Starting place: _____

Destinations: _____

Route to be taken: _____

Places to stop: _____ No. of days _____ No. of nights _____

_____ No. of days _____ No. of nights _____

_____ No. of days _____ No. of nights _____

Average speed on interstates: _____ m.p.h.

Average speed on roads: _____ m.p.h.

Approximate gas mileage of car: _____ m.p.g.

Approximate cost of each gallon of gas: _____

Scale of map: _____

Measure distance between first two stopping places: _____

a. Convert to miles: _____

b. Between next two stopping places: _____

Convert to miles: _____

c. Between next two stopping places: _____

Convert to miles: _____

d. Between the other stopping places, if any: _____

e. Total miles: _____

How much gas will you need for the trip? _____

How much will gas cost for the entire trip? _____

How long will it take to drive between the first and second stop? _____

Between the second and third stop? _____

Between the third and fourth stop? _____

Between the rest of the stops, if any? _____

Expense for motels or camping: How much for one night? _____

Times number of nights on trip? _____

Expense for food: _____

Breakfasts? _____

Lunches? _____

Dinners? _____

Use with "Plan a Trip" activity. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

518

Assessment Activities

SKILL Multiplying a one-digit number by multiples of 10 and 100

3.6-1

Have the students find the answers to the following problems. They may use modeling materials if they need them.

1.
$$\begin{array}{r} 20 \\ \times 4 \\ \hline \end{array}$$

2. $100 \times 7 = \underline{\hspace{2cm}}$

3. $5 \times 30 = \underline{\hspace{2cm}}$

4.
$$\begin{array}{r} 700 \\ \times 2 \\ \hline \end{array}$$

SKILL Multiplying two- and three-digit numbers by a one-digit number

3.6-2

Have the students solve the following problems. They may use models or draw pictures to help them solve the problems.

1.
$$\begin{array}{r} 15 \\ \times 5 \\ \hline \end{array}$$

2. $29 \times 2 = \underline{\hspace{2cm}}$

3.
$$\begin{array}{r} 129 \\ \times 2 \\ \hline \end{array}$$

4. $222 \times 3 = \underline{\hspace{2cm}}$

5. $34 \times 3 = \underline{\hspace{2cm}}$

6.
$$\begin{array}{r} 21 \\ \times 4 \\ \hline \end{array}$$

7. $154 \times 2 = \underline{\hspace{2cm}}$

8.
$$\begin{array}{r} 413 \\ \times 2 \\ \hline \end{array}$$

SKILL Dividing a two-digit number by a one-digit number

3.6-3

Have the students solve the following problems. They may use models or draw pictures to help them solve the problems.

1. $17 \div 4 = \underline{\hspace{2cm}}$

2. $2 \overline{)56}$

3. $60 \div 6 = \underline{\hspace{2cm}}$

4. $7 \overline{)25}$

5. $24 \div 5 = \underline{\hspace{2cm}}$

6. $8 \overline{)24}$

7. $17 \div 3 = \underline{\hspace{2cm}}$

8. $9 \overline{)99}$

SKILL Multiplying by multiples of
10 and 100

4.6-4

Have the students solve the following problems.

$$1. 40 \times 6 = \underline{\quad\quad\quad} \quad 2. \begin{array}{r} 52 \\ \times 30 \\ \hline \end{array} \quad 3. 23 \times 60 = \underline{\quad\quad\quad} \quad 4. \begin{array}{r} 500 \\ \times 39 \\ \hline \end{array}$$

$$5. \begin{array}{r} 80 \\ \times 40 \\ \hline \end{array} \quad 6. 8 \times 700 = \underline{\quad\quad\quad} \quad 7. \begin{array}{r} 400 \\ \times 17 \\ \hline \end{array} \quad 8. 32 \times 90 = \underline{\quad\quad\quad}$$

SKILL Multiplying two-digit numbers by
two-digit numbers

4.6-5

Have the students solve the following problems. Remind them to show their work.

$$1. \begin{array}{r} 97 \\ \times 14 \\ \hline \end{array} \quad 2. 57 \times 61 = \underline{\quad\quad\quad} \quad 3. 43 \times 84 = \underline{\quad\quad\quad} \quad 4. \begin{array}{r} 93 \\ \times 27 \\ \hline \end{array}$$

SKILL Dividing two- and three-digit
numbers by a one-digit number

4.6-6

Have the students work the following division problems. They may use models if they need them.

$$1. 5 \overline{)65} \quad 2. 552 \div 6 = \underline{\quad\quad\quad} \quad 3. 7 \overline{)714} \quad 4. 58 \div 4 = \underline{\quad\quad\quad}$$

$$5. 145 \div 5 = \underline{\quad\quad\quad} \quad 6. 9 \overline{)307} \quad 7. 3 \overline{)76} \quad 8. 833 \div 4 = \underline{\quad\quad\quad}$$

572

SKILL Multiplying four-digit numbers by two-digit numbers

5.6-7

Have the students solve the following multiplication problems by applying a rule they have learned.

1.
$$\begin{array}{r} 1,103 \\ \times 12 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 2,314 \\ \times 40 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 6,874 \\ \times 98 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 6,401 \\ \times 76 \\ \hline \end{array}$$

SKILL Dividing two- to four-digit numbers by one-digit numbers

5.6-8

Have the students solve the following division problems by applying a rule they have learned.

1. $4 \overline{)332}$

2. $2 \overline{)817}$

3. $6 \overline{)182}$

4. $3 \overline{)288}$

5. $9 \overline{)8,000}$

6. $7 \overline{)6,879}$

7. $6 \overline{)4,956}$

8. $8 \overline{)1,707}$

SKILL Dividing two- to four-digit numbers by two-digit numbers

5.6-9

Have the students solve the following division problems by applying a rule they have learned.

1. $43 \overline{)2,924}$

2. $20 \overline{)3,248}$

3. $5,785 \div 28 = \underline{\hspace{2cm}}$

4. $6,272 \div 32 = \underline{\hspace{2cm}}$

Record Keeping

On the following page is a sample record form for an individual child. The purpose of the form is to provide data on the student's mastery in computation. By checking the code for the skills mastered, the teacher has information for a student report which shows the progress and levels of mastery. This information is helpful in individual educational progress reports and parent reports.

Example:

Topic 2: Basic Facts for Addition and Subtraction

☐ 1.2-1 ☒ 2.2-2 ☒ 2.2-3 ☒ 3.2-4 ☒ 3.2-5

These checks show that the student started on the 2nd grade skill and mastered through the 3rd grade skill in basic facts. If more specific information on these skills is necessary, the skills are stated on the tab pages (i.e., tab for topic 2 will give the specific skill statements).

Regardless of the student's grade in school, this record card will show where he is as he works toward grade level. It gives a quick accountability of skills learned to help evaluate the progress made during a year's time. It is also invaluable when writing the next year's IEP.



Individual Record Card

Student Name _____

Grade Level _____

Coding _____

grade level topic number skill number

Topic 1 Modeling Addition and Subtraction

Check skills mastered

- ☐ K.1-1 ☐ K.1-2 ☐ K.1-3 ☐ 1.1-4
☐ 1.1-5 ☐ 1.1-6 ☐ 2.1-7 ☐ 2.1-8

Example 3 4 3
 Grade 3 Topic 4 Skill 3

Topic 2 Basic Facts for Addition and Subtraction

Check skills mastered

- ☐ 1.2-1 ☐ 2.2-2 ☐ 2.2-3 ☐ 3.2-4 ☐ 3.2-5

Topic 3 Addition and Subtraction with Regrouping

Check skills mastered

- ☐ 2.3-1 ☐ 2.3-2 ☐ 3.3-3 ☐ 4.3-4 ☐ 5.3-5

Topic 4 Modeling of Multiplication and Division

Check skills mastered

- ☐ 2.4-1 ☐ 2.4-2 ☐ 3.4-3 ☐ 3.4-4

Topic 5 Basic Facts for Multiplication and Division

Check skills mastered

- ☐ 3.5-1 ☐ 3.5-2 ☐ 4.5-3 ☐ 5.5-4

Topic 6 Algorithms for Multiplication and Division

Check skills mastered

- ☐ 3.6-1 ☐ 3.6-2 ☐ 3.6-3 ☐ 4.6-4 ☐ 4.6-5
☐ 4.6-6 ☐ 5.6-7 ☐ 5.6-8 ☐ 5.6-9

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

533



The Universal Game Board

A Universal Game Board can be used in support of many objectives. The game board has a theme and is used with game cards, game markers, and dice or spinners.

Purpose: To give the student practice on a specific skill, usually for drill, review, or enrichment.

Game board: May be made on tagboard or file folders and laminated.

Suggested themes:

Animals	Holidays	Farm
Fish	Space	Underwater Scenes
Dinosaurs	Monsters	Snoopy
Racetrack	Records	Airplanes
Circus	Sesame Street	Bugs
Cars	Winnie the Pooh	etc.

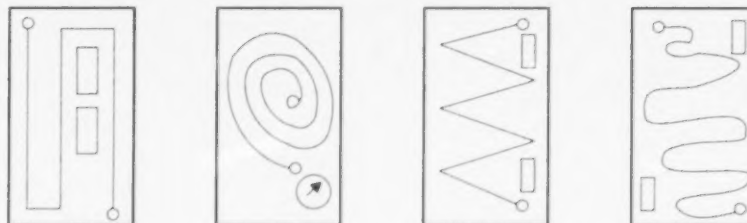
**Die/dice
and
spinners:**

Standard dice may be used (two for a short game, one for a longer game). Use dice numbered 1–3 to make the game last longer. Polyhedra Dice from Creative Publications are different shapes and offer variety.

A spinner may be used instead of dice. It may be mounted on the board or separately. Spinners are available commercially or can be made with cardboard, a brad, and a paper clip.

Cards: A deck of cards that contains problems that reinforce one or more objectives will be needed. (Students, for enrichment or extra practice, can generate the problems that are on the cards.) Index cards, tagboard, or blank playing cards may be used.

Formats: (The lines represent paths to follow.)



Players move from "Start" to "Finish."

A game using a board containing 20 spaces will last about 5–7 minutes. A game using a board with 36 spaces will last about 10–12 minutes. You can also make a continuous game board and count the laps.

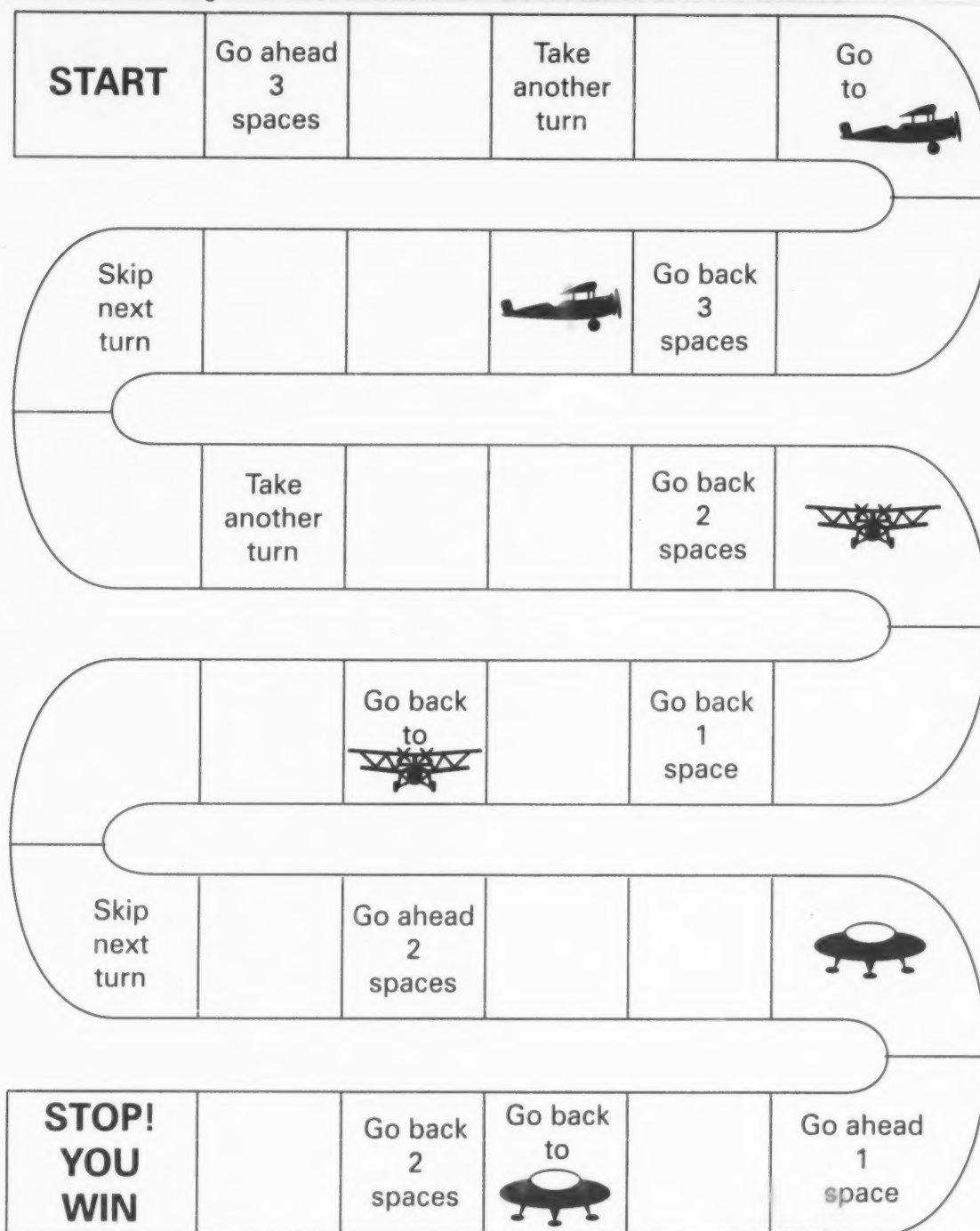
Lettering: Letraset, rub-on letters, stamps, or your own (or students') writing may be used.

Markers: Try to match the theme of the game. You need only two or three per game board. Look for erasers, miniature toys, or cake decorations at card, specialty, or craft stores.

Decorations: Stickers can be placed in each space to carry out a particular theme. Books of stickers for a particular theme may be found at card shops or discount stores. Bubblegum cards may be used instead of stickers. Another alternative is to use pictures from coloring books or magazines as a background and plain stickers for spaces or hand-marked spaces.



Example of Universal Game Board



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

556



Play Dough Recipes

Standard

Play Dough

1-1/4 cups flour
1 cup salt
2-1/4 tablespoons powdered alum
3/4 cup boiling water
2 tablespoons vegetable oil

Mix flour, salt, and alum thoroughly. Add water and oil. Knead thoroughly. For colors, add food coloring to water before mixing.

Cooked Play Dough

1 cup flour
1/4 cup salt
1 cup water
1 tablespoon vegetable oil
2 teaspoons cream of tartar

Combine all ingredients. Heat until ingredients form a ball. Add food coloring.

Potter's Clay

1/4 cup flour
1/4 cup cornstarch
1 cup salt
3-3/4 cups water
6-8 cups flour

Blend 1/4 cup flour and 1/4 cup cornstarch with enough water to make a paste. Boil the water and salt. Add the flour mix and cook until clear. Cool overnight. Add 6 to 8 additional cups of flour and knead until desired consistency is achieved.

Metric

Boiling-Water Play Dough

550 ml flour
250 ml salt
15 ml powdered alum
60 ml vegetable oil
375 ml boiling water

Mix flour, salt, and alum in a medium bowl. Add vegetable oil. Stir in boiling water. Mix or knead until smooth. Add more flour if mixture is too soft. Add food coloring if desired and knead again. Store in airtight container.

Cooked Play Dough

250 ml flour
125 ml salt
10 ml cream of tartar
30 ml vegetable oil
250 ml boiling water

Mix flour, salt, and cream of tartar in a pan. Add vegetable oil and boiling water. Cook about 3 minutes on a stove or hot plate. Mixture will "pull away" from the edges of the pan. Turn out on a washable surface or waxed paper. Knead when cool enough to touch. Knead in food coloring if desired. Store in airtight container.

Uncooked Play Dough

500 ml flour
125 ml salt
175 ml water
few drops food color
few drops oil of clove or mint extract
(optional)
Measure and mix. Knead until desired consistency.

537

Colored Pasta Recipe

1. Fill an aluminum pie pan with approximately 1" of rubbing alcohol.
2. Add several drops of the desired food coloring.
3. Place some pasta in the colored alcohol.
4. Remove the pasta and place on a paper towel to dry.

Edible Clay Recipes

Recipe 1

- 1 cup peanut butter
- 1 can prepared frosting
- 1-1/2 cups nonfat milk

Recipe 2

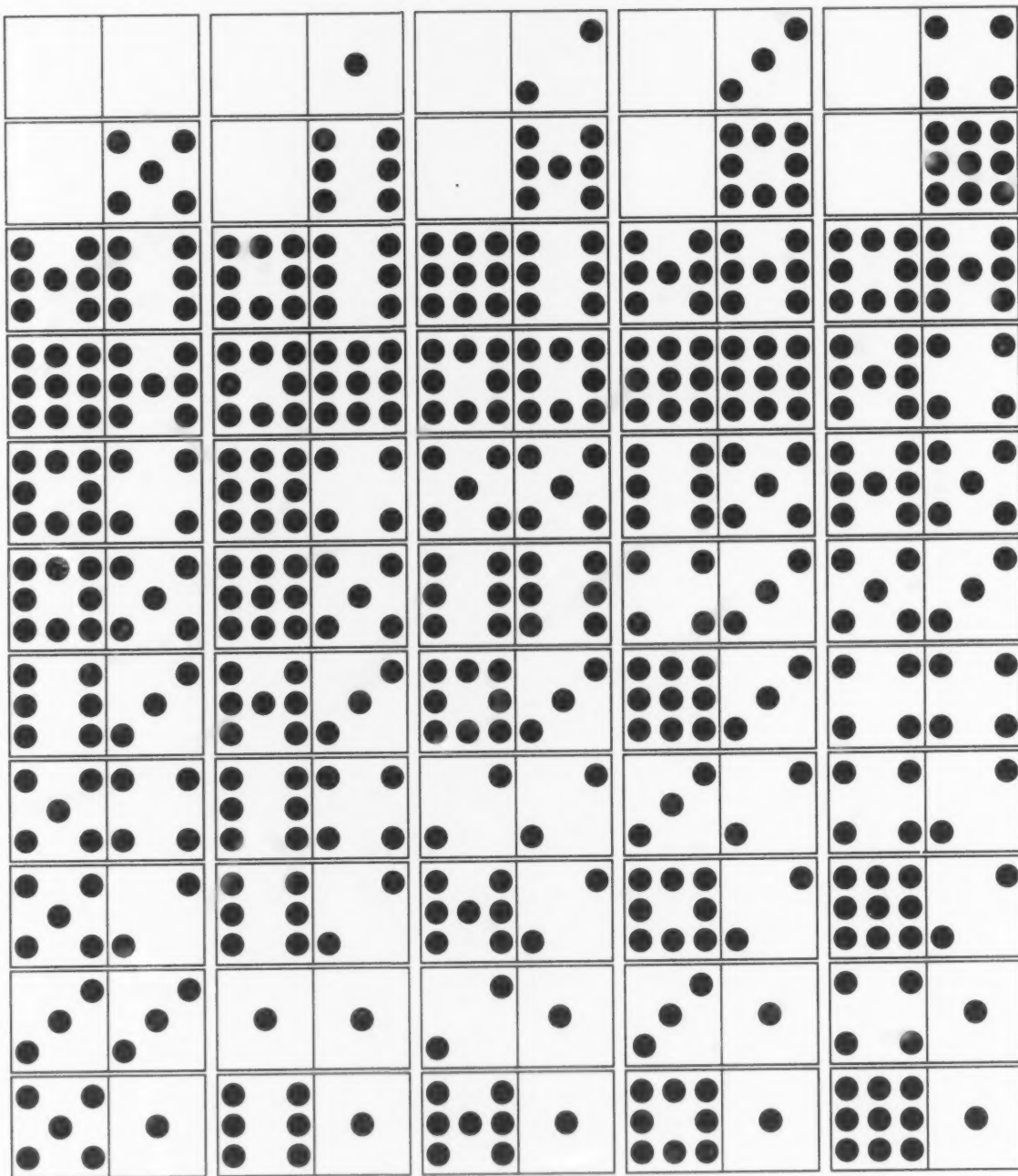
- 1 part peanut butter
- 1 part powdered sugar
- 1 part Karo syrup
- 1 part powdered milk

Recipe 3 (The Healthy Version)

- 1 part peanut butter
- 1 part powdered sugar
- 1 part wheat germ
- 1/2 part honey



Dominoes

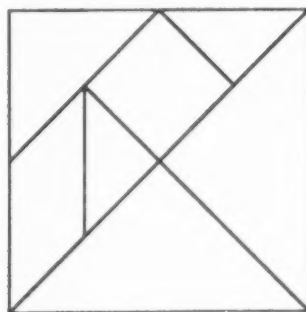
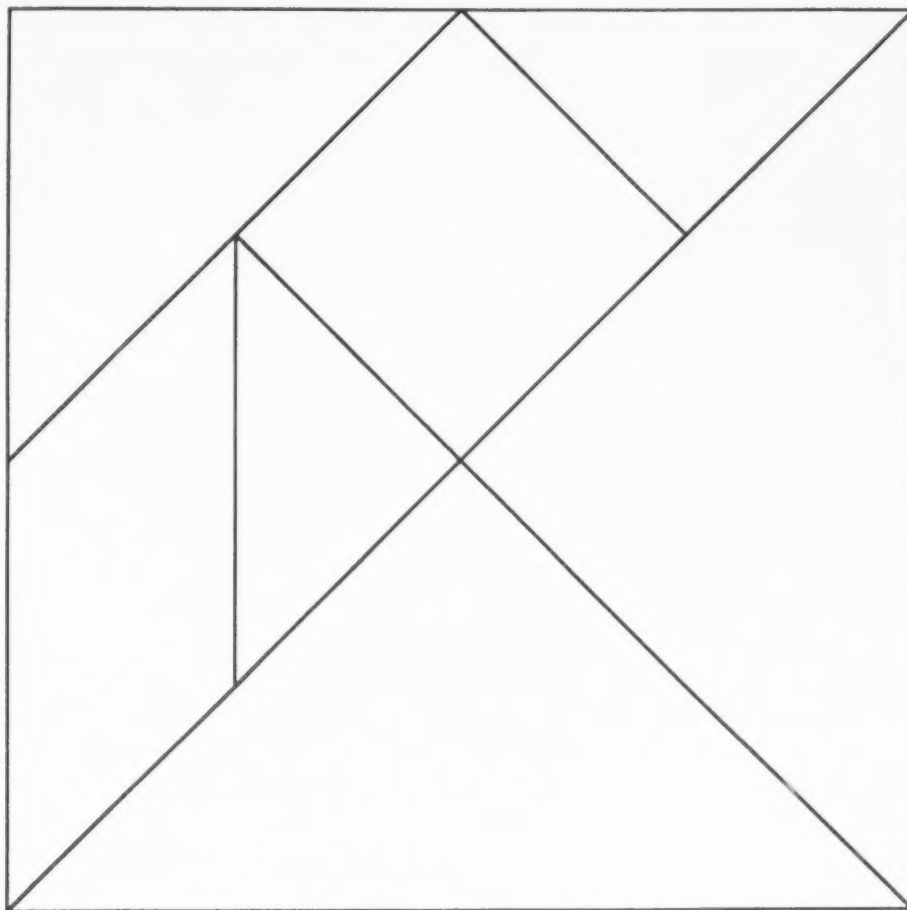


Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

559



Tangram Pieces

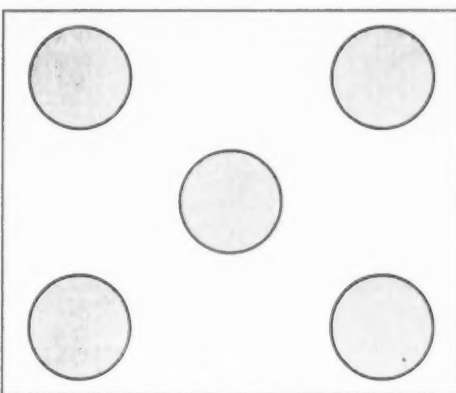
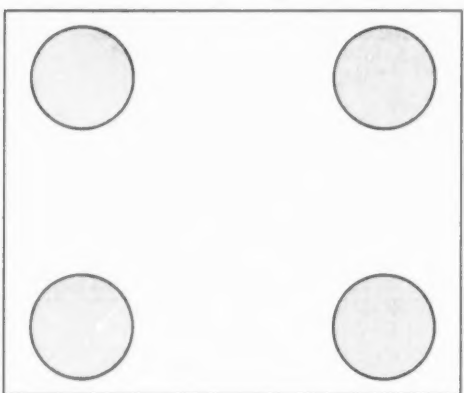
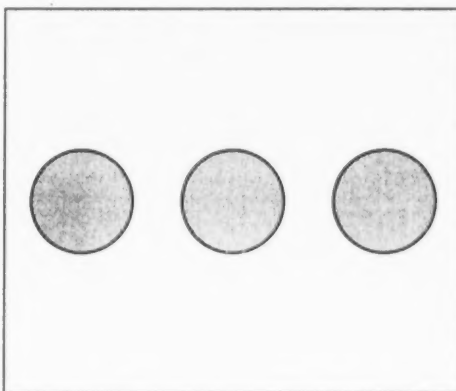
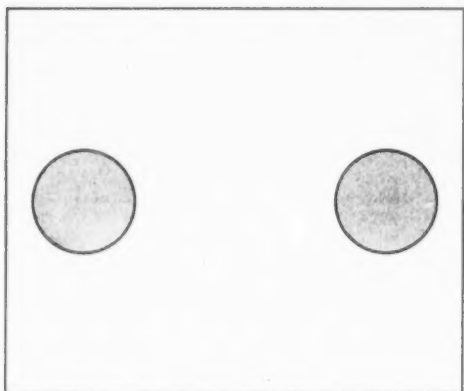
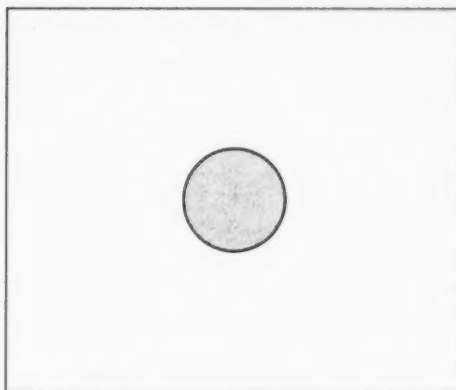


Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

500



Dot Cards

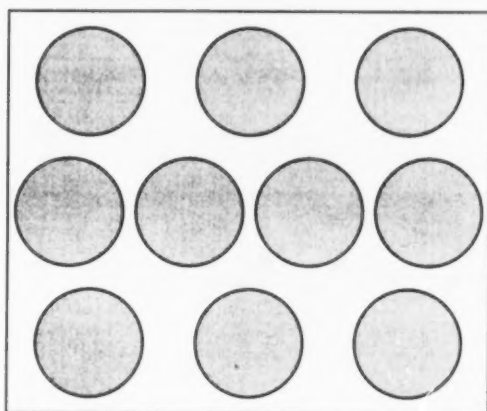
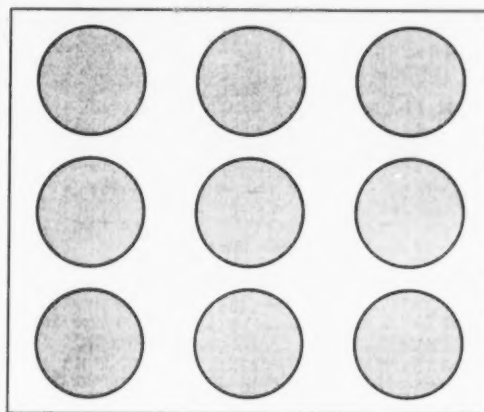
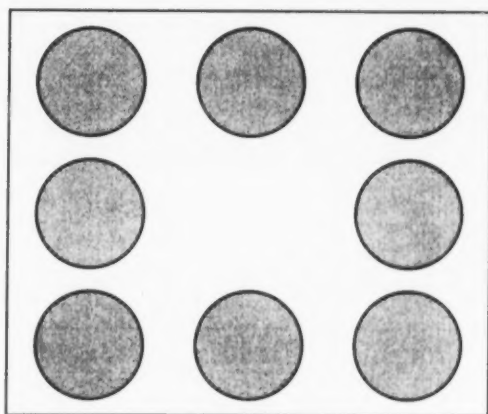
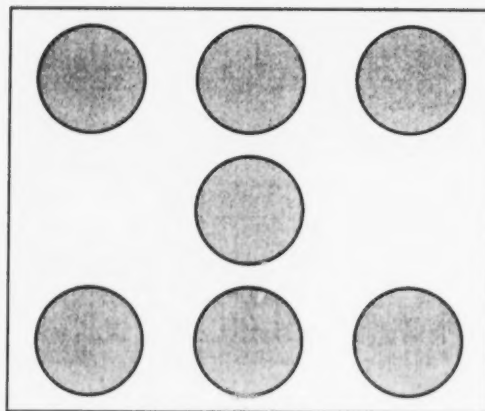
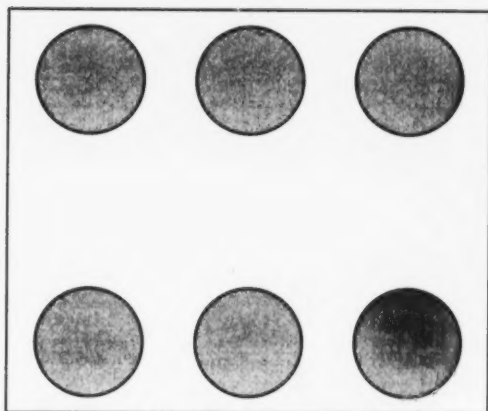


501

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Dot Cards



562

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Money

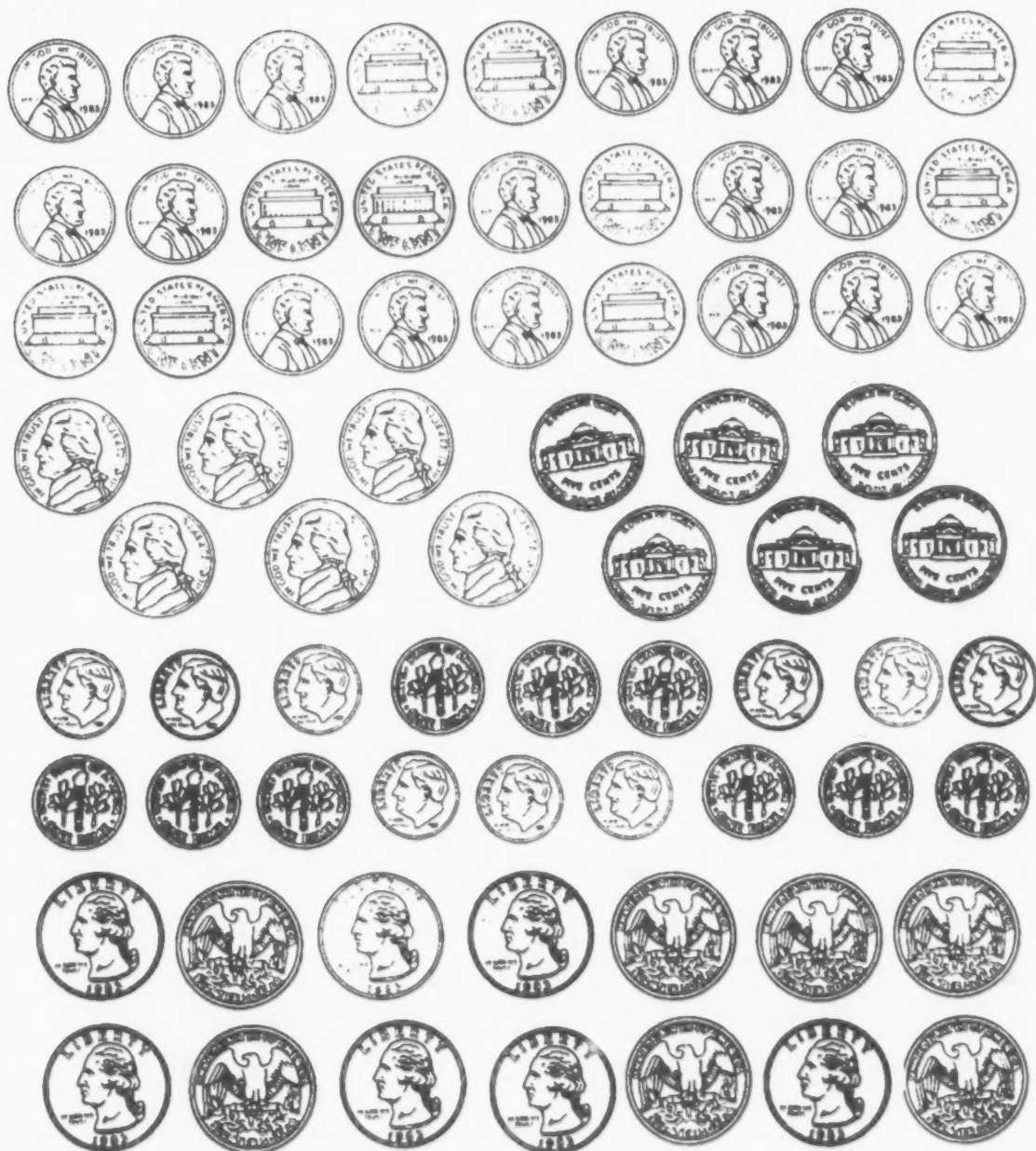


563

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



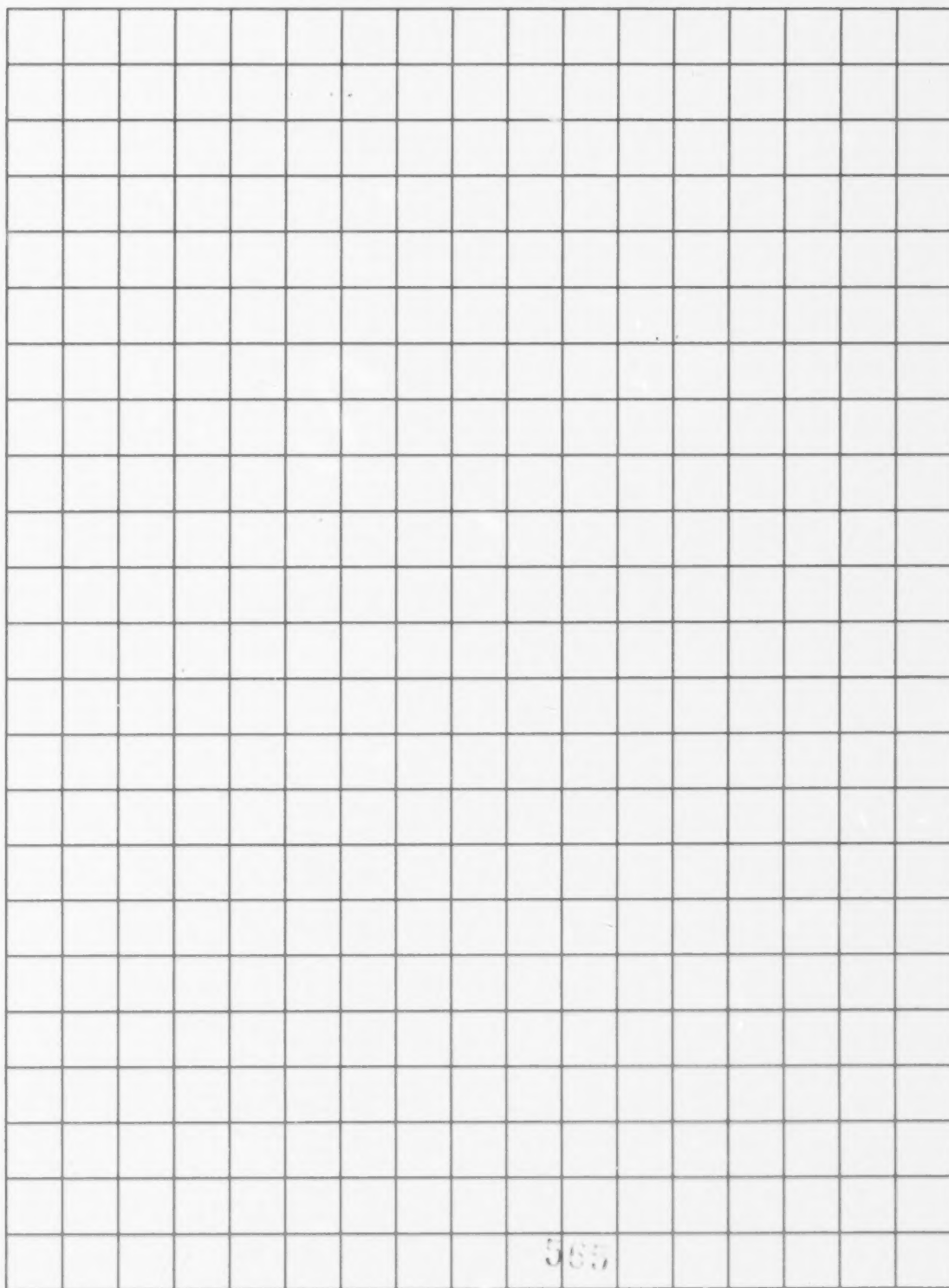
Coin Samples



564

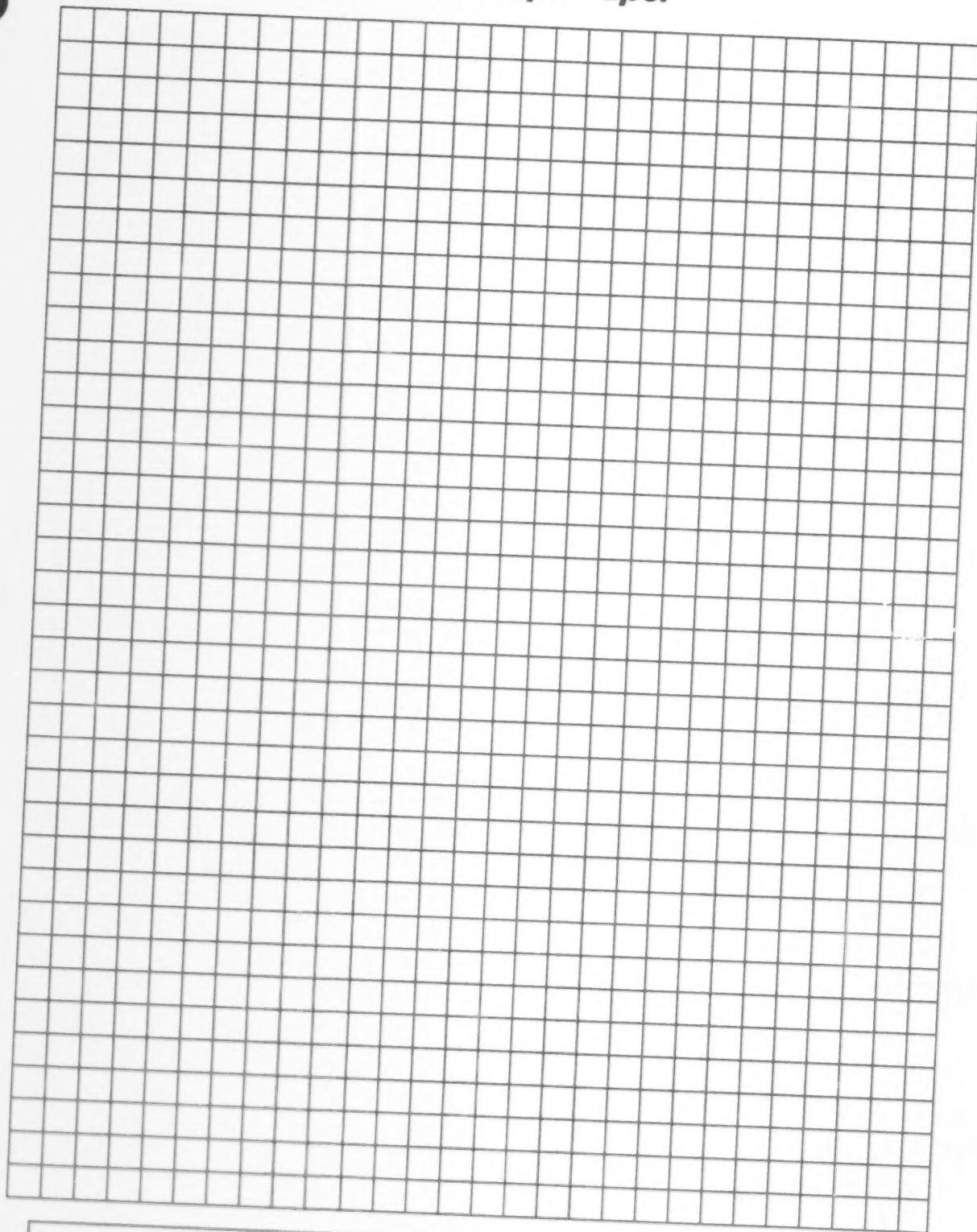
Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

Centimeter Graph Paper



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

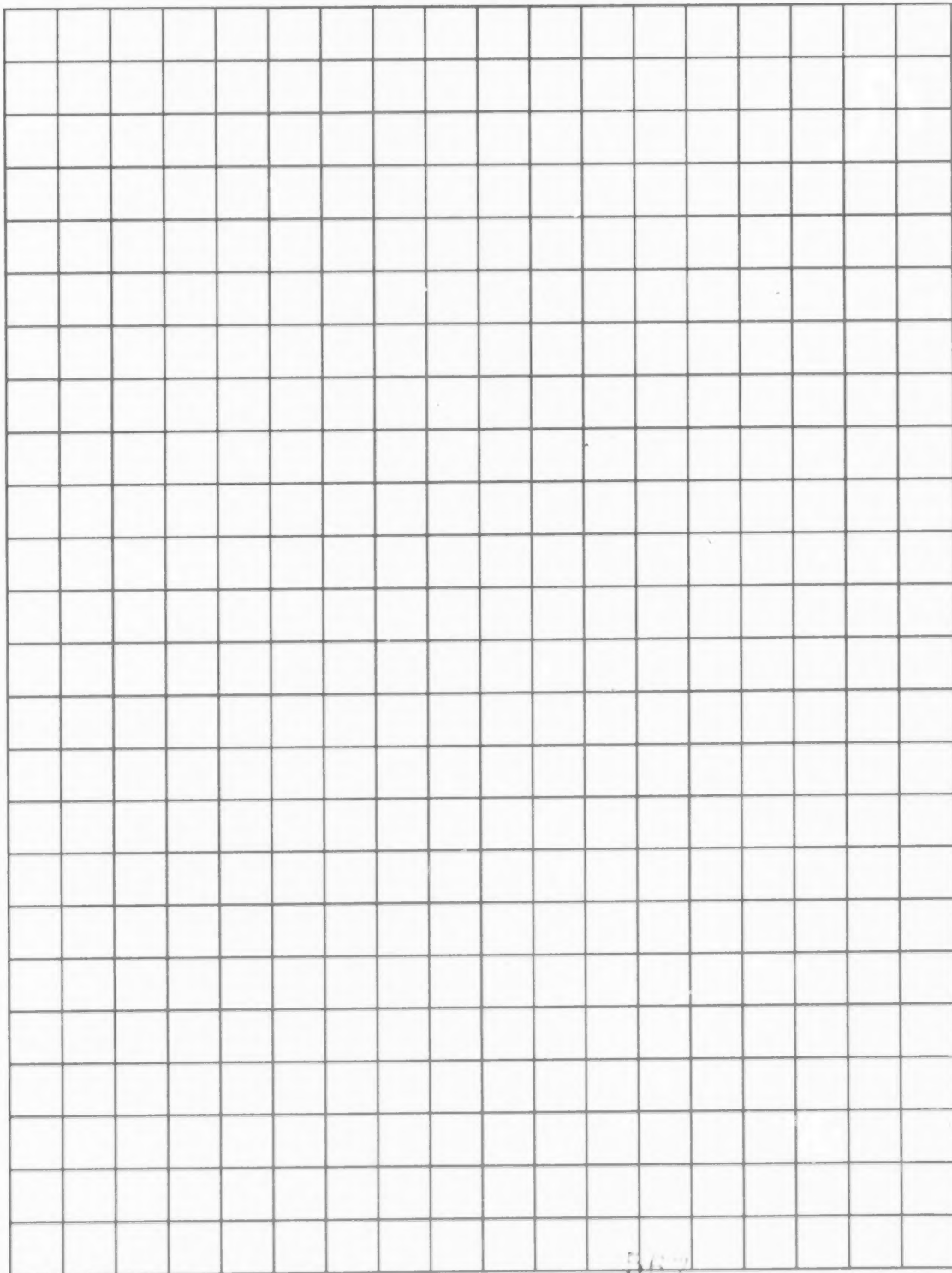
1/4" Graph Paper



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

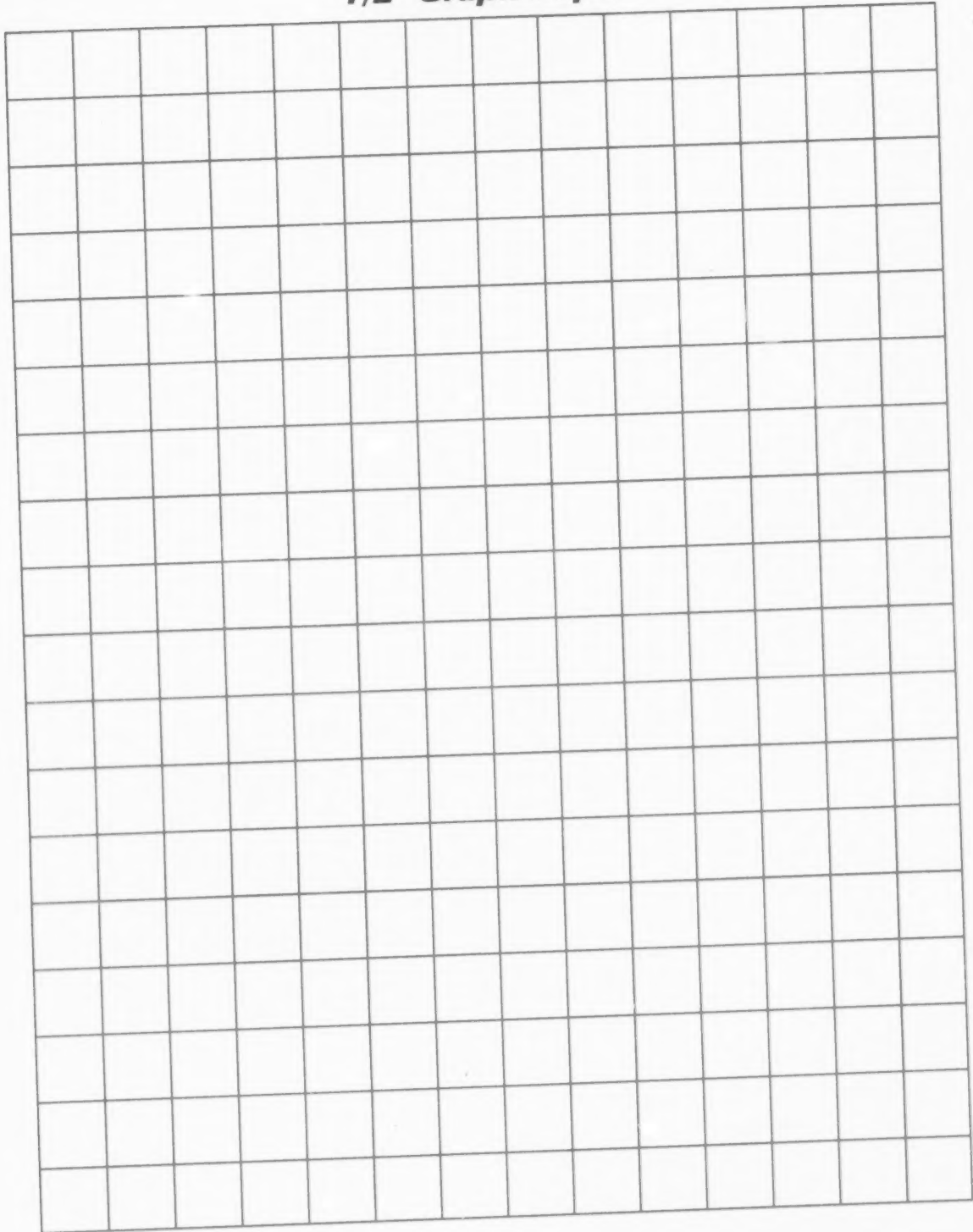
566

3/8" Graph Paper



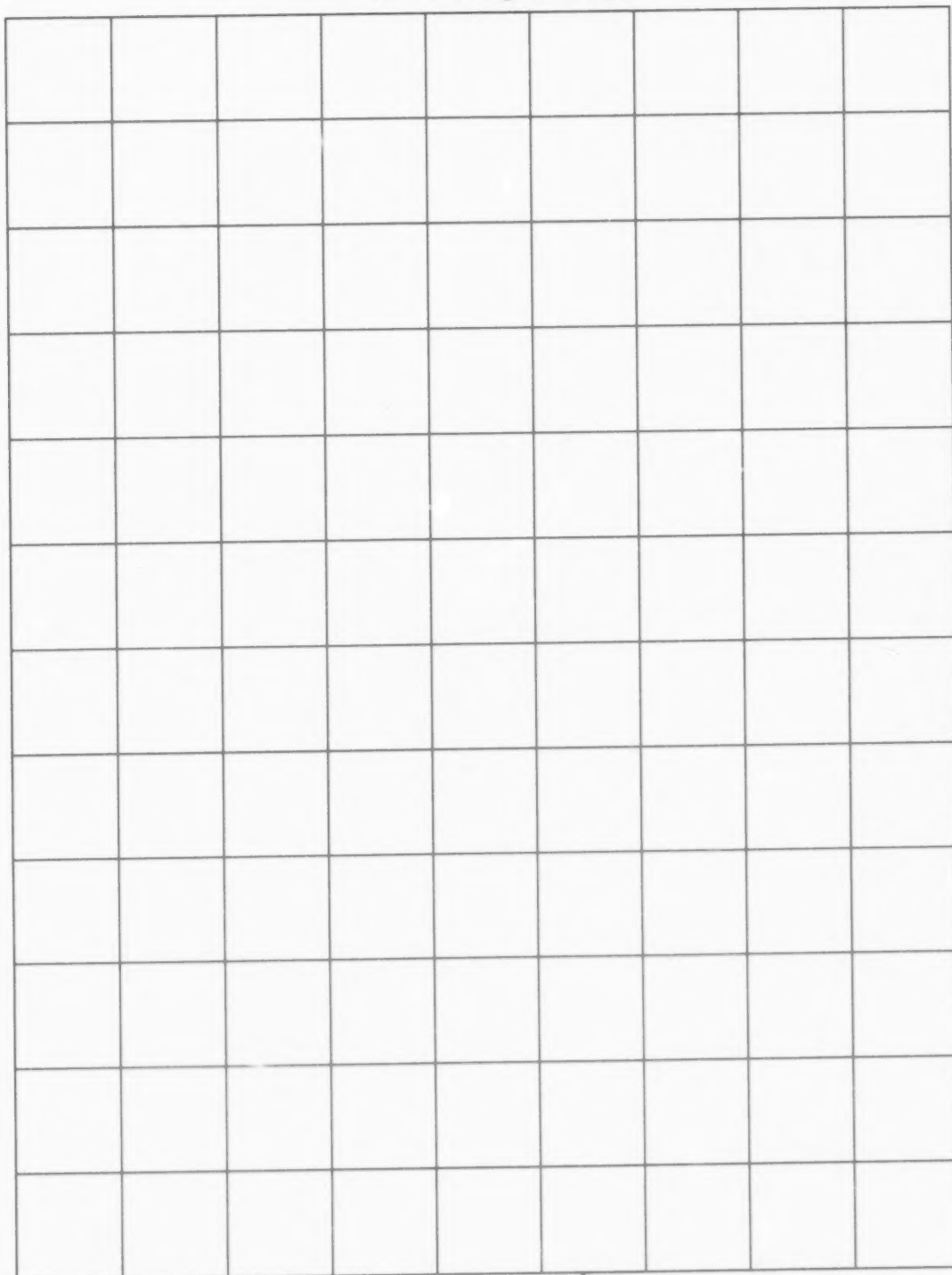
Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

1/2" Graph Paper



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved. 563

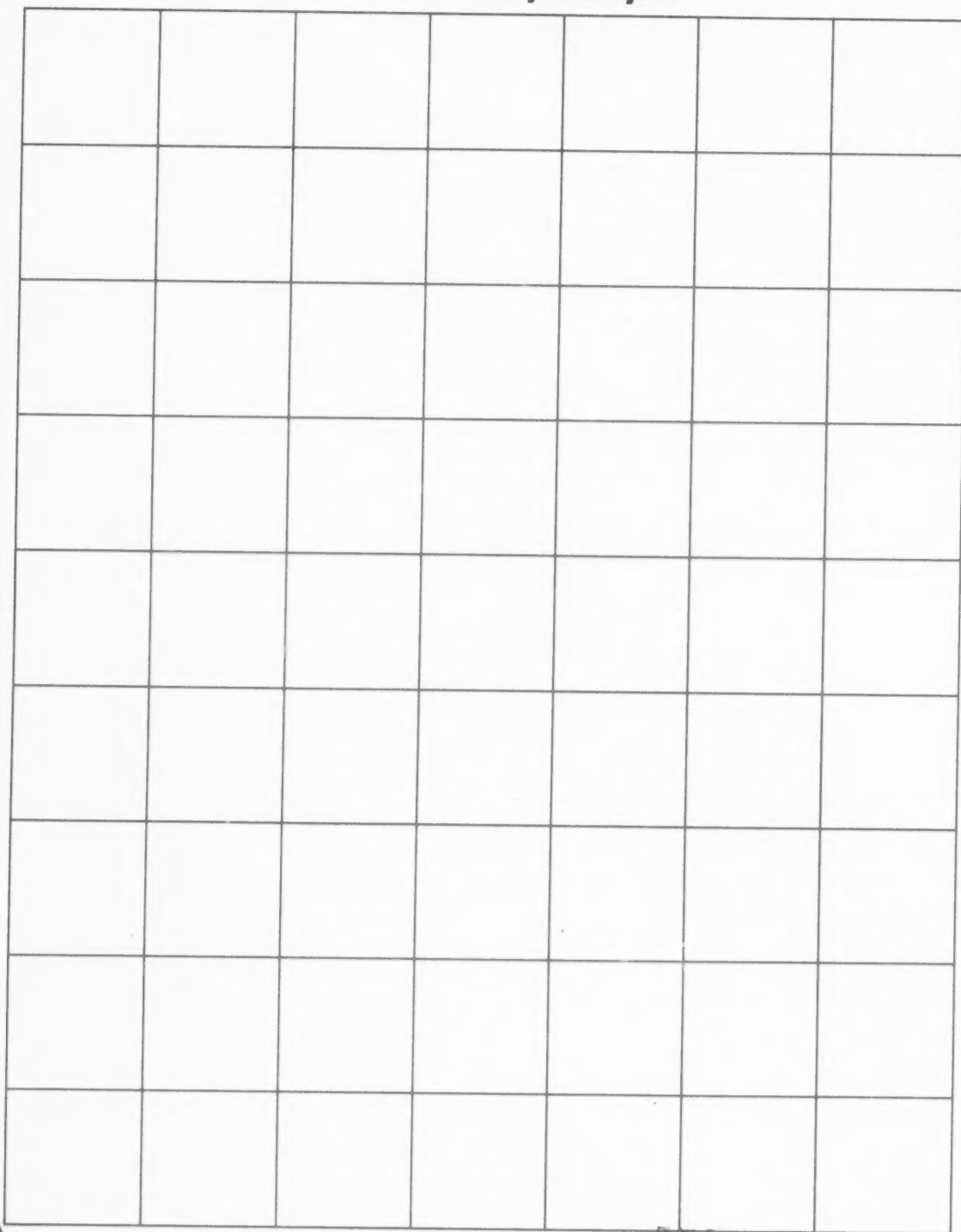
3/4" Graph Paper



589

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

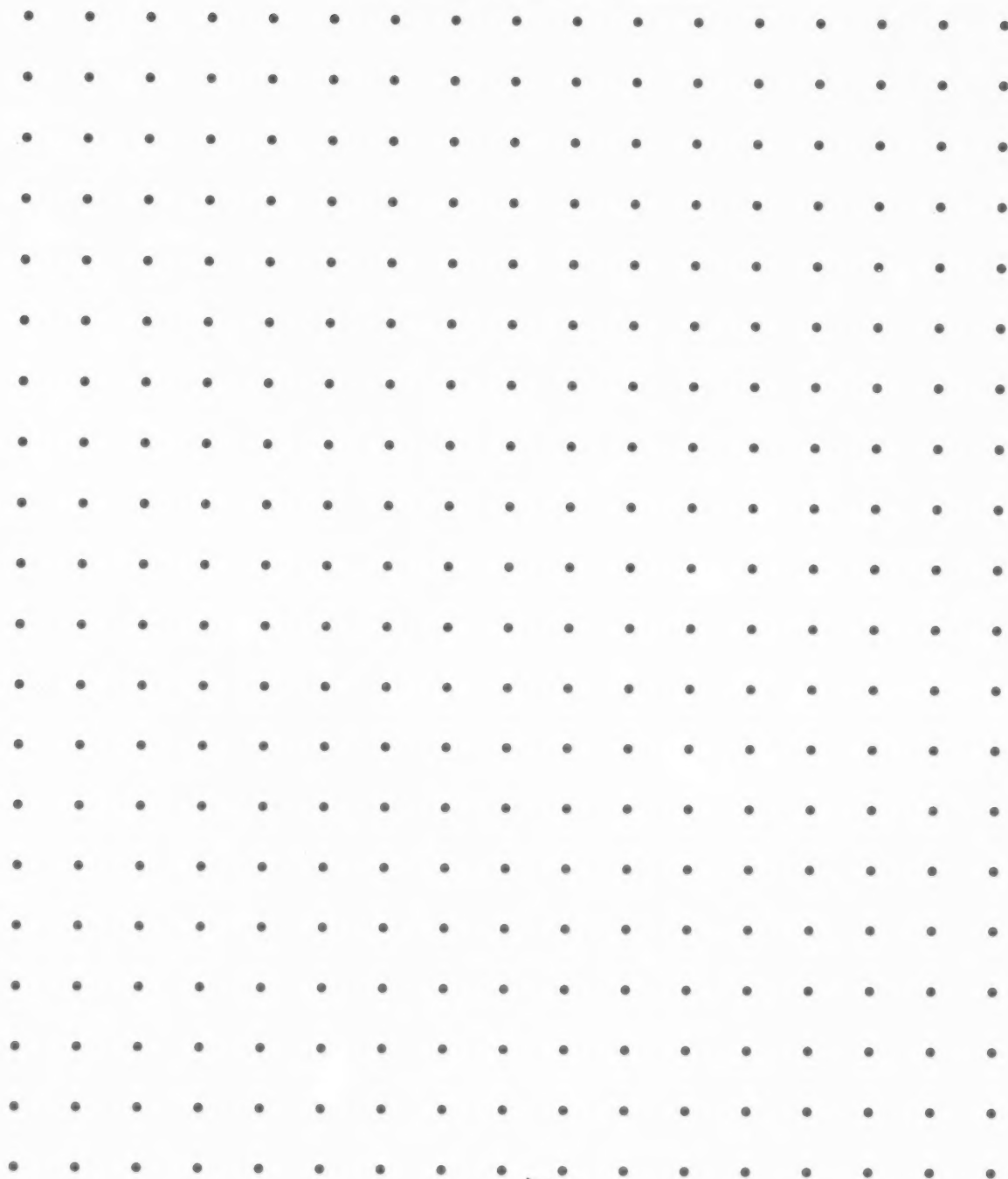
1" Graph Paper



570

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

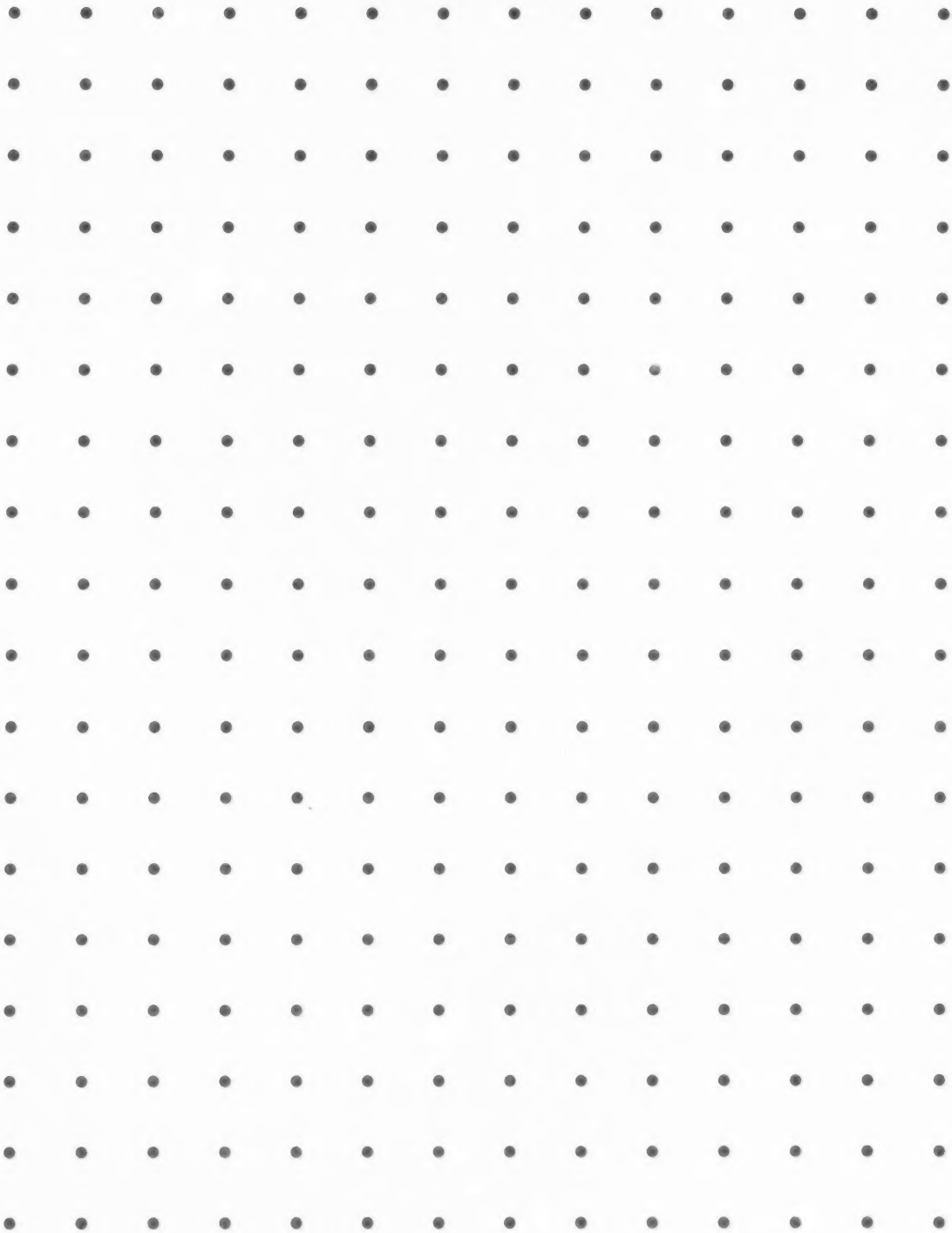
Centimeter Dot Paper



571

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

1/2" Dot Paper



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

572

Geoboard Paper

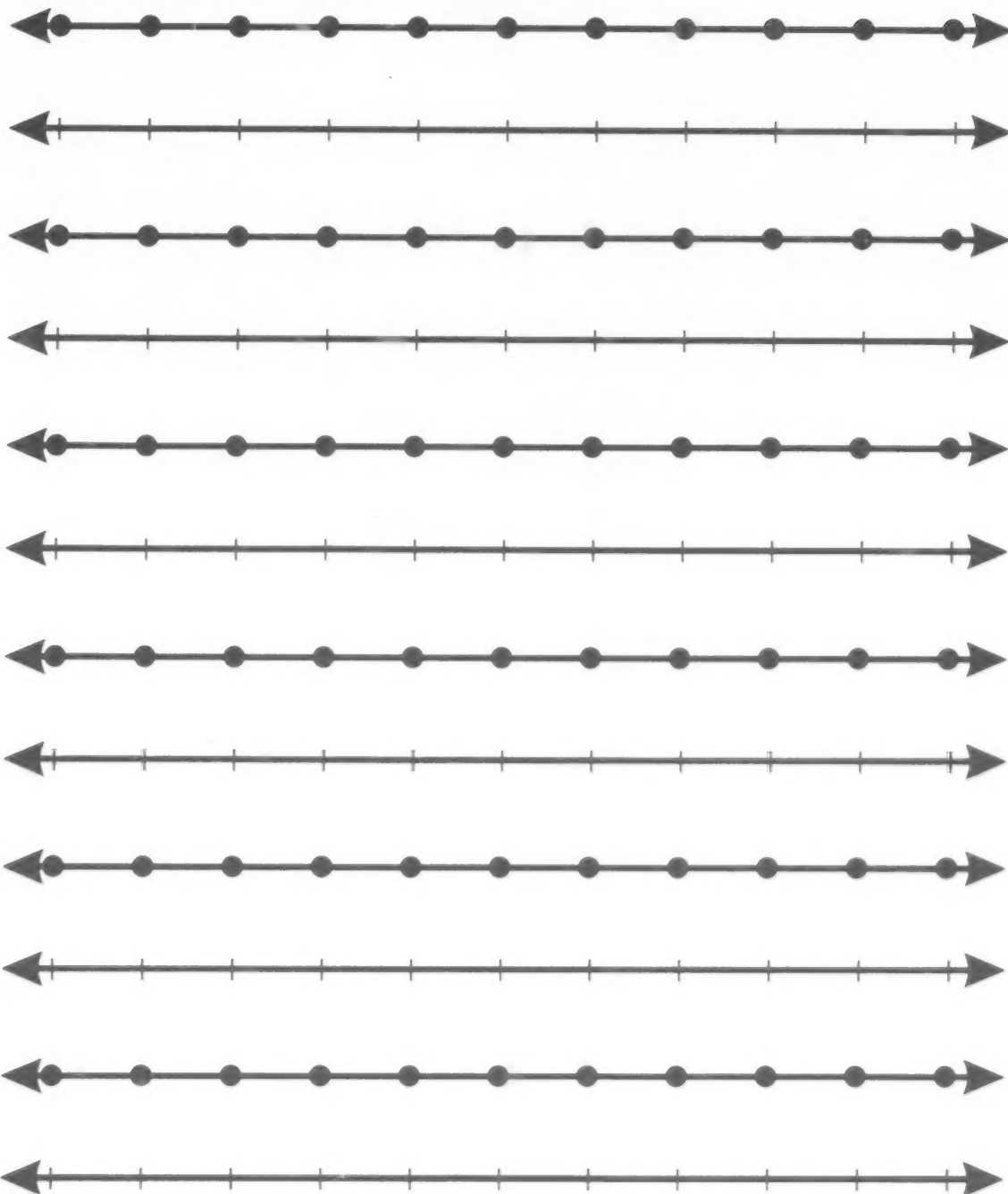


575

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Blank Number Lines



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

574



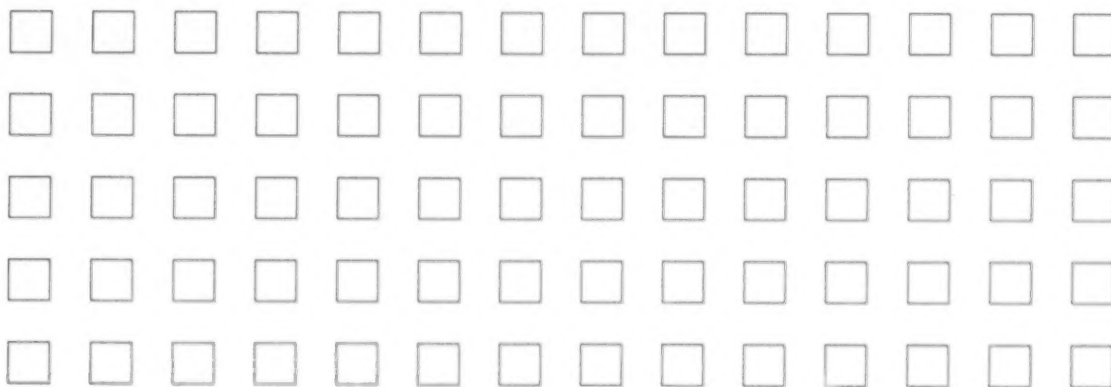
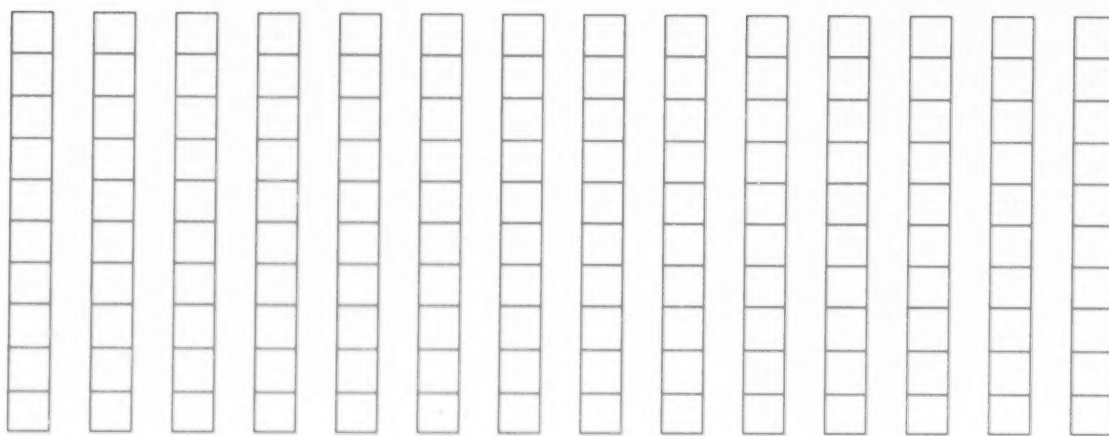
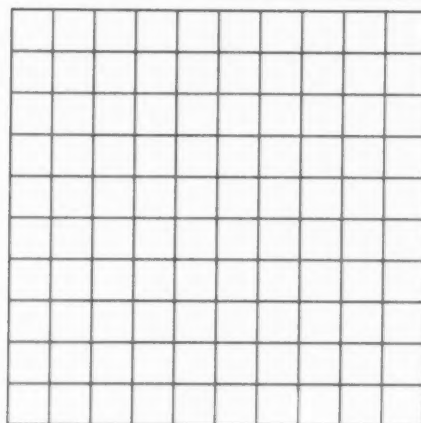
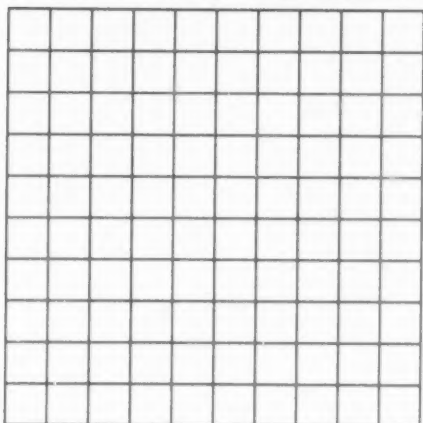
Hundred Grid

575

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Small Grids

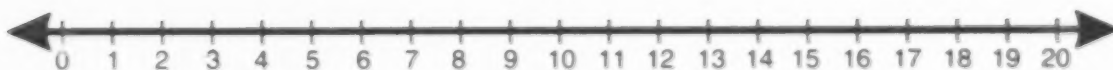
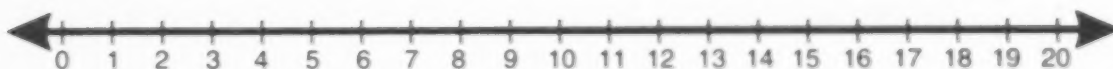
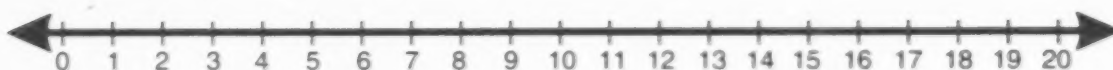
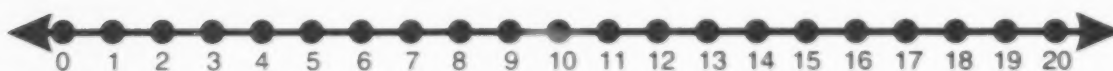
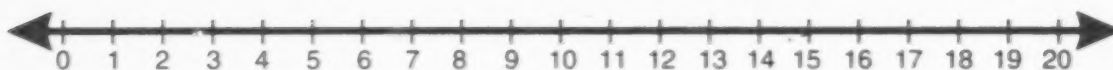


Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

576



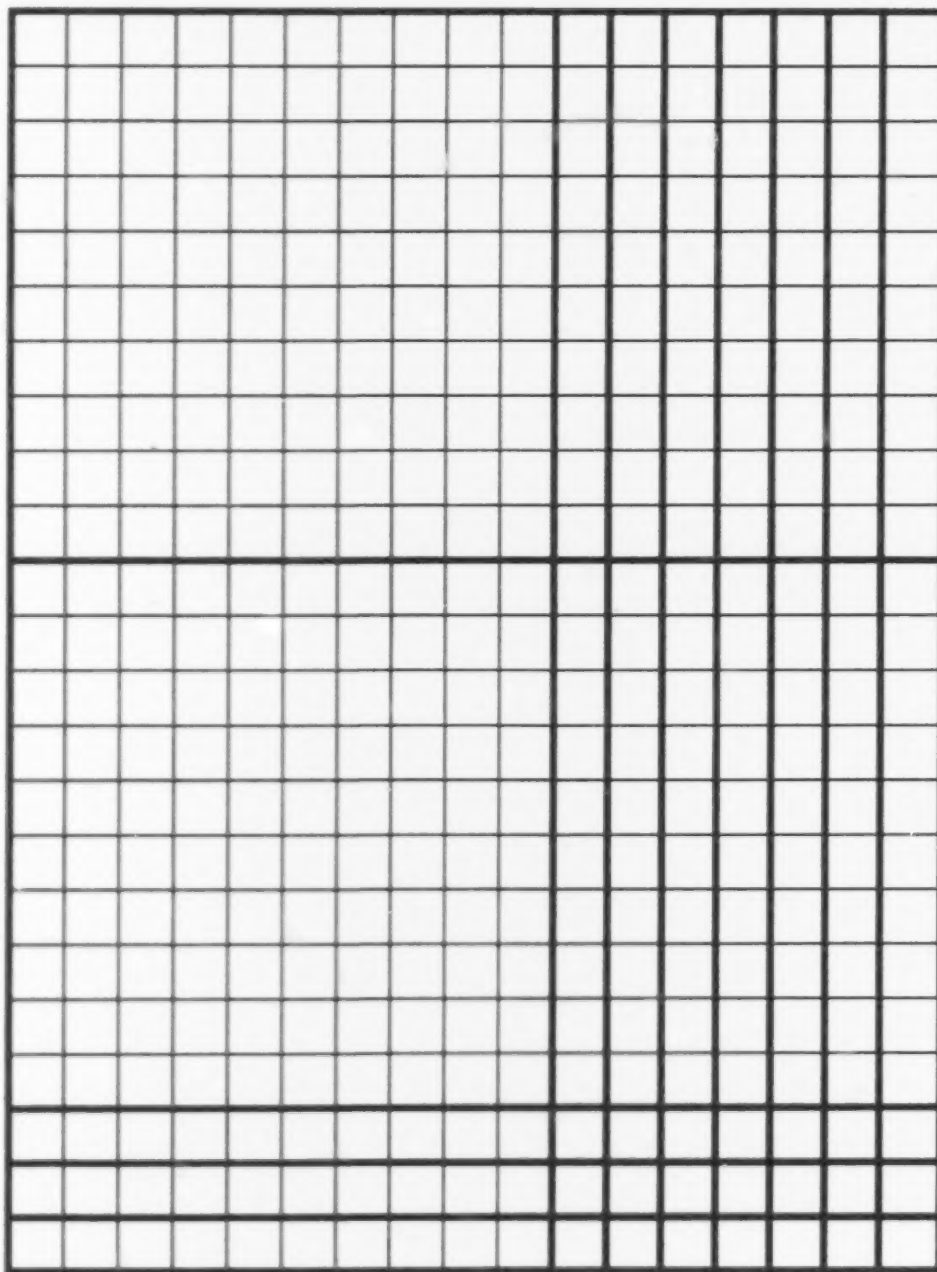
Number Lines



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Base Ten Patterns

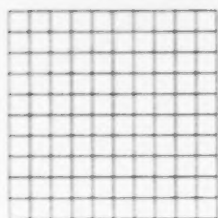
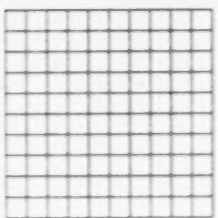
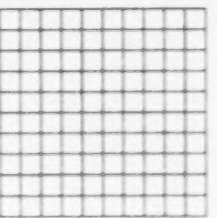
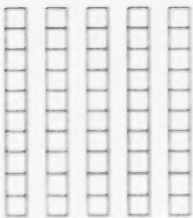
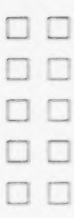
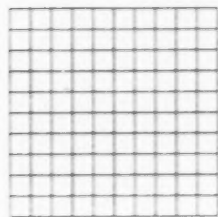
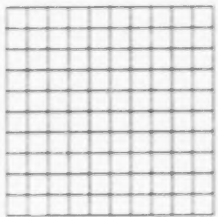
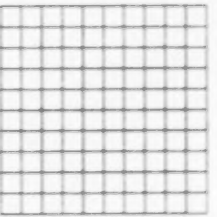
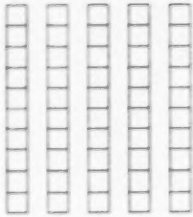
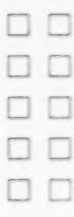
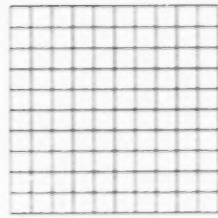
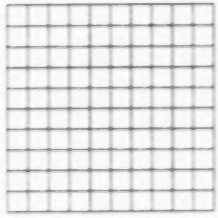
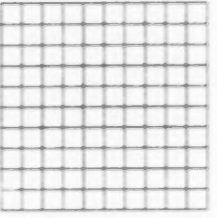
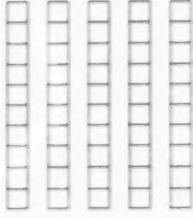
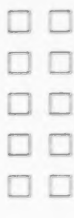
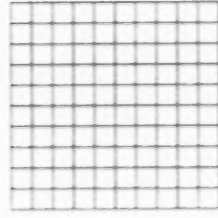
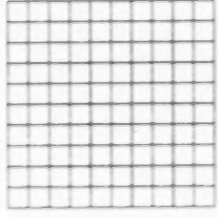
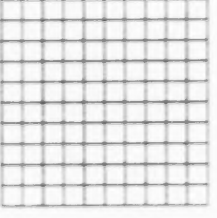
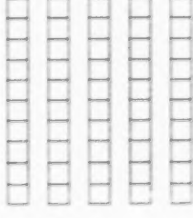
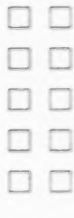
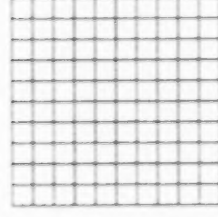
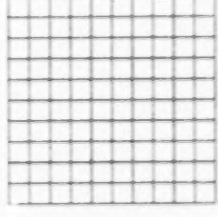
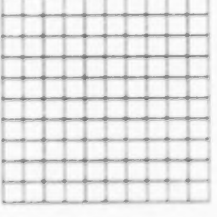
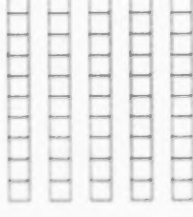


573

Directions: Cut along the thick lines to make 2 hundreds, 17 tens, and 21 ones. Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Base Ten Patterns—Grids

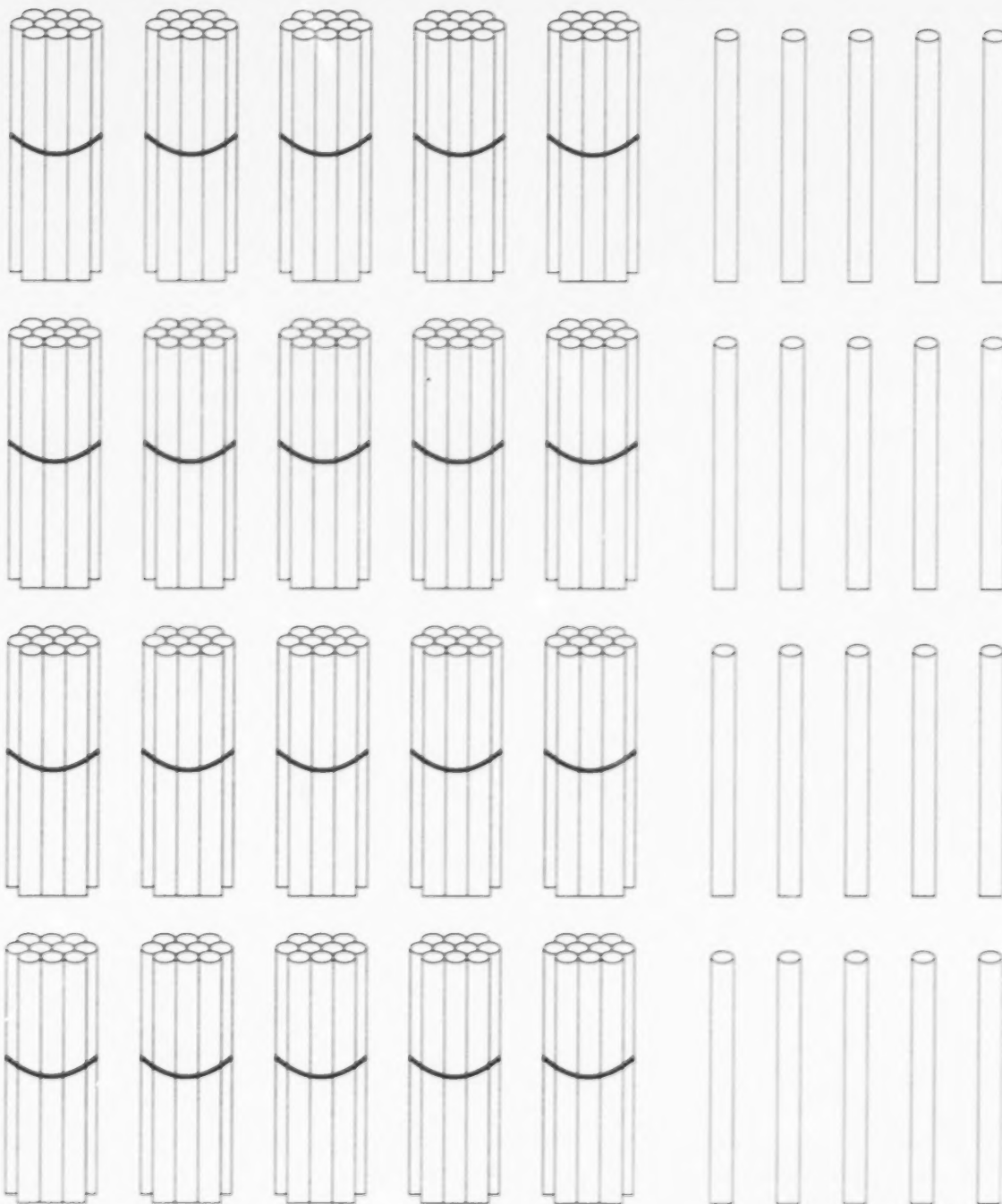
				
				
				
				
				

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

573



Base Ten Patterns—Bundles



Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved. 580



Hundred Square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

581

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Blank Multiplication Matrix

582

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Multiplication Table

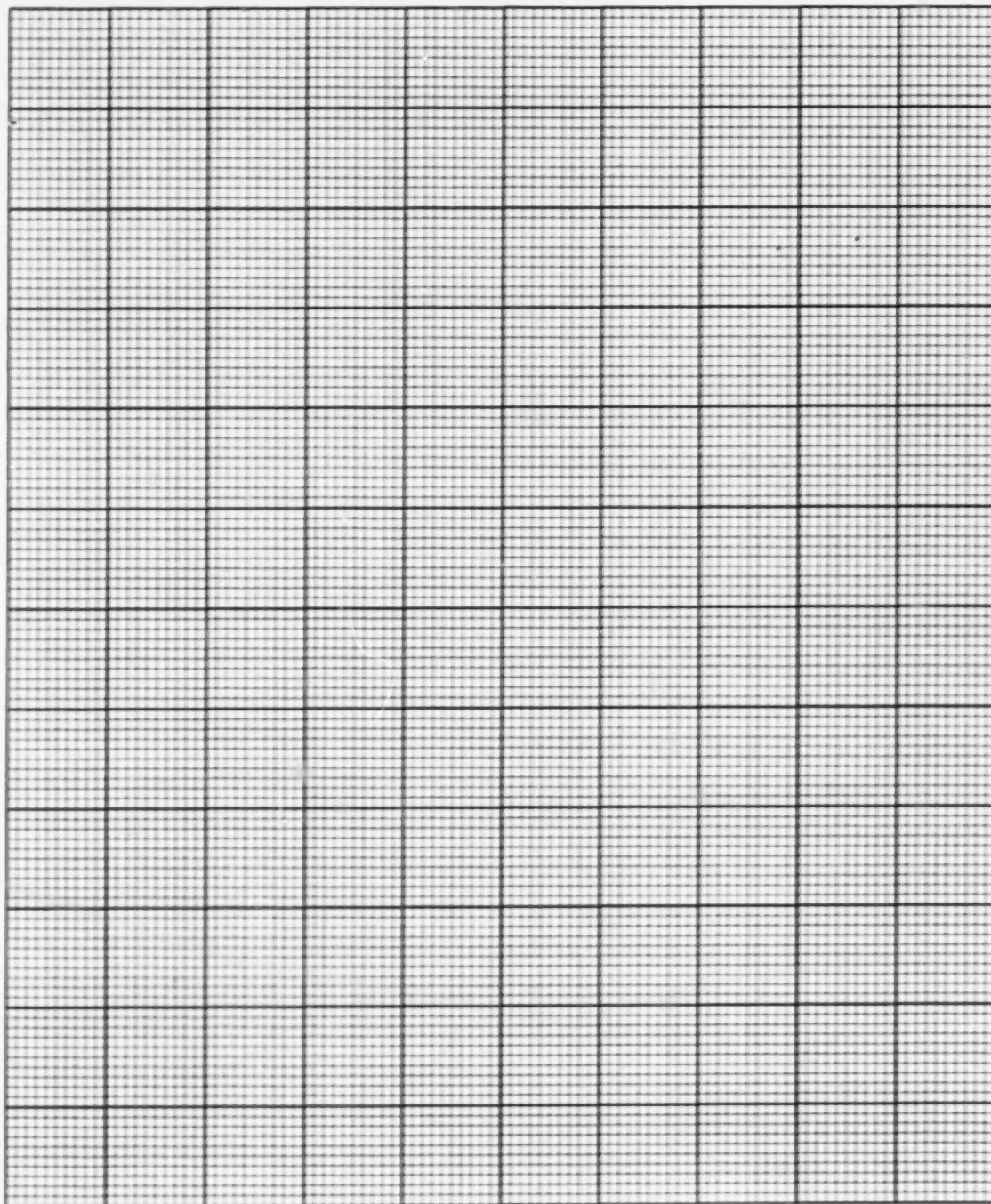
X	0	1	2	3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10
2	0	2	4	6	8	10	12	14	16	18	20
3	0	3	6	9	12	15	18	21	24	27	30
4	0	4	8	12	16	20	24	28	32	36	40
5	0	5	10	15	20	25	30	35	40	45	50
6	0	6	12	18	24	30	36	42	48	54	60
7	0	7	14	21	28	35	42	49	56	63	70
8	0	8	16	24	32	40	48	56	64	72	80
9	0	9	18	27	36	45	54	63	72	81	90
10	0	10	20	30	40	50	60	70	80	90	100

583

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.



Base Ten Graph Paper

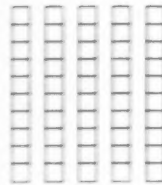
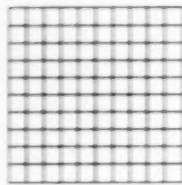
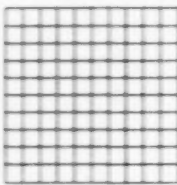
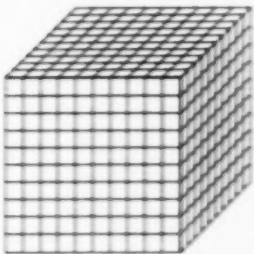
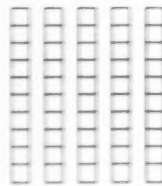
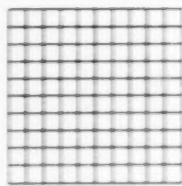
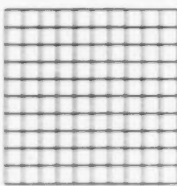
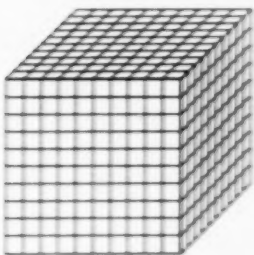
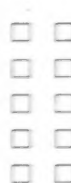
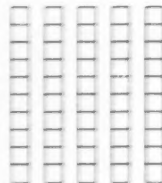
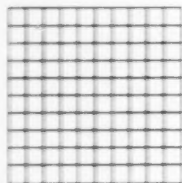
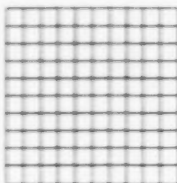
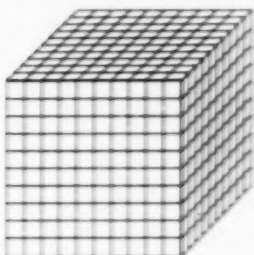
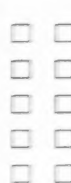
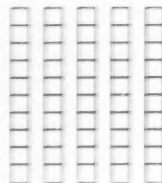
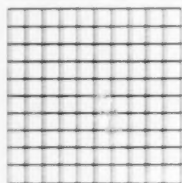
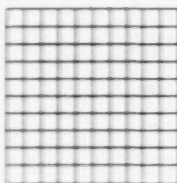
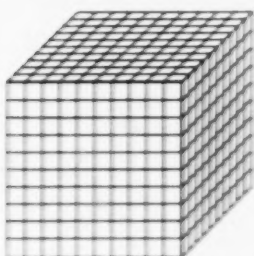
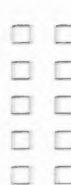
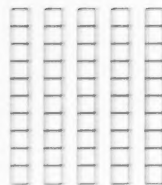
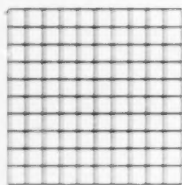
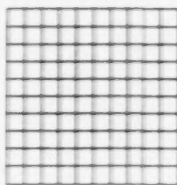
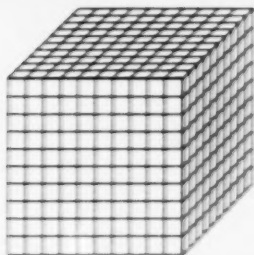


Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.

584



Base Ten Patterns



585

Copyright © 1998 by Larry and Shirley Bradsby. All rights reserved.